A New Expression of the Curve S-N in Fatigue based on the Concept of the "Weakest Link" of Weibull

By N. Ngarmaïm, B. Tikri, B. Bassa, N. Kimtangar, F. Pennec & J-L. Robert

University of N'Djamena, Chad

Abstract- A new expression of the SN curve based on the concept of "weakest link" is proposed. It is a probabilistic model introducing a new parameter N_c, the number characteristic of cycles corresponding to the failure probability equal to 1. His confrontation with the terms of the curve most used SN, particular those Basquin, the Wöhler and the Stromeyer, the fatigue tests and martensite steels P220 100C6 data provided errors of about 5% maximum for models Basquin, Wöhler and that proposed. Expression takes into account the volume of the Seine part of the test, which will help explain the number variations of cycles for a given level of stress. The search for a correlation relationship between the fatigue limit $\sigma^0$ and the breaking strength $R_m$ will help prevent the fatigue life of the work-hardened materials.

GJRE-A Classification : FOR Code: 091399

Strictly as per the compliance and regulations of:

© 2014. N. Ngarmaïm, B. Tikri, B. Bassa, N. Kimtangar, F. Pennec & J-L. Robert. This is a research/review paper, distributed under the terms of the Creative Commons Attribution-Noncommercial 3.0 Unported License http://creativecommons.org/licenses/by-nc/3.0/), permitting all non commercial use, distribution, and reproduction in any medium, provided the original work is properly cited.
A New Expression of the Curve S-N in Fatigue based on the Concept of the "Weakest Link" of Weibull

N. Ngarmaim °, B. Tikri °, B. Bassa °, N. Kimtangar °, F. Pennec § & J-L. Robert §

Abstract- A new expression of the SN curve based on the concept of "weakest link" is proposed. It is a probabilistic model introducing a new parameter Nc, the number characteristic of cycles corresponding to the failure probability equal to 1. His confrontation with the terms of the curve most used SN, particular those Basquin, the Wöhler and the Stromeyer, the fatigue tests and martensite steels P220 10006 data provided errors of about 5% maximum for models Basquin, Wöhler and that proposed. Expression takes into account the volume of the Seine part of the test, which will help explain the number variations of cycles for a given level of stress. The search for a correlation relationship between the fatigue limit and the breaking strength Rm will help prevent the fatigue life of the work-hardened materials.

I. Introduction

The static essay, serving to determine the ultimate resistance to break, may be considered as a very particular case of a essay of fatigue, the number of cycles with break was equal at the most to 1. This ultimate resistance with break is the minimal value in which the break of the material occurs in a certain way.

The problem of dispersal of the trial results, as well in statics (for the determination of the ultimate resistance with break) that in fatigue (for the determination of the number of cycles with break or for the determination endurance limit) is the consequence of the structural heterogeneity of the defects since the elaboration of material [1]-[11]. The density of these defects vary from specimen to another, yet all taken from the same sample. In the case of fatigue, for a given stress level σi, one can also observe a dispersion value of the number of cycles to failure N, [12]-[13]. The number of cycles to failure is related to the material structure and thus the defects that contain. That these defects are difficult to measure, it is advisable to use as a random variable [12], for a given stress level, the number of cycles to failure to explain the existence of defects by a probabilistic approach. We rely on the concept of the "weakest link" Weibull to formulate a new expression of the SN curve can take into account the actual volume of a flawless material considered. In the absence of a means of assessing defect in the structure, the results for two types of materials are comparable with those of Wöhler and Basquin. A simulation made by the variation of the tensile strength of a material that shows this new expression for describing the fatigue behavior of a hardened material.

II. Concept of "Weakest Link" Weibull

According to this concept, for juxtaposed volumes, subjected to monotonic loading, the failure of one is independent of the other, if one of the volumes fails, the system is considered failed [14]. The survival probability of a system consisting of n elements (or volume) is given by:

\[ R_n = 1 - e^{-\int f dv} \]  

(1)

Where f is the probability density of failure and dv is the elementary volume constituting the reference volume V.

III. Specification of the Probability Density Function f

We will consider as a random variable the number of cycles to failure for a given stress level by the fact that the number of cycles determines the fatigue. Dependent on the dispersion, the number of cycles is a variable representing statistically the stress applied from the viewpoint fatigue (damage to a given level of stress). We will consider the failure probability density f as a function of number of cycles to failure; it is given in the following form:

\[ f(N) = \beta N^m \]  

(2)

Where \( \beta \) is a parameter to be determined, and the module m, within the meaning of Weibull [14].

The parameter \( \beta \) can be determined when we know a characteristic value of the number of cycles \( N_c \) to a level corresponding stress. At this number of cycles \( N_c \) characteristic density of default probability take the value 1. It then pulls:

\[ \beta = \frac{N_c^m}{N^m} \]  

(3)

or,

\[ f(N) = \left( \frac{N}{N_c} \right)^m \]  

(4)
In this case, the probability of survival given equation (1) becomes:

\[ R_n = 1 - e^{-\int_{Nc}^{N} \sigma^m dv} \]  

(5)

IV. PROPOSAL FOR A NEW EXPRESSION OF THE S-N CURVE

It is obvious that there is a relationship between the probability of failure \( P \) (function the number of cycles) and the level of responsible of this fatigue failure stress. We admit that there is a function \( g \) dependent on the characteristics of fatigue \( \sigma^0 \) and static \( R_m \) linking the probability of failure \( P \) stress responsible for this failure.

Since \( \sigma^0 \) and \( R_m \) are constants for a given material, then \( g \) is a constant function. A simple relation linking the stress at failure probability is that the stress is proportional to the probability of failure; with as the proportionality coefficient function \( g \). This relationship is expressed by:

\[ \sigma = g(\sigma^0, R_m)P(N) \]

(6)

\[ = g(\sigma^0, R_m)[1-R(N)] \]

(7)

For a tension loading when the load is uniform, the expression (7) becomes:

\[ \sigma = g(\sigma^0, R_m)e^{-\int_{Nc}^{N} \sigma^m dv} \]

(8)

Thus, two limiting cases are possible according to the value of the stress \( \sigma \):

- For \( \sigma = R_m \), ultimate resistance to break, \( N = 0 \), the relation (8) becomes:
  \[ R_m = g(\sigma^0, R_m) \]
  (9)

- For \( \sigma = \sigma^0 \), endurance limit, \( N = N^0 \), the relation (8) becomes:
  \[ \sigma^0 = g(\sigma^0, R_m)e^{-\int_{Nc}^{N} \sigma^m dv} \]
  (10)

\[ g(\sigma^0, R_m) = \sigma^0 e^{\int_{Nc}^{N} \frac{\sigma^m}{\sigma^0} dv} \]

(11)

The equality of two expressions \( g \), given by equations (9) and (11) gives \( R_m \) depending on \( \sigma^0 \)

\[ R_m = \sigma^0 e^{\int_{Nc}^{N} \frac{\sigma^m}{\sigma^0} dv} \]

(12)

V. EXPERIMENTAL STUDY OF THE NEW EXPRESSION

To validate the sensitivity of the proposed fatigue behavior of a material model for \( V = 1 \), we have experimental data from two types of materials: steel martensite P220 and 100C6. A comparative study is made, from these data, with models of the most utilized S-N curves [15]-[16] such that Basquin, the Wöhler and the Stromeyer:

\[ \begin{align*}
\log(N) &= -a_B \log(\sigma) + b_B \quad : \text{Basquin} \\
\log(N) &= -a_W \sigma + b_W \quad : \text{Wöhler} \\
\log(N) &= -a_S \log(\frac{\sigma - \sigma^D}{\sigma}) + b_S \quad : \text{Stromeyer}
\end{align*} \]

(14)

Where, for the proposed model

\[ \begin{align*}
a_N &= \frac{1}{m} \\
b_N &= \log(N_C)
\end{align*} \]

The curves parameters are determined for each fatigue testing each of the two steels result, by the method of least square.

a) Test Data

i. P220 Steel

The data are taken from the steel work performed LaMI (Laboratory of Mechanics and Engineering) in order to investigate the influence of cutting processes on the fatigue behavior of parts. Two procedures were used, laser and shears, it, with several execution qualities (good, standard and bad) corresponding to different sets of machines settings. Table.1 gives the geometry of the specimen and the fatigue test at constant amplitude tension-compression alternating statement.
Table 1: Specimen geometry and fatigue testing of the P220 cut steel with the standard laser process result

<table>
<thead>
<tr>
<th>Material: P220</th>
<th>Type of solicitation: tension-compression at constant amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solicitation frequency: 20Hz</td>
<td></td>
</tr>
</tbody>
</table>

Dimensions and shape of the specimen

![Specimen geometry](image)

<table>
<thead>
<tr>
<th>σ (MPa)</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>344</td>
<td>10596.5</td>
</tr>
<tr>
<td>321.8</td>
<td>296828</td>
</tr>
<tr>
<td>319.9</td>
<td>281248</td>
</tr>
<tr>
<td>319.8</td>
<td>320467</td>
</tr>
<tr>
<td>311.5</td>
<td>371162</td>
</tr>
<tr>
<td>311.1</td>
<td>305053</td>
</tr>
<tr>
<td>309.5</td>
<td>204238</td>
</tr>
<tr>
<td>309.19</td>
<td>309469</td>
</tr>
<tr>
<td>305.9</td>
<td>533570</td>
</tr>
<tr>
<td>304.238</td>
<td>520089</td>
</tr>
<tr>
<td>303.5</td>
<td>662397</td>
</tr>
<tr>
<td>302.467</td>
<td>390994</td>
</tr>
<tr>
<td>301.9</td>
<td>533570</td>
</tr>
<tr>
<td>290</td>
<td>752259</td>
</tr>
<tr>
<td>280</td>
<td>6026,5</td>
</tr>
<tr>
<td>275</td>
<td></td>
</tr>
<tr>
<td>270</td>
<td></td>
</tr>
<tr>
<td>235</td>
<td></td>
</tr>
</tbody>
</table>

(*) no rupture

ii. Martensite Steel 100C6

The tableaux.2 and 3 below give, respectively, the chemical composition and the fatigue test result of the 100C6 steel [11].

Table 2: Chemical composition (% by mass) of the martensite steel 100C6

<table>
<thead>
<tr>
<th>Si</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
<th>Cr</th>
<th>Cu</th>
<th>Ni</th>
<th>Mo</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.242</td>
<td>0.339</td>
<td>0.012</td>
<td>0.008</td>
<td>1.461</td>
<td>-</td>
<td>0.147</td>
<td>0.032</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 3: Fatigue test of martensite steel 100C6 result

<table>
<thead>
<tr>
<th>σ (MPa)</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>975</td>
<td>0.1183</td>
</tr>
<tr>
<td>960</td>
<td>0.1019</td>
</tr>
<tr>
<td>960</td>
<td>0.1639</td>
</tr>
<tr>
<td>900</td>
<td>9.86</td>
</tr>
<tr>
<td>900</td>
<td>1.67</td>
</tr>
<tr>
<td>860</td>
<td>4.66</td>
</tr>
<tr>
<td>880</td>
<td>664.4</td>
</tr>
<tr>
<td>870</td>
<td>542.1</td>
</tr>
<tr>
<td>860</td>
<td>1663.3</td>
</tr>
<tr>
<td>850</td>
<td>6026.5</td>
</tr>
</tbody>
</table>

The value of the ultimate resistance to break \( R_m \) of the steel P220 is between 600 and 800 MPa, it will take three values in this range for the simulation thus allowing to highlight the sensitivity of the proposed model.

b) Comparison

The comparison will be made by tracing curves in the plane \((\log(\sigma), \log(N))\). Parameter values, determined by the method of least squares, corresponding to different models, given by Equations (14), for the two steels, are given in the following Table 4:

Table 4: Value of parameters of the models used for the two steels

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>P220 steel</td>
<td></td>
</tr>
<tr>
<td>Basquin</td>
<td>(a_\text{B} = 23.66; b_\text{B} = 148.2)</td>
</tr>
<tr>
<td>Wöhler</td>
<td>(a_\text{W} = 0.078; b_\text{W} = 36.65)</td>
</tr>
<tr>
<td>Stromeyer</td>
<td>(a_\text{S} = 1.52; b_\text{S} = 17.73)</td>
</tr>
<tr>
<td>Proposed</td>
<td>(R_m = 600\text{MPa}; \sigma^D = 271.24\text{MPa}; m = 0.062; N_c = 21403756)</td>
</tr>
<tr>
<td></td>
<td>(R_m = 700\text{MPa}; \sigma^D = 271.57\text{MPa}; m = 0.051; N_c = 15622128; a_n = 19.61; b_n = 16.56)</td>
</tr>
</tbody>
</table>
VI. Results and Discussion

- P220 steel

(a) The four models and the linear trend of the experiment;

(b) Models Basquin of Wöhler, those proposed and the linear trend of the experience.

Figure 1: SN curves in the plane \((\ln(\sigma), \ln(N))\) of the models studied for P220 steel

- 100C6 steel

(a) The four models and the linear trend of the experiment;

(b) Models Basquin of Wöhler, those proposed and the linear trend of the experiment.

Figure 3: SN curves in the plane \((\ln(\sigma), \ln(N))\) of the models studied, for steel 100C6
Basquin models and the proposed are approaching significantly from the linear trend of the experiment while that of Stromeyer does not follow this trend for large values of the constraint (figure 3.a). Wöhler model, meanwhile, is shifted in translation relative to this trend (figure 3.a and b). The relative error on the values of constraint provided by the curves Basquin of Wöhler and that proposed not exceeding 5% there too compared with the values of the experiment.

VII. Conclusion

The sensitivity of the proposed model to the values of $R_m$, is reflected in an angular shift in direction of rotation hands of a watch while $R_m$ increases (figure 2). The limit of $\sigma^0$ fatigue increases substantially, but the value of the number of cycles $N_c$ diminish characteristic significantly well as the modulus value $m$ while $R_m$ increases, the appreciable increase in the endurance limit with that of the breaking strength reflecting the inflection the S-N curve traced in the plane $\{\sigma; N\}$, thus modifying the scope of limited field of fatigue.

The proposed model introduces a new parameter, the number of cycles characteristic $N_c$ for which the failure probability density function takes the value 1 and therefore the smallest number of cycles which the rupture occurs with certainty. It takes also into account the volume of the material. All the calculations are done by taking $V=1$. To take into account the defects can be considered as a fraction $V$ corresponds to useful volume of the material constituting the piece to explain the variations of cycles number to failure for a given level of stress. The model allows, given its sensitivity to the change $R_m$, to track the fatigue behavior of work hardened materials, a phenomenon for which the value of its breaking strength $R_m$ varies notably.

VIII. Acknowledgement

We thank the Committee CONCOFORT CNAR (Supports National Center for Research in Chad) who funded our trip to France for the finalization of this article. Faculty of Exact Sciences at the University of N'Djamena (Chad) and the Polytechnic University of Mongo to be accepted to release us this work. All of our gratitude to the responsible of the Laboratory MMS Pascal Montlucon Institute for their technical support (France).

Bibliography

15. B. Tikri, N. Nadjitonon, J. L. Robert. Nouvelle loi non linéaire d’endommagement par fatigue basée sur la courbe de Bastenaire-IUT de Montlucon- Département Génie Mécanique et Productique-