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Analysis of Electro-Thermal Characteristics of a Conductive Layer with Cracks and Holes

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ANALYSISOFELECTROTHERMALCHARACTERISTICSOFACONDUCTIVELAYERWITHCRACKSANDHOLES

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Palash Mondal [α] & Md. Abdullah Al Ahad [σ]

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I. Introduction

Today's world is extensively and rapidly inclining to miniaturizing of electronic components to meet the demands and flexibility. Miniaturization of electronic devices has led to tremendous integration levels, with complicated network of conductive layer assembled together on a chip area no larger than a few square centimeters [1]. The thermal characteristics of the incorporated conductive layer undergo a huge amount of current [2]. This phenomenon is unavoidable in modern electronics, so it is indeed an urge of time to investigate the electro-thermal problems and determine the associated resultant temperature fields properly, as far as the reliability of electronic devices is concerned. When an electrically conducting material is subjected to a current flow, Joule heating is induced, which eventually leads to the generation of heat in the conductor. This electrical and thermal conduction ultimately causes thermal stress in the materials, which is considered to be one of the major reasons of metal line failure in electronic packaging [3].

The problem of heat conduction in a layer under the influence of current flow has been explained theoretically by Carslaw and Jaeger [4]. Steady temperature distribution near the tip of a crack in a homogeneous isotropic conductive plate was analyzed by Saka and Abe [5] under a direct current field with the help of pathindependent integrals. Further, the analysis was extended by Sasagawa et al. [6] to determine the current density and temperature distributions near the corner of an angled metal line subjected to direct current flow. Greenwood and Williamson [7] treated the case of a conductor subjected to a direct current flow, in which temperature dependent material properties were considered, and showed that equipotential were isothermals under the assumption that the relationship between the temperature and electrical potential at the positions of current input and output satisfied the condition of zero electro-thermal heat flux vector [8], and
the remaining portion of the boundary was insulated both electrically and thermally.

This paper represents a study on the electrical and thermal conduction through a conductive metal layer having edge cracks or internal holes subjected to constant current density. The method of Greenwood and Williamson was extended by Jang [9] is used to obtain a solution to the coupled nonlinear problem of steady-state electrical and thermal conduction across a crack in a conductive layer for which material properties were assumed to be functions of temperature.

The main aim of the current work is focused on conducting a comprehensive numerical modeling of potential distribution and thermal gradient due to the effect of various relevant parameters like varying crack depth and internal holes of different geometrical shapes.

II. Physical Model

The physical model considered in the present study is a two-dimensional rectangular film of copper as shown in Fig. 1. The length and width of the layer is 0.2m and 0.1m respectively. The constant current density, J_n =30MAm^{-2} is injected from the left boundary (1) which passes through the whole layer and exists from the right boundary (4). The left and right boundaries are kept at temperature T_i=328K and T_o=273K respectively while the top (3) and bottom (2) edges are kept both thermally and electrically insulated.

We considered the current and heat balances in a two dimensional approximation of the real geometry, as the thickness considered is negligible.

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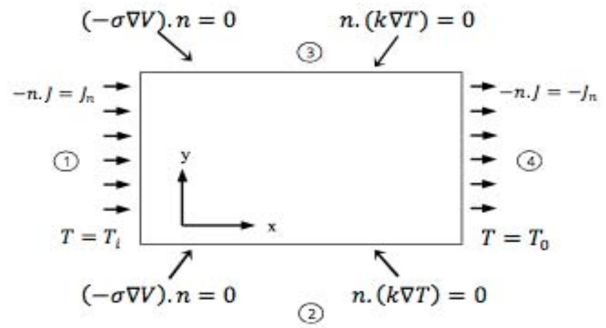
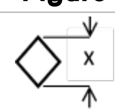
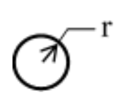
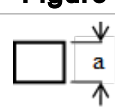
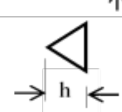
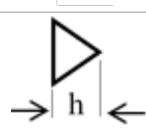


Figure 1 : Schematic diagram of the physical domain with boundary conditions

a) Parameters

Table 1 : List of parameters

PARAMETER	UNIT	VALUE
Common Parameters		
Length (L)	m	0.2
Width (W)	m	0.1
Thickness (t)	m	0.001
Current Density (J)	MAm^{-2}	30
Thermal Conductivity (k)	$Wm^{-1}K^{-1}$	400
Electrical Resistivity (ρ)	Ωm	1.754 e-8
Ambient Temperature	K	298
Inlet Temperature (T_i)	K	328
Outlet Temperature (T_o)	K	273
Edge Cracks		
Crack Width (w)	m	0.001
Varying Crack Depth (d)	m	0.01, 0.02, 0.03, 0.04, 0.05
Internal Holes		
Hole Shape	**Dimension**	**Figure**
Diamond	x= 0.02 m	
	x= 0.04 m	
Circle	r = 0.008m	
	r = 0.016m	
Hole Shape	**Dimension**	**Figure**
Rectangle	a= 0.014m	
	a= 0.028m	
Triangle (J enters from the vertex)	h=0.0187m	
	h=0.0374m	
Triangle (J enters from the base)	h=0.0187m	
	h=0.0374m	

III. Mathematical Formulation

When a conductive layer is subjected to a constant current, there will be potential difference throughout the layer. The electric potential strongly depends on electrical resistivity of the layer material and the value of injected current. That potential difference affects heat generation, which finally involves in calculating thermal distribution. The energy generation term in the governing equation is calculated based on the distribution of electrical potential, there by leading to a coupled analysis of heat transfer problem with electrical problem.

a) Electrical Problem

At a point in space, the electric potential is the potential energy per unit of change that is associated with a static (time invariant) field. The electric field at a point is equal to the negative gradient of the electric potential over there. In symbols,

$$E = -\nabla\varphi \tag{1}$$

Where, $\varphi(x, y, z)$ is the scalar field representing the electric potential at a given point and E is the corresponding electric field.

The continuum form of Ohm's law is

$$\nabla\varphi = -\rho J \tag{2}$$

Which is only valid in the reference form of the conductive material. Equation (2) can be written for orthographic / isotropic material as,

$$\mathrm{i}J_x + \mathrm{j}J_y + \mathrm{k}J_z = -\frac{1}{\rho}\left[i\frac{\partial\varphi}{\partial x} + j\frac{\partial\varphi}{\partial y} + k\frac{\partial\varphi}{\partial z}\right] \tag{3}$$

If the material is anisotropic, we can conclude as,

$$J_x = -\frac{1}{\rho_x}\frac{\partial\varphi}{\partial x},\ J_y = -\frac{1}{\rho_y}\frac{\partial\varphi}{\partial y},\ J_z = -\frac{1}{\rho_z}\frac{\partial\varphi}{\partial z} \tag{4}$$

According to conservation of current law, for stationary current and no free charge in that region,

$$\frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z} = 0 \tag{5}$$

From (4) and (5), for the two dimensional analysis,

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = 0 \tag{6}$$

The boundary conditions for prescribed electrical potential distribution are:

$$\varphi(0, H) = \varphi_1, \qquad \varphi(L, H) = \varphi_2$$

$$\varphi(L, 0) = 0, \qquad \varphi(0, H) = 0$$

And for electrical insulation or prescribed current density,

$$\text{At } x = 0, \frac{d\varphi}{dx} = -\rho J \qquad \text{At } x = L, \frac{d\varphi}{dx} = \rho J$$

$$\text{At } y = 0, \frac{d\varphi}{dy} = 0 \qquad \text{At } y = b, \frac{d\varphi}{dy} = 0$$

b) Thermal Problem

The temperature distribution of a conductive layer is easily understood when the wire is subjected to both-endfixed temperature, open to atmosphere at both-ends etc. But if the layer is subjected to Joule heating or heating due to current, temperature distribution largely depends on thermal conductivity, convection co-efficient of a material.

The energy balance equation for the volume element is stated as-

$$\begin{pmatrix} Net\ rate \\ of \\ heat\ gain \\ by \\ conduction \\ (\mathrm{I}) \end{pmatrix} - \begin{pmatrix} Net\ rate \\ of \\ heat\ loss \\ by \\ convection \\ (\mathrm{II}) \end{pmatrix} + \begin{pmatrix} Rate\ of \\ energy \\ generation \\ (\mathrm{III}) \end{pmatrix} = \begin{pmatrix} Rate\ of \\ increase \\ of \\ thermal \\ energy \\ (IV) \end{pmatrix}$$

So arranging we get,

$$-\frac{1}{A}\frac{[Aq]_x - [Aq]_{x+\Delta x}}{\Delta x} - \frac{hP}{A}[T(x) - T_\infty] + g = \rho C_p \frac{\partial T}{\partial t} \qquad (7)$$

Applying, $\Delta x \rightarrow 0$ with the flux by Fourier law

$$\frac{1}{A}\frac{\partial}{\partial x}\left(Ak\frac{\partial T}{\partial x}\right) - \frac{hP}{A}[T(x) - T_\infty] + g = \rho C_p \frac{\partial T}{\partial t} \qquad (8)$$

So, the basic governing equation for two dimensional heat problems-

$$\frac{1}{A}\frac{\partial}{\partial x}\left(Ak_x\frac{\partial T}{\partial x}\right) + \frac{1}{A}\frac{\partial}{\partial y}\left(Ak_y\frac{\partial T}{\partial y}\right) - \frac{hP}{A}[T(x) - T_\infty] + g = \rho C_p \frac{\partial T}{\partial t} \qquad (9)$$

For variable area and conductivity,

$$\frac{1}{A}\frac{\partial}{\partial x}\left(Ak_x\frac{\partial T}{\partial x}\right) + \frac{1}{A}\frac{\partial}{\partial y}\left(Ak_y\frac{\partial T}{\partial y}\right) - \frac{hP}{A}[T(x) - T_\infty] + g = 0 \qquad (10)$$

For constant area and variable conductivity,

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) - \frac{hP}{A}[T(x) - T_\infty] + g = 0 \qquad (11)$$

For constant area and constant conductivity,

$$k\left(\frac{d^2T}{dx^2} + \frac{d^2T}{dy^2}\right) - \frac{hP}{A}[T(x) - T_\infty] + g = 0 \qquad (12)$$

For variable area, variable conductivity and insulated layer,

$$\frac{1}{A}\frac{\partial}{\partial x}\left(Ak_x\frac{\partial T}{\partial x}\right) + \frac{1}{A}\frac{\partial}{\partial y}\left(Ak_y\frac{\partial T}{\partial y}\right) + g = 0 \qquad (13)$$

The boundary conditions are,

$$\left.\begin{matrix} x=0,\ T(x)=T\_\infty \\ x=L,\ T(x)=T\_L \end{matrix}\right\} \text{(For prescribed temperature)!}$$

$$\left.\begin{matrix} x=0,\ q_0=-kA\frac{dT}{dx} \\ x=L,\ q_0=kA\frac{dT}{dx} \end{matrix}\right\} \text{(For prescribed heat flux)}$$

$$\left.\begin{matrix} x=0,\ \square A\frac{dT}{dx}=h(T-T_\infty) \\ x=L,\ kA\frac{dT}{dx}=h(T-T_\infty) \end{matrix}\right\} \text{(For convection)!}$$

Coupling Electrical and thermal problems,

$$\frac{1}{A}\frac{\partial}{\partial x}\left(Ak_x\frac{\partial T}{\partial x}\right) + \frac{1}{A}\frac{\partial}{\partial y}\left(Ak_y\frac{\partial T}{\partial y}\right) - \frac{hP}{A}[T(x) - T_\infty] + g = \rho C_p \frac{\partial T}{\partial t} \qquad (13)$$

$$\text{And, } g = \rho\left(J_x^2 + J_y^2\right) = \left\{\left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial y}\right)^2\right\} \qquad (14)$$

Here, potential distribution, which is found by electrical analysis of a conductive layer, is put in above equation and the heat generation is calculated. Then heat generation is used in thermal analysis where temperature distribution of that conductive layer is evaluated.

c) Electro-Thermal Heat Flux

Electro-thermal heat flux of a conductive layer is the difference between the thermal heat flux and the flux generation by the potential distribution of that layer. Electro-thermal Heat Flux for two dimensional electrothermal problem,

$$P = q - \frac{1}{2k\rho}\varphi\left(\frac{\partial\varphi}{\partial x} + \frac{\partial\varphi}{\partial y}\right) \qquad (15)$$

d) Solution Procedure

In this study, the Galerkin weighted residual method of finite-element formulation is used as a numerical procedure. The finite element method begins by the partitions of the continuum area of interest into a number of simply shaped regions called elements. These elements may be of different size and shapes. Within each element, the dependent variable is approximated using interpolation functions.

The coupled governing equations (14)-(15) are transformed into sets of algebraic equations using the finite element method to reduce the continuum domain into discrete triangular domains. The system of algebraic equations is solved by iteration technique. The solution process is iterated until the subsequent convergence is satisfied.

$|\psi^{m+1} - \psi^m| \leq 10^{-4}$ Where m is the number of iteration and is the general dependent variable.

IV. Results and Discussion

In this study, the electro-thermal characteristics of thin conductive metal layer are measured through a simulation study using the COMSOL Multiphysics software by finiteelement- method (FEM).

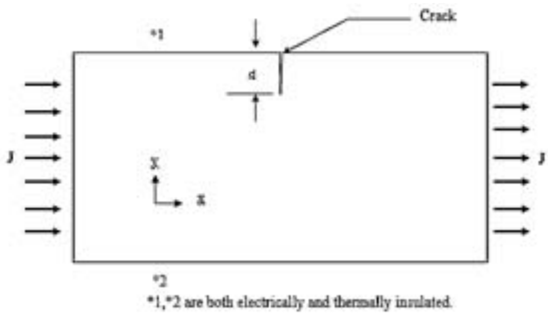


Figure 2 : Geometry of a rectangular copper plate having an edge crack at mid-length position of the layer

Figure 2 shows the geometric configuration of the copper plate and Figure-3 visualizes the generated temperature distribution profile. Figure-6 illustrates the effect of crack depth on the electrical potential. Electrical potential is a function of axial position and it follows a linear shape. We can see that, changing the crack depth increases the maximum electrical potential. When the copper layer with an edge crack at mid-length position of the layer, with constant material property and unchanged boundary conditions is subjected to uniform current density, then Figure 7 depicts the trend of generated temperature along the length.

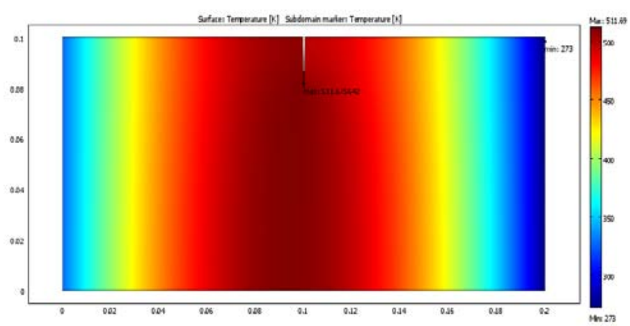


Figure 3 : Temperature profile of a rectangular copper plate having an edge crack at mid-length position of the layer

As the generated temperature is the function of position, so the figure is dome shaped. The temperature increases along length until the center and decreases thereafter. Increasing the crack depth obviously increases the maximum temperature as at the points of crack, the current density increases which raises the temperature. Therefore, the position of maximum generated temperature is at the position of the crack, more appropriately around the tip of the crack. From Figure-7, it is obvious that generated maximum temperature increases proportionally with the ratio of crack depth to crack width, which ultimately depicts that temperature is proportional to crack depth, as crack width is constant.

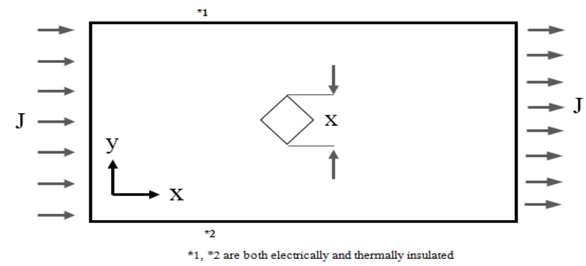


Figure 4 : Geometry of a rectangular copper plate having a right-angled diamond-shaped hole at its center

Figure 8 shows the variation of heat flux when varying the crack depth. It should be noticed that at the position of crack, the shape changes. The heat flux sharply decreases at the crack as the sectional area decreases. Moreover, the minimum heat flux is at the top of the crack.

In case of holes, Figure 5 shows the general trend of temperature distribution of a rectangular copper plate having a right-angled diamond-shaped hole at its center. Figure 10 represents the change of temperature with change of hole shape keeping the boundary condition unchanged.

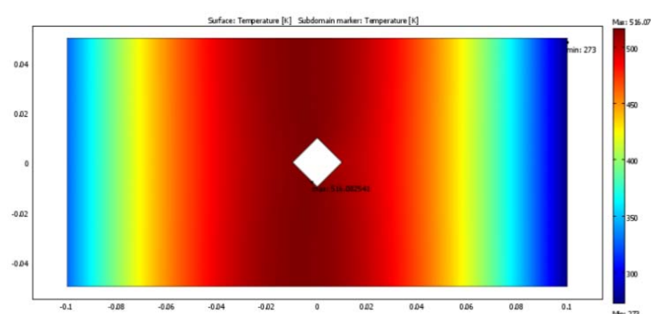


Figure 5 : Temperature profile of a rectangular copper plate having a right-angled diamond-shaped hole at its center

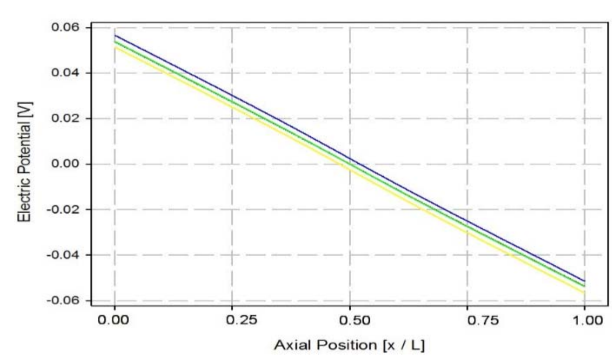


Figure 6 : Comparison of electric potential at the center line along X- axis for various crack depth of a rectangular copper plate having an edge crack at midlength position of the layer

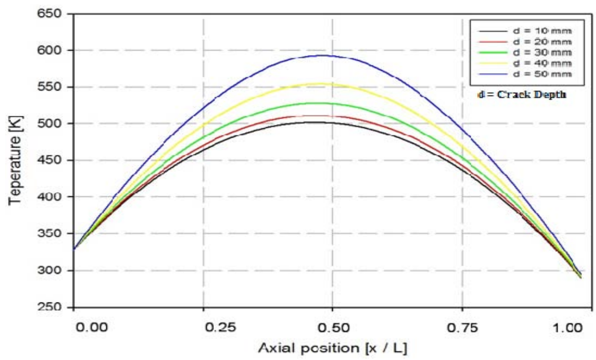


Figure 7 : Comparison of temperature at the centerline along X- axis for various crack depth of a rectangular copper plate having an edge crack at mid-length position of the layer

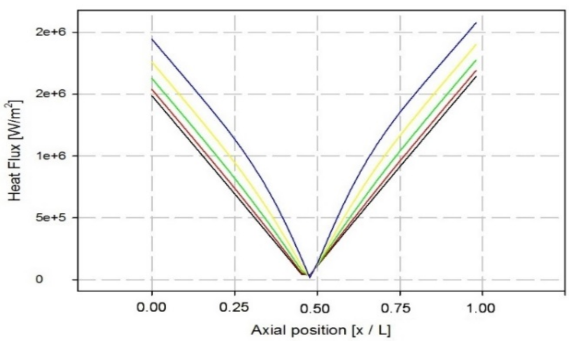


Figure 8 : Comparison of heat flux at the centerline along X- axis for various crack depth of a rectangular copper plate having an edge crack at mid-length position of layer

The temperature is dependent on heat generation and g∝ $|\nabla V|$ i.e. g is dependent on axial position. The figure shows that the shape is almost same for circle, diamond and rectangle. But with reversed triangle (Vertex is opposite to current input direction) the temperature is little lower.

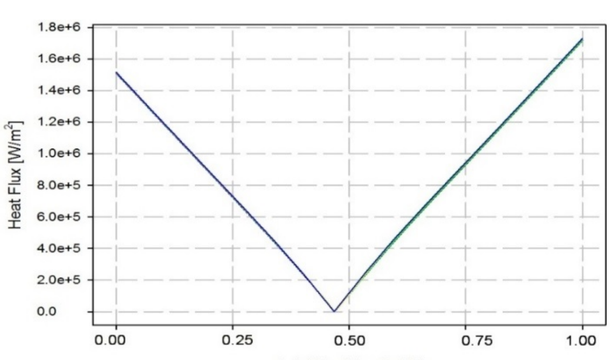


Figure 9 : Comparison of electric potential (developed at the upper edge along X-axis) of a rectangular copper plate having various types of holes at its center

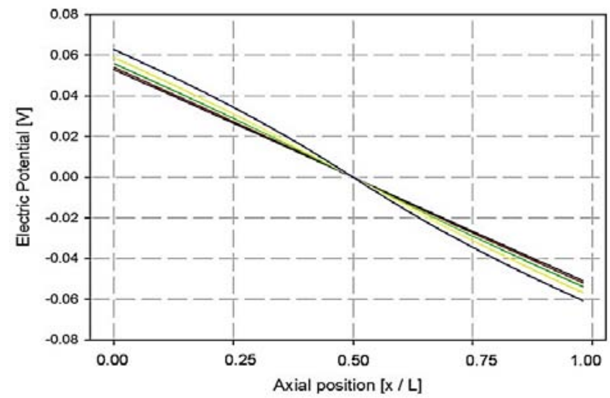


Figure 10 : Comparison of temperature (at the upper edge along X-axis) of a rectangular copper plate having various types of holes at its center

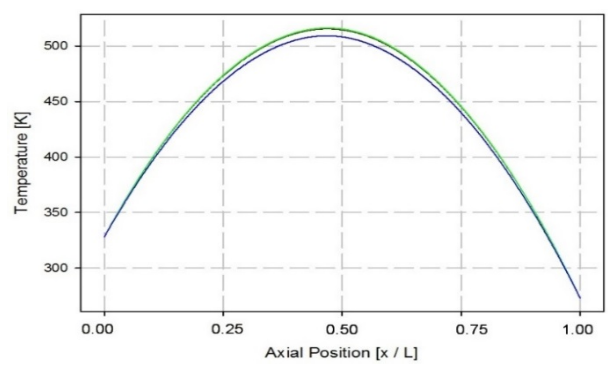


Figure 11 : Comparison of heat flux (generated at the upper edge along X-axis) of a rectangular copper plate having various types of holes at its center

Making the hole diameter as a variable quantity, which shows that with the increment of the size ratio, temperature increases as it withstand the same current input within a smaller region, makes another analysis. Here temperature is a function of size ratio (the ratio of hole diameter to plate diagonal)

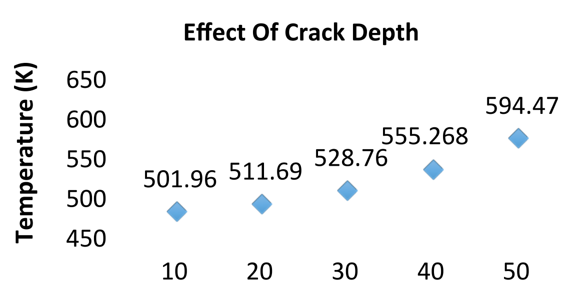


Figure 12 : Temperature (at the center line along X- axis) variation with the ratio of crack depth to crack width of a rectangular copper plate having an edge crack at midlength position of the layer

Table 2 : Effect of varying Size Ratio of the Hole

Shape	Size Factor	Size Ratio	Maximum Temperature (K)
Diamond	Hole Diagonal/ Plate Diagonal	0.089	516.08
		0.134	528.76
		0.179	548.06
		0.224	577.84
Rectangle	Hole Diagonal/ Plate Diagonal	0.089	516.21
		0.179	547.89
Circle	Hole Diameter/ Plate Diagonal	0.071	515.51
		0.143	544.57
Triangle (J enters from the topmost point)	Hole Height/ Plate Diagonal	0.084	499.31
		0.167	535.28
Triangle (J enters from the base)	Hole Height/ Plate Diagonal	0.084	499.32
		0.167	535.03

V. Conclusion

Thermal and electrical characteristics of uniform and non-uniform conductive layers are studied in this research and the analysis was done by computer simulation using COMSOL Multiphysics. In this paper, conductive layers with different types and shapes of internal holes as well as edge cracks are studied.

When analyzing edge crack, it is also seen that crack on conductive layer affects electrical potential and the temperature distribution. An interesting result is found that increasing the crack depth has more significant effect than changing the crack position when other parameters are unchanged. Sharp drop of electrical potential at crack position indicates the sudden change of temperature at that position.

It has been investigated that any kind of hole in a conductive layer increases the temperature. Impacts of circular, diamond-shaped, rectangular and triangular holes are studied. Changing the shape of the internal hole creates little fluctuation on characteristic curves.

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Nomenclature

Symbol	Meaning	Unit
J	Current density	Am^{-2}
k	Thermal conductivity	$Wm^{-1}K^{-1}$
h	Convective HT co-efficient	$Wm^{-2}K^{-1}$
C_p	Specific heat capacity	$KJKg^{-1}{}^{\circ}C^{-1}$

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