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Numerical Study of Natural Convection in a Vertical Channel Partially Filled with a Porous Medium

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Keywords: *finite volume method; natural convection; porous medium; vertical channel.*

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NUMERICALSTUDYOFNATURALCONVECTIONINAVERTICALCHANNELPARTIALLYFILLEDWITHAPOROUSMEDIUM

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Numerical Study of Natural Convection in a Vertical Channel Partially Filled with a Porous Medium

Samir Amraqui [α], Nadia Dihmani [σ] & Ahmed Mezrhab [ρ]

Abstract- A numerical study of natural convection in a vertical channel partially filled with a porous matrix is performed. The flow is described using Brinkman-extended Darcy model. The governing equations are solved by using the finite-volume method. The pressure-velocity coupling is provided by the SIMPLER algorithm. The objective of this work is to determine heat transfer and fluid flow characteristics for Prandtl number Pr = 0.71. A parametric study was conducted by varying the thicknesses of the porous layers and Darcy Number for two Rayleigh number $Ra = 10^6$ and $Ra = 10^7$. The isotherms, streamlines, average Nusselt number and dimensionless temperature are presented for different parametric study.

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- Nomenclature

A	aspect ratio, $A = L/b$
B	channel width [m]
Da	Darcy number, $Da = K/b^2$
e_p	porous layer thickness, [m]
e_p^*	dimensionless Porous layer thickness, e_p/b
K	thermal conductivity [$W\,m^{-1}\,K^{-1}$]
K	permeability
L	channel length [m]
Nuw	average Nusselt numbre
P	pression, [Pa]
P	dimensionless pression, $(p+\rho_o g y)b^2/\rho_o\alpha^2$
Pr	Prandtl number, ν/α
Ra	Rayleigh number, $g\beta(T_h - T_a)b^3/\nu\alpha$
R_k	thermal conductivity ratio, k_s/k_f
T	temperature, [K]
T_o	average temperature, $(T_h + T_a)/2$, [K]
u, v	velocity component along x, y [$m\,s^{-1}$]
U, V	dimensionless velocity components along x, y. $U = ub/\alpha$, $V = vb/\alpha$
x, y	cartesian coordinates [m]
X, Y	dimensionless cartesian coordinates, $X = x/b$, $Y = y/b$
α	thermal diffusivity [$m^2\,s^{-1}$]
β	volumetric expansion coefficient [K^{-1}]
ΔT	maximal difference temperature, $\Delta T = (T_h - T_a)$
λ	dimensionless viscosity ratio, $\lambda = \mu_s/\mu_f$
θ	dimensionless temperature, $\theta = (T - T_0)/(T_C - T_F)$
Θ	dimensionless temperature, $\Theta = T/T_C$
μ	dynamic viscosity

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I. Introduction

The study of the natural convection mode in vertical porous channel is particularly developed in recent years because it relates to various applications such as cooling of electronic equipment, nuclear reactors, thermal insulation heat exchangers, building industry, geophysical flows and crystal growth [Chu and Hwang (1977), Nield, Bejan (1992), Mezrhab (1997)].

In recent years, a large number of experimental and numerical researches have been devoted to study the heat transfer in fully or partially porous vertical channels. [Debbissi (2000)] studied the water evaporation by natural convection between two flat plates. A uniform heat flux is imposed on one wet wall whereas the other plate is insulated or heated and supposed impermeable and is kept at a constant flow by taking into consideration the radiation plates.

[Yan and Lin (2001)] studied the combined effects of buoyancy forces and heat and mass diffusion in a laminar natural convection flow inside a vertical pipe. These authors investigated the effects of wet walls temperatures, air humidity and the aspect ratio on the flow and heat and mass transfer. Recently, [Orfi, Debbissi, Belhaj and Nasrallah (2004)] examined the thin liquid film evaporation flowing down on the inner face of a vertical channel plate. The wet wall is subjected to a uniform heat flux and the second plate is insulated and impermeable.

The purpose of this work is to study numerically the natural convection in a vertical channel with two porous layers arranged vertically by examining the effect of the porous layers thickness on the flow structure, the average Nusselt number and the temperature distribution within the channel.

II. Formulation Mathématique du Problème

The configuration of the problem studied is depicted in Figure 1. It shows the geometry of a vertical parallel-plates channel partially occupied by two porous layers. The vertical plates are isothermal and kept at the hot temperature T_h. The analysis assumes that the porous media is homogeneous and isotropic. The fluid flow is incompressible, laminar and two-dimensional. The momentum equations are simplified using Boussinesq approximation, in which all fluid properties are assumed constant except the density in its contribution to the buoyancy force.

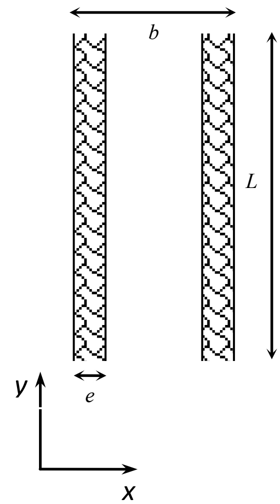


Figure 1 : Schematic configuration of the problem

The two-dimensional governing equations based on the Brinkman–Darcy model can be written in the following dimensionless form:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \quad (1)$$

$$\frac{\partial U}{\partial \tau}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\lambda Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)-\frac{Pr}{Da}\frac{U}{K_x} \quad (2)$$

$$\frac{\partial V}{\partial \tau}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\lambda Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)-\frac{Pr}{Da}\frac{V}{K_y}+RaPr\theta \quad (3)$$

$$\sigma\frac{\partial \theta}{\partial \tau}+U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=R_k\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \quad (4)$$

Where $\lambda=1$, $R_k=1$, $\sigma=1$ in the fluid region

$\lambda=\infty$, $R_k=k_s/k_f$, $\sigma=\dfrac{(\rho c)_p}{\varepsilon(\rho c)_f}$ in the porous region.

The boundary conditions corresponding to the considered problem are as follows:

- Solid plates boundary conditions: $0\le Y\le 1$, $X=0$ and $X=1$

$$U=0,\ V=0,\ \theta=1,\ \frac{\partial P}{\partial X}=0$$

- Inlet boundary conditions: $0\le X\le A$ and $Y=0$

$$U=0,\ \frac{\partial V}{\partial Y}=0,\ \frac{\partial \theta}{\partial Y}=0,\ P=\frac{-Q_{in}^2}{2}$$

Where Q_{in} is the mass flow rate at the channel inlet.

$$Q_{in}=\int_0^1 [V(X)]_{Y=0}\,dX$$

- Outlet boundary conditions: $0\le X\le A$ and $Y=1$

$$U=0,\ \frac{\partial V}{\partial Y}=0,\ \frac{\partial \theta}{\partial Y}=0,\ P=0$$

III. Numerical Procedure

A series of calculations was done for the set of parameters given in Table 1, to determine the optimum non-uniform grid (i.e. the best compromise between accuracy and computational costs). A numerical simulations of the problem considered for two Rayleigh number $Ra=1\times10^6$ and 10^7, $A=5$ and $Pr=0.71$ were performed with six different grids. From the table 1, it was found that the difference in Nusselt number obtained with a 30×140 grid and a 30×160 grid is only 0.1% percent. Therefore, all the computations in the present study were done with a non-uniform 30x140 grid.

Table 1 : Results of the grid independence study for $Pr = 0.71$: $Ra = 10^6$ and $Ra = 10^7$

	$Ra = 10^6$			$Ra = 10^7$		
	Nuw	ψ_{max}	Q	Nuw	ψ_{max}	Q
(14x90)	16.885	389.01	$0.41495.10^3$	18.282	783.18	$0.83540.10^3$
(14x100)	16.891	390.49	$0.41653.10^3$	18.287	790.92	$0.84365.10^3$
(20x100)	17.575	408.209	$0.43542.10^3$	19.125	853.69	$0.91060.10^3$
(20x120)	17.582	410.489	$0.43786.10^3$	19.132	896.143	$0.93509.10^3$
(24x120)	17.847	413.260	$0.44081.10^3$	19.513	882.758	$0.94161.10^3$
(24x130)	17.849	414.15	$0.44176.10^3$	19.517	896.14	$0.95589.10^3$
(28x130)	18.046	415.27	$0.44296.10^3$	19.797	895.557	$0.95526.10^3$
(28x140)	18.047	416.06	$0.44380.10^3$	19.800	908.96	$0.96956.10^3$
(30x140)	18.155	417.57	$0.44541.10^3$	19.907	915.19	$0.97621.10^3$
(30x160)	18.156	418.705	$0.44662.10^3$	19.914	941.68	$1.0045.10^3$

Furthermore, the numerical code has been validated by taking into account some numerical studies available in the literature. Firstly, it was validated on the problem of natural convection in a square porous cavity [Beckermann, Vistkanta and Ramadhyani (1986)]. The results found for the average Nusselt number for different Rayleigh number *Ra*, the Prandlt number *Pr* and Darcy number *Da* are in good agreement with the present results as shown in Table (2).

Table 2 : Average Nusselt number for differents controls parameters and for $A = 1$

Ra	Da	Pr	Beckermann et al.(1986)	Nos Résultats
10^5	10^{-1}	1.0	4.724	4.648
10^5	10^{-1}	0.01	4.724	4.648
10^8	10^{-4}	1.0	24.97	24.891
10^8	10^{-4}	0.01	24.97	24.891
10^{12}	10^{-8}	1.0	48.90	48.854
10^{12}	10^{-8}	0.01	48.90	48.854

Secondly, the code has been tested on the problem of natural convection in an asymmetrically heated vertical channel partially filled with a porous medium. A comparison of the velocity profile for three fluid layer thickness and for $Da = 10^{-2}$ between the predicted results and those obtained by [Paul, Jha, Singh (1998)] is shown in figure 2. Results show an excellent agreement.

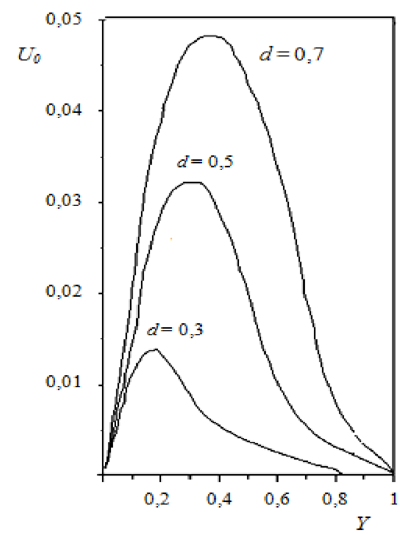


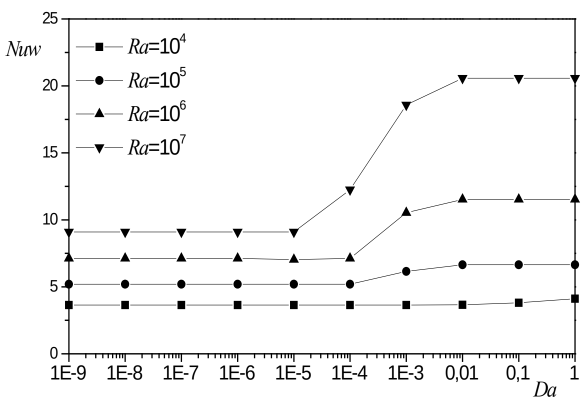
Figure 2 : Velocity profiles for $Da = 10^{-2}$

IV. Results and Discussion

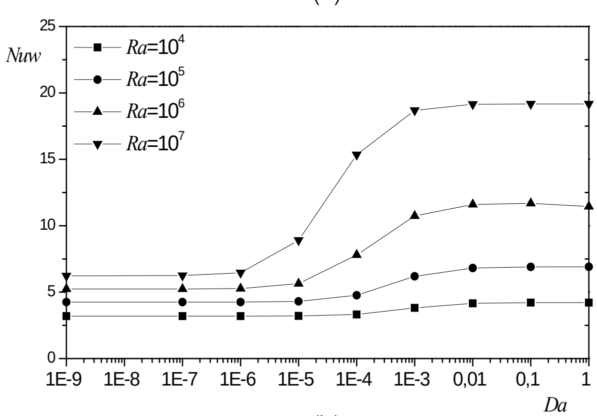
Numerical simulations are performed for $Pr = 0.71$, $A = 5$, $R_k = 1$ and for Darcy number and porous layer thickness ranged respectively between $10^{-8} \leq Da \leq 1$ and $0.05 \leq e_p^* \leq 0.3$. A survey of the heat transfer and flow overall results obtained is presented and discussed. Some local results are also reported by means of isotherm, streamline, average Nusselt number and dimensionless temperature.

Figure 3 shows the variation of the average Nusselt number *Nuw* according to Darcy number for different Rayleigh number and for three porous layer thickness: $e_p^* = 0.05$, $e_p^* = 0.1$, $e_p^* = 0.2$.

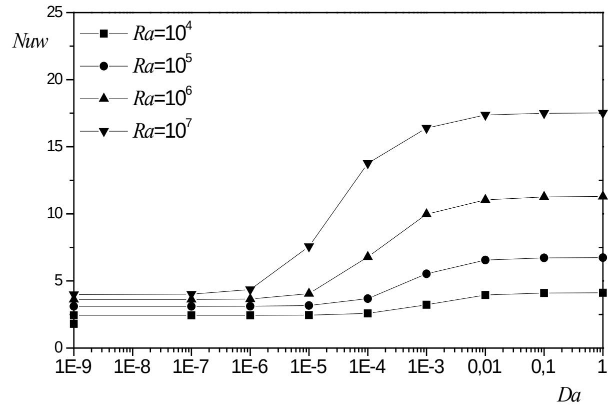
In general, we observe that for a fixed Darcy number, heat transfer increases with increasing Rayleigh number. However, for a given *Ra*, there is a significant decrease of Nusselt number *Nuw* as the permeability decreases. Moreover, from these figures, we can distinguish three zones: A first zone with low permeability $Da \leq 10^{-6}$: the Nusselt number remains almost constant, in this range of Da, and the porous layers acts as an impermeable wall where the flow is almost zero. A second zone characterized by high Darcy numbers $Da \geq 10^{-3}$: the average Nusselt number is higher and practically independent of Da. The porous layers do not hinder the fluid flow and the heat transfer is more important as Rayleigh number increases. This is due to the fact that the Darcy number becomes negligible compared to the convective terms in the governing equations system. A final intermediate zone $10^{-5} \leq Da \leq 10^{-3}$: we note that the average Nusselt number increases significantly with the permeability. This increase is even higher for $Ra = 10^6$ and $Ra = 10^7$ and for low porous layers thicknesses.



(a)



(b)

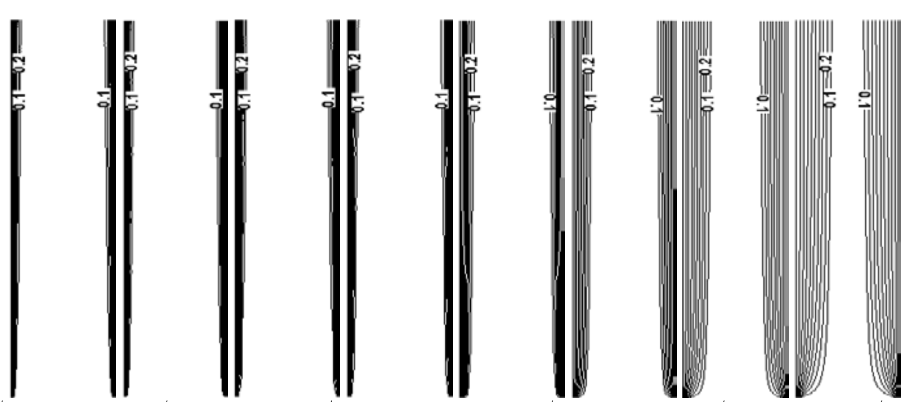


(c)

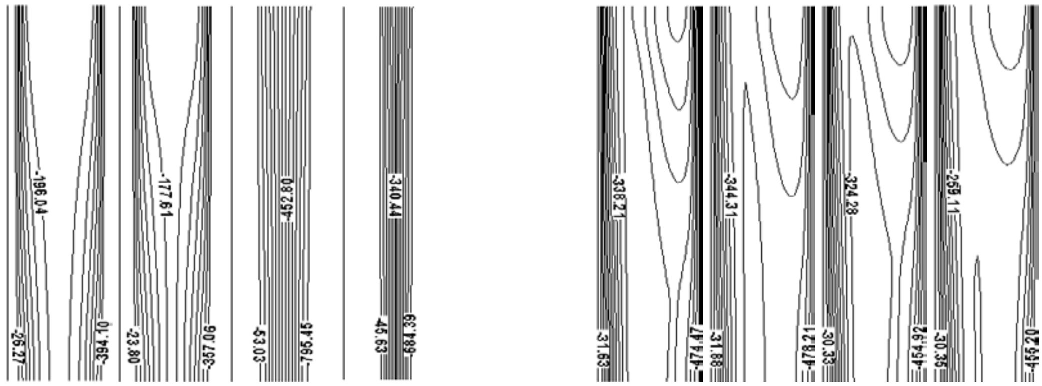
Figure 3 : Evolution of the average Nusselt number as a function of Darcy numberfor different Ra : (a) $e_p^* = 0.05$, (b) $e_p^* = 0.1$, (c) $e_p^* = 0.2$

Figure 4 displays the isotherms and the streamlines obtained for various porous layer thickness and respectively for two Darcy number $Da = 1$ and $Da = 10^{-6}$.

Let us note that for $Da = 1$, which corresponds to a permeable porous layer (Figure 4. a), the variation of the thickness has no significant effect on the streamlines shape. However, for $Da = 10^{-6}$, which corresponds to the porous layers of low permeability (Figure 4. b), we note the absence of flow circulation in the porous media witch behaves like solid walls. The flow is confined in the fluid medium and becomes lower as the thickness increases. The isotherms are parallel to the channel walls and are thinner for $Da = 1$.



$e_p^*=0.05$ $e_p^*=0.1$ $e_p^*=0.2$ $e_p^*=0.3$ $e_p^*=0.05$ $e_p^*=0.1$ $e_p^*=0.2$ $e_p^*=0.3$

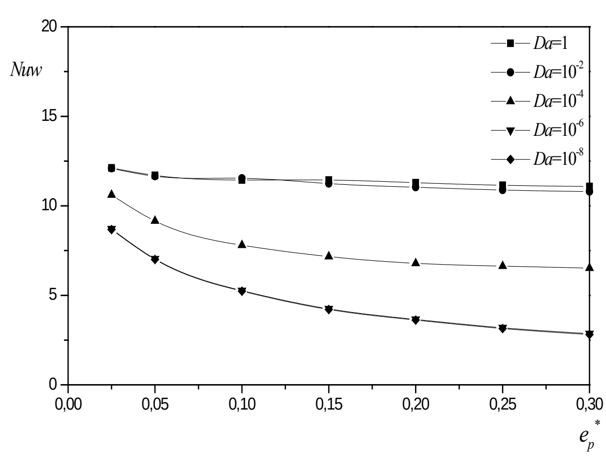


(a) (b)

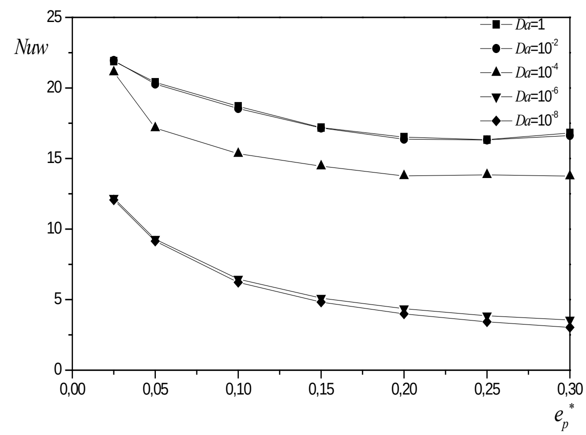
Figure 4 : Isotherms and streamlines for various e_p^* for $Ra = 10^7$: (a) $Da = 1$, (b) $Da = 10^{-6}$

Figure 5 gives the variation of the average Nusselt number *Nuw* versus the porous layers thickness e_p^* for five different Darcy numbers and for two Rayleigh number $Ra = 10^6$ and $Ra = 10^7$.

As can be seen the different curves are located between two limiting curves corresponding to the fluid and solid behavior of the porous material $Da = 1$ and $Da = 10^{-8}$. It is observed that the heat transfer decreases with increasing the porous layers thickness, which is more important as the Darcy number decreases. Indeed, we note that for sufficiently large permeability ($Da = 1$ and $Da = 10^{-2}$) the influence of the porous layer thickness on heat transfer is negligible and the average Nusselt number remains constant. However, for relatively low permeability the Nusselt number decreases with increasing e_p^*. It should be noted that it is sufficient to introduce porous layers of thickness less than 0.1 to reduce significantly heat transfer.



(a)

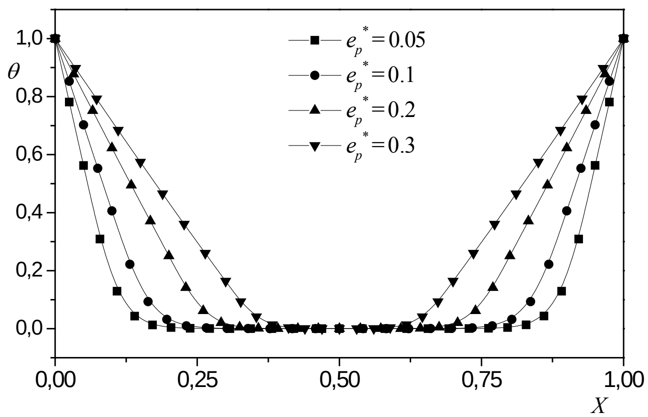


(b)

Figure 5 : Evolution of the Nusselt number as a function of e_p^* for different Darcy number: (a) $Ra = 10^6$, (b) $Ra = 10^7$

To understand the shape of the isotherms, we plotted in Figure 6 the evolution of the dimensionless temperature profiles θ in the horizontal median plane of the channel for different porous layers thickness and for two Darcy number $Da = 1$ and $Da = 10^{-6}$.

As we have previously reported, for large Darcy number (Figure 6. a), the introduction of porous layers has a negligible effect on dimensionless temperature whatever their thickness. However, for $Da = 10^{-6}$ (Figure 6. b), note that the curves slope decreases significantly for large thickness values and θ is much smaller that the thickness e_p^* is low.



(b)

Figure 6 : Evolution of the dimensionless temperature as a function of X for differents e_p^* and for $Ra = 10^7$: (a) $Da = 1$, (b) $Da = 10^{-6}$

V. Conclusion

This paper presents a numerical study of natural convection in a vertical parallel-plate channel partially filled with porous medium. Numerical calculations were performed to investigate the effect of Rayleigh numbers *Ra*, Darcy number *Da* and porous layers thickness e_p^* on the flow field and heat transfer. We examined the Rayleigh number effect characterizing the convection intensity and we concluded that the heat transfer increases with increasing Rayleigh number *Ra*. We also showed that the variation of the porous layers permeability, through the Darcy number, affects significantly the heat transfer. Indeed, we have identified three zones and we found that for a fixed Rayleigh number and high value of *Da*, the Nusselt number *Nuw* is nearly constant and the flow is similar to that observed in a fluid channel. Whereas, for small Darcy numbers, the average Nusselt number decreases until it reaches a minimum for $Da = 10^{-6}$ where there is no convective exchange in the porous layers. Results show also that the heat transfer decreases significantly when the porous layer thickness is less than 0.1 (e_p^* □0.1). Finally, note that the isotherms and streamlines are very sensitive to variation of the porous layers thickness for relatively large Darcy number.

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