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On the Calculation of Crack Width in RC Linear Elements under Eccentric Load

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On the Calculation of Crack Width in RC Linear Elements under Eccentric Load

A. Pisanty [α] & R. Farhat [σ]

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I. Notation

A_s - the area of reinforcement close to the tension face of the section

A_s' - the area of reinforcement close to the compres-sion face of the section

d - effective height of the section

d_s - distance from the center of the tensile rein-forcement As to the extreme fiber in tension

d_s' - distance from the center of the compression reinforcement As' to the extreme fiber in com-pression

e_d - eccentricity of the normal force relative to sec-tion center

E_{cm-} concrete modulus of elasticity

E_s - reinforcing bars modulus of elasticity

f_{ctm} - the mean tensile strength of the concrete

f_{yk} - yield strength of reinforcing bars

K_1-K_2 – coefficients for calibration of S_{rm} (bond and stress distribution) [ENV 1992-1-1:dec. 1991]

M_{dser}- service moment acting on the section result-ing from static analysis

$M_{sd,ser}$ - moment acting on the section after normal force being transferred to A_s

$N_{d,ser}$ - service normal force acting on the section resulting from static analysis

s_{rm} - average final crack spacing [ENV 1992-1-1:dec. 1991]

w_k - the design crack width

y - distance from the extreme fiber in tension to the section center

y' - distance from the extreme fiber in compression to the section center

β - coefficient relating the average crack width to the design crack width [ENV 1992-1-1:dec. 1991]

ε_{sm} - mean strain in the reinforcement at the crack allowing for tension stiffening

ϕ - bar's diameter (or the average scaled bars diameters)

ρ_r - reinforcement ratio relative to the effective concrete section in tension Ac,eff.

σ_{sr} - stress in the tensile reinforcement under the cracking moment Mcr [ENV 1992-1-1:dec. 1991]

σ_s - stress in the tensile reinforcement under the service moment including the axial force transferred to the tensile reinforcement

II. Introduction

Limiting crack width is one of the two basic condi-tions (but not only) for securing suitable perfor-mance in serviceability limit state: deformation and cracking limitation. The later is no less important since crack width requirements are more relaxed than in the past, but the need of verification is essen-tial. Some codes, like the ACI [ACI 318M-05], have given up calculating crack width, assuming that con-trol may be attained indirectly. The EN 2 in its for-mer [ENV 1992-1-1:dec. 1991] and present [BS EN 1992-1-1:2004] versions, has pursued in providing procedures for calculating the crack width, however, focusing on pure bending mainly. Considering ec-centrically loaded sections is important in both RC and PC elements. A simple procedure is offered here that allows a straightforward crack width calculation in linear concrete elements, eccentrically loaded. The results are compared with the limitations im-posed by EN2 [1992, 2004] and with the stress state of sections eccentrically loaded obtained by nonli-near material analysis [Farhat, R., 1995] and found to be in very good agreement.

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III. Calculation of Crack Width According to En 2 [ENV 1992]

EN 2 [ENV 1992] offered the following procedure for calculating crack width:

$$w_k = \beta\, s_{rm}\, \varepsilon_{sm} \tag{1}$$

The average final crack spacing defined as:

$$s_{rm} = 50 + 0.25\, k_1\, k_2\, \phi / \rho_r \tag{2}$$

Note: k_2 = 0.5 for bending and 1.0 for pure tension with possible interpolation for intermediate cases according to:

$k_2 = (\varepsilon 1 + \varepsilon 2)/2\ \varepsilon 1$ with $\varepsilon 1$ & $\varepsilon 2$ being the greater and the lesser tensile strains at the boundaries of the sec-tion considered (quote).

Though this definition leaves the impression that ec-centric load is dealt with, it appears not to be the case, as eccentric compression is not included in this consideration and in a cracked section under eccen-tric tension there hardly is any possibility to calcu-late ε_1, while undoubtedly ε_2 will be in compression.

The mean strain in the reinforcement defined as:

$$\varepsilon_{sm} = \frac{\sigma_s}{E_s}\left[1 - \beta_1 \beta_2 \left(\frac{\sigma_{sr}}{\sigma_s}\right)^2\right] \tag{3}$$

This procedure EN 2 [ENV 1992] was modified in EN 2 [BS EN 1992-1-1:2004] to:

$$w_k = s_{r,max}\,(\varepsilon_{sm} - \varepsilon_{cm}) \tag{4}$$

Essentially there is difference in the cracks spacing and the strains, however the final calculation results according both renders almost identical results.

IV. Proposed Method for Calculation of Crack Width Under Eccentric Load

The proposed herein method, follows the procedures as given in [ENV 1992-1-1:dec. 1991] (detailed above) or [BS EN 1992-1-1:2004], except for a transformation suggested that allows for easy and simple consideration of the eccentricity in loading. Only the procedure given in EN2 [ENV 1992-1- 1:dec. 1991] is discussed in the following, however in the numerical examples that follow crack width is calculated according both EN2 versions.

A symmetrical with reference to vertical axis section is given in Figures 1a&2a (see notation). On the section acts a normal force in service $N_{d,ser}$ at eccen-tricity e_d vs. the section center, as obtained from elastic static analysis. The case of $N_{d,ser}$ in compres-sion with e_d is given at Figure 2a and $N_{d,ser}$ in tension with e_d is given in Figure 1a.

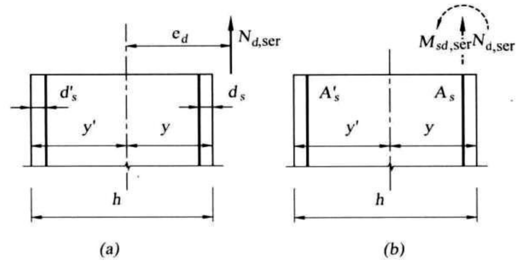

Figure 1 : Eccentric normal force in tension acting on a section

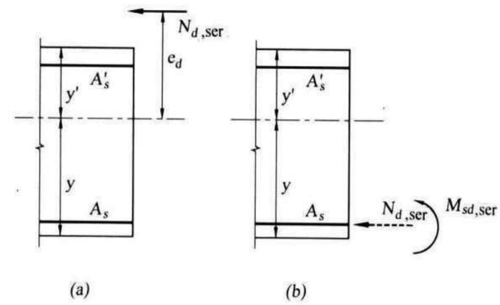

Figure 2 : Eccentric normal force in compression acting on a section

1. It is proposed to transfer the load to the center of the tensile (or the less compressed) reinforcement in the section - As. In order to maintain equili-brium, after transfer, the moment will be:

$$M_{sd\ ser} = N_{d\ ser}\,[e_d + (y - d_s)] \tag{5}$$

for eccentric compression - see Fig. 2b

$$M_{sd,ser} = N_{d,ser}\,[e_d - (y - d_s)] \tag{6}$$

for eccentric tension - see Fig. 1b

From here on the section analysis for cracking will be conducted under the action of $M_{sd,ser}$ and $N_{d,ser.}$

2. The stress in the tensile face is to be checked as-suming uncracked section. If it exceeds f_{ctm} (the mean tensile strength of the concrete) the section is cracked.
3. The stress in the tensile reinforcement will be:

for eccentric tension

$$\sigma_s = \frac{M_{sd,ser}}{0.87\, d\, A_s} + \frac{N_{d,ser}}{A_s} \tag{7}$$

$$\sigma_s = \frac{M_{sd,ser}}{0.87\, d\, A_s} - \frac{N_{d,ser}}{A_s} \tag{8}$$

for eccentric compression

4. The stress σ_{sr} - in the tensile reinforcement under the cracking moment M_{cr} is (ignoring the normal force):

$$\sigma_{sr} = \frac{M_{cr}}{0.87\,d\,A_s} \tag{9}$$

5. The average strain in the tensile reinforcement is calculated as given in (3) above. β_1 and β_2 remain as recommended there.
6. The average distance between cracks srm is calculated according to (2) above, with k1 = 0.8 for high bond bars and k_2 = 0.5 for pure bending. ϕ and ρr as defined in EN2 [ENV 1992].
7. Finally the maximum crack width, according to EN2 [ENV 1992] is:

$$w_{max} = 1.7\, s_{rm}\, \varepsilon_{sm} \tag{10}$$

V. Numerical Examples

The examples given in the following aim to cover a variety of problems that may rise applying the of-fered procedure. In all examples the concrete type is f_{ckcyl}=25Mpa with mean concrete tensile strength - f_{ctm} = 2.6 MPa and E_{cm} = 31000 MPa. The reinforcement consists of ribbed single bars (ϕ) with f_{yk} = 400 MPa and/or welded mats of high strength welded bars (ψ) with f_{yk} = 500 MPa.

Example 1

A section of a wall, 300 mm thick, contains 2000 mm^2/m tensile reinforcement in the form of Φ16@100mm at a distance d_s= 50 mm from the in-ner face of the wall, (See Figure 3).

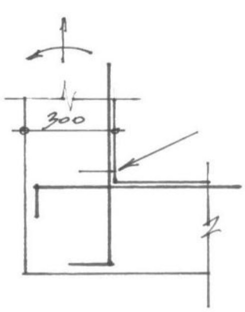

Figure 3 : Wall section, 300 mm thick, eccentrically loaded

The calculated maximum loading on the wall at this section produces:

$$M_{d,ser} = 75.3\text{kNm/m} \quad N_{d,ser} = 115.9\text{kN/m}$$

The section effective depth d is 250mm.

Solution:

The section is under tensile load with eccentricity

e_d = 75.3/115.9 = 0.65m

Transferring the load to the center of the tensile rein-forcement results in a moment:

$$M_{sd,ser} = 115.9[0.65 - (0.15 - 0.05)] = 63.75\text{kNm/m}$$

The stress at service in the tensile reinforcement will be:

$$\sigma_s = \frac{63.75\,10^6}{0.87\;250\;2000} + \frac{115900}{2000} = 204.5\,\text{MPa}$$

With a cracking moment M_{cr} = 39.0 kNm/m

$$\sigma_{sr} = \frac{39.0\,10^6}{0.87\;250\;2000} = 89.7\,\text{MPa}$$

The mean strain in the reinforcement is:

$$\varepsilon_{sm} = \frac{204.5}{2\,10^5}[1 - 0.5(\frac{89.7}{204.5})^2] = 0.924\,10^{-3}$$

An estimate of $A_{c,eff}$ gives 80000 mm^2, therefore

$$\rho_r = 2000/80000 = 0.025$$

The average distance between cracks will be:

$$s_{rm} = 50 + 0.25\;0.8\;0.5\;16/0.025 = 114\,\text{mm}$$

Therefore the calculated maximum crack width is:

$$w_{max} = 1.7\;114\;0.924\;10^{-3} = 0.179\text{ mm}$$

The maximum crack width assessed according to EN2 [BS EN 1992-1-1:2004] is 0.196 mm.

The result was reviewed with the aid of:

a. Nonlinear section analysis developed by Farhat [Farhat, 1995] wherefrom the stresses and strains in the cracked section are as follows:

$$\varepsilon_c = -0.332\,10^{-3} \qquad \varepsilon_s = 0.989\,10^{-3}$$
$$\sigma_c = -7.6\text{MPa} \qquad \sigma_s = 197.8\text{ MPa}$$

The difference between the suggested here analysis and the nonlinear analysis for σ_s is 3.3% - within very reasonable level of accuracy.

b. According to Table 7.2N [BS EN 1992-1-1:2004] for a maximum bar diameter of 16 mm and at a stress level of 200 MPa the crack width to be ex-pected will be approximately 0.2 mm. Also, accord-ing to Table 7.3N [BS EN 1992-1-1:2004] when the maximum bars spacing does not exceed 150 mm and the stress level is about 200 MPa the maximum ex-pected crack width is 0.2 mm. Here the distance be-tween the bars 100 mm therefore a crack width of less than 0.2 mm should be expected.

Example 2

A 400 mm thick section of a floor is given (Figure 4) where the tensile reinforcement at a distance 50 mm from the upper

$$\Phi 14@125mm + \#\psi 12@125mm$$

Any bottom floor reinforcement is ignored for the purpose of this analysis.

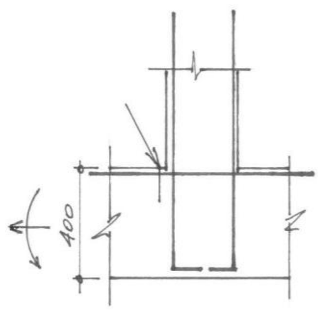

Figure 4 : Floor section, 400 mm thick, eccentrically loaded

The reinforcement placed in the form of a matt is not fully embedded in the support therefore one half of the amount is considered active, however scaling the amount in terms of strength to an equivalent of ribbed bars the total amount of reinforcement is 1760 mm^2 (1200+560).

Due to the most extreme load combination the fol-lowing was obtained:

$$M_{d,ser} = 75.9 kNm/m \qquad N_{d,ser} \quad 150.9 kN/m$$

Solution:

The section is under tensile load with eccentricity

e_d = 75.9/150.9 = 0.50m.

Transferring the load to the center of the tensile rein-forcement results in a moment:

$$M_{sd,ser} = 150.9[0.50-(0.20-0.05)] = 52.82 kNm/m$$

The stress at service in the tensile reinforcement will be :

$$\sigma_s = \frac{52.82\ 10^6}{0.87\ 350\ 1760} + \frac{150900}{1760} = 184.2 MPa$$

The cracking moment is 69.33 kNm/m

$$\sigma_{sr} = \frac{69.33\ 10^6}{0.87\ 350\ 1760} = 129.4 MPa$$

Therefore the mean strain in the reinforcement is:

$$\varepsilon_{sm} = \frac{184.2}{2\ 10^5}[1-0.5(\frac{129.4}{184.2})^2] = 0.694\ 10^{-3}$$

An estimate of $A_{c,eff}$ gives 113300 mm^2, therefore

$$\rho_r = 1760/1133000 = 0.0155$$

The average distance between cracks will be (with an average bars diameter – 13mm):

$$s_{rm} = 50 + 0.25\ 0.8\ 0.5\ 13/0.0155 = 133.9 mm$$

Therefore the calculated maximum crack width is:

$$w_{max} = 1.7\ 133.9\ 0.694\ 10^{-3} = 0.158\ mm$$

w_{max} calculated according to EN2 [BS EN 1992-1-1:2004] is 0.160 mm.

Discussion of the results:

a. Stresses and strains resulting from nonlinear sec-tion analysis [Farhat, 1995] produce:

$$\varepsilon_c = -0.203\ 10^{-3} \qquad \sigma_c = -4.81 MPa$$
$$\varepsilon_s = 0.883\ 10^{-3} \qquad \sigma_s = 176.6\ MPa$$

Again the difference between σs from the analysis offered and the nonlinear analysis [Farhat, 1995] is 4.1% - a very fair level of accuracy.

b. According to table 7.2N [BS EN 1992-1-1:2004] for a stress level of 180 MPa in the reinforcement a maximum bar size of over 16 mm is allowed for li-miting crack width to 0.2 mm.

According to Table 7.3N [BS EN 1992-1-1:2004] for the stress level of 180 MPa a maximum bar spac-ing exceeds 150 mm, but in the current example the distance is 125 mm, therefore it may be concluded that the max. crack width is lower than 0.2 mm (here – 0.158 mm).

Example 3

A portion of the ceiling of a buried underground structure is given, 400 mm thick, having

$$\phi 14@125mm + \#\psi 12@125mm$$

2320 mm^2/m at distance 50 mm from the bottom face and 1111 mm2/m at distance 50 mm from the upper face.

The effective depth d is 350 mm. See Figure 5.

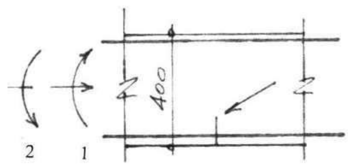

Figure 5 : The ceiling of a buried underground structure section, 400 mm thick, eccentrically loaded

Two different loading combinations are considered:

1. $M_{d,ser}$ = 120.3 kNm/m causing tension at the bottom face together with a compressive force

$$N_{d,ser} = -123.7 \text{ kN/m}$$

2. Md,ser = 30.9 kNm/m causing tension at the upper face together with a tension force

$$N_{d\ ser} = 137.7 \text{ kN/m}$$

Solution:

Addressing loading combination 1:

The section is under compressive load with eccen-tricity e_d = 120.3/123.7 = 0.973m

Transferring the load to the center of the tensile rein-forcement results in a moment:

The stress at service in the tensile reinforcement will be

$$\sigma_s = \frac{138.92\,10^6}{0.87\;350\;2320} - \frac{123700}{2320} = 143.3\text{MPa}$$

The cracking moment is 69.33 kNm/m

$$\sigma_{sr} = \frac{69.33\,10^6}{0.87\;350\;2320} = 98.1\text{MPa}$$

Therefore the mean strain in the reinforcement is:

$$\varepsilon_{sm} = \frac{143.3}{2\,10^5}[1 - 0.5(\frac{98.1}{143.3})^2] = 0.549\,10^{-3}$$

With:

The average distance between cracks will be:

$$A_{c,eff} = 113300 \text{ mm}^2,\ \rho_r = 2320/113300 = 0.0205$$

Therefore the calculated maximum crack width is:

$$s_{rm} = 50 + 0.25\;0.8\;0.5\;13/0.0205 = 113.4\text{mm}$$

w_{max} according to EN2 [BS EN 1992-1-1:2004] is 0.109 mm

Addressing load combination 2:

Checking stresses assuming uncracked state under eccentric tension proves that in the upper face the stress is 1.50 MPa and at the bottom face the stress is -0.82 MPa.

Discussion of the results for load combination 1:

a. The stresses and strains obtained in nonlinear analysis [Farhat, 1995] are:

$$\varepsilon_c = -0.33\,10^{-3} \qquad \sigma_c = -7.55 \text{ MPa}$$

$$\varepsilon_s = 0.69\,10^{-3} \qquad \sigma_s = 137.4 \text{ MPa}$$

The stress in the reinforcement for the proposed analysis is 143.3 MPa and the difference is again only 4.1%.

b. According to Table 7.2N [BS EN 1992-1-1:2004] at stress level 140 MPa the crack width to be ex-pected is way below 0.2 mm. According to Table 7.3N [BS EN 1992-1-1:2004] for stress level 140 MPa the bars spacing for limiting the cracks to 0.2 mm is 200 mm but we have here only 125 mm, therefore the result obtained is acceptable.

VI. Conclusions

A simple procedure is presented; modifying slightly the proposed procedure in EN2 in it's both versions, allowing calculating directly crack width in linear concrete members, with sections under eccentric tensile or compressive normal force. Several exam-ples offered demonstrate the simplicity and practi-cality in application of the procedure. Nonlinear section analysis proves a very good cor-respondence with the results obtained with the sim-plified method offered. A good correspondence is obtained also with forecasts from EN2 [ENV 1992- 1-1:dec. 1991] and EN2 [BS EN 1992-1-1:2004] based themselves on similar calculations.

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