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I. Formulation of a Problem

Analysis of the literature [1 – 5] shows that the dynamic model of the car from rolling down the hill with built correctly. The recommended formulas for determination of the distance traveled in **скатывании** car with slides [1] do not take into account the motion of the car on the profile of the hills in pure rolling wheels of wheel pairs of carriages and the rolling elements of bearing on the inner and outer rings, if not believe that such a movement indirectly taken into account the concept of the main specific resistance wo, which found empirically. Estimated rate of dissolution of the car with the roller coaster is found by the formula freely falling body, which does not comply with the physical sense of the problem being solved, as it should be defined as a result of solution of a differential equation of motion of the car, how this is done in [6 – 9]. The work is not yet covers the solution of a number of practical problems getting the car to the hill.

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Proceeding from this, in [10, 11] shows accounting moments of friction (net rolling wheel on the rail thread and in the bearings **буксовых** nodes front and rear bogies of a wagon with their subsequent replacement of contingent slip friction. It was noted that, if the active force in the form of the projection of the force of gravity and the forces of aerodynamic drag to the direction of rolling the car, acting on the car, the more marginal the friction force at the same time with rolling it is also possible movement of wheels on the rail lines.

Thus, up to the present time out of mind researchers were missed construction of the model of the car from rolling down the hill with in strict accordance with the provisions of the classical theoretical mechanics and, accordingly, the definition of speed and covered distance from rolling on the profile of the hills in the counter and/or wind.

II. Man-Made Assumption

Consider the General case, when the car with the sorting slides forward with a specified initial velocity v0 (usually 4÷5 km/h or 1.1 ÷ 1,38 m/s, and the maximum value of 9 km/h or 2.5 m/s). When wagon rolling down the hump with the roller coaster car will be face-the impacts are mostly external forces in the form of the force of gravity of the car with or without cargo $\overline{G}$ and the forces of aerodynamic drag (where $\overline{F}_{rw} \in \overline{F}'_{rwx}, \overline{F}'_{rwy}$). Distribution of the forces of gravity of the car body with a weight on the front and rear bogie depend on the technology of placing of cargo (symmetrically, or not symmetric about the axes of symmetry of the car) on the car.

Let the car roll off the sorting slides forward with portable speed $\overline{v}_e = \overline{v} = \overline{v}_w$ (unknown value) [12]. Wind speed in relation to the top of the hill (the earth) (i. e., the absolute velocity of the particles of air) $\overline{V}_{a.w}$ (the value specified by [13]) is directed along the horizontal axis *Oxyz*. Find the projection of the relative velocity of

Global Journal of Researches in Engineering (A) Volume XIV Issue I Version I Year 2014

the particle air (wind speed) $\bar{v}_{rw}$ (concerning the moving axes $O_1x_1y_1z_1$, connected with the wagon).

Show the dependence of the projection of the relative velocity of the particle air (wind speed) $\bar{v}_{rw}$ (the calculated value) of wind speed in relation to the top of the hill (the earth) (i. e., the absolute velocity of the particles of the air) $\bar{v}_{a.w}$ and the speed of the car $\bar{v} = \bar{v}_w$. Let us assume that the car rolls down from the top of the hill with a velocity $\bar{v}_e = \bar{v} = \bar{v}_w$ relative to the fixed coordinate system *Oxyz*, associated with the top of the hill. Take that with a wagon rigidly connected mobile coordinate system $O_1x_1y_1z_1$, and the particles of air, in turn, are moving at velocity $\bar{v}_{rw}$ relative to the moving system of coordinates $O_1x_1y_1z_1$ (i .e., cars) (Fig. 1) [12].

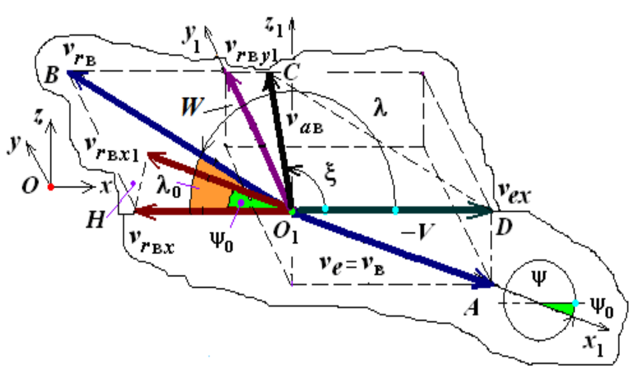

Figure 1 : Vector diagram of the speed of the car and the wind

In Fig. 1 indicated by: *O* the beginning of the motionless coordinate system *Oxyz*, rigidly connected with the top of the hill; O_1 the beginning of the moving coordinate system $O_1x_1y_1z_1$, rigidly connected to the wagon; (*H*, *V* and *W* horizontal, vertical and frontal plane; the angle of descent (in accordance with the profile of the hills value given); $\bar{v}_{rw}$ the relative velocity of the particles of air (wind speed) in relation to the moving reference system $O_1x_1y_1z_1$ (car) value (calculated); λ guide corner of the vector of relative velocity of the air particles along the axis *Ox* (calculated value); $\bar{v}_{a.w}$ the absolute velocity of the particles of the air against the earth (to the top of the hill) (value of the defined); ξ guide the angle of the vectors of absolute soon-of air particles along the axis *Ox* (the value given).

We believe that the relative velocity of the particles of air (wind speed) $\bar{v}_{r.w}$ is located on a horizontal plane *H* and is directed at an angle λ (or λ_0) to the horizontal axis (axis *Ox*), and drive speed (the speed of the car) $\bar{v}_e = \bar{v} = \bar{v}_w$ in the vertical plane *V* and is directed at an angle of descent of rolling down ψ (or ψ_0) to the horizontal axis (axis *Ox*).

According to the theorem of addition of velocities at the complex motion [14, 15], we write

$$\bar{v}_{a.w} = \bar{v}_{ex} + \bar{v}_{r.w}, \tag{1}$$

where $\bar{v}_{a.w}$ is the absolute velocity of a particle air (wind speed); $\bar{v}_{ex} = \bar{v}_x = \bar{v}_{wx}$ projection of portable speed (the speed of the car) $\bar{v}_e = \bar{v} = \bar{v}_w$ on the axis *Ox*:

$$v_{ex} = v_x = v_{wx} = v_e \cos(\psi_0), \tag{2}$$

Taking into account the fact that ψ (or ψ_0) the tilt angle of the hill to the horizontal axis (axis *Ox*);

$\bar{v}_{r.w}$ the relative velocity of the particles of air (wind speed) in relation to the car.

Keep in mind that if the wind direction is opposite to the direction of movement of the car (i. e. wind, see Fig. 1), the vector equation (1), in accordance with rule subtract of vectors [12, 14], can be written as:

$$\bar{v}_{aw} = \bar{v}_{ex} + (-\bar{v}_{rw}),$$

where

$$\bar{v}_{rw} = \bar{v}_{ex} - \bar{v}_{aw}. \tag{3}$$

Projection (3) on the axis *Ox* when the wind is of the form [12, 14]:

$$v_{rw.x} = v_e \cos(\psi_0) - v_{aw} \cos(\xi), \tag{4}$$

where ξ the angle between the vector $\bar{v}_{aw}$ (the absolute velocity of a particle air (wind speed)and longitudinal axis *Ox*, rad.

In accordance with the expression (4) the force of aerodynamic drag $\bar{F}_{rw}$ for oncoming wind determined depending on the speed of the car from rolling on the profile of slides, *N*:

the axis *Ox*

$$F'_{rwx}(v_e) = 0{,}5c_{\mathrm{w}}\rho_{\mathrm{w}}A_{\mathrm{t}}\left(v_e\cos(\psi_0) - v_{aw}\cos(\xi)\right)^2; \tag{5}$$

the axis *Oy*

$$F'_{rw.y} = 0{,}5c_{\mathrm{w}}\rho_{\mathrm{w}}A_{\mathrm{b}}\left(v_{rw}\sin(\lambda_0)\right)^2. \tag{5, a}$$

In the last formulas [14]: c_{w} – dimensionless experimental coefficient resistor of air, depending on the shape of the body and how it is directed at the movement (usually take depending on the shape of the surface in the range from 0,55 up to 1.2, for example, the cylindrical body, having in the cross-section of a circle (trumpet) c_{w} = 0,6; for the flat surface of the c_{w} = 1,1); ρ_{w} – average air density (kg/m^3) (usually take 1,26 of 1.29); A_{t} – area of the front surface of the car with a cargo of, m^2: $A_{\mathrm{t}} = 2B \times 2H$ (where $2B$ and $2H$ – the width and height of the windward surfaces of wagons loaded with, m); A_{b} – area the side of the wagon loaded with: $A_{\mathrm{b}} = 2L \times 2H$ (where $2L$ – the length of the windward side surfaces of wagons with cargo, m), m^2.

In (5, a), and guide the corner λ the relative velocity of the particle air (wind speed) $\overline{v}_{r.\mathrm{w}}$ are according to the theorem of sinus

$$\sin(\lambda) = \frac{v_{aw}}{v_{\mathrm{rw}}}\sin(\xi). \tag{6}$$

Note that according to (5) of the projection of the forces of aerodynamic drag $\overline{F}_{rw}$ in the direction of the sliding carriages for the wind is a function that depends on the projection of the load speed of the wagon $\overline{v}_{ex} = \overline{v}_x = \overline{v}_{\mathrm{wx}}$ (see. (2)), i. e. $F'_{rwx}(v_e)$.

III. Formation of a Design Model of the Car from Rolling

A simplified model of the car from rolling down the hill with, taking into account the friction rolling wheels car with a slip, take the model shown in Fig. 2, and the calculation model - in Fig. 3 [3 – 5, 11].

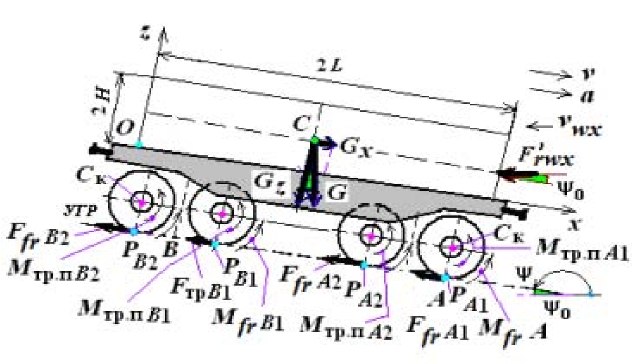

Figure 2 : A simplified model of the car from rolling down the hill with

In Fig. 2. marked: $M_{fr.bA}$ ($M_{fr.bA} \in \{M_{fr.bA1}, M_{fr.bA2}, M_{fr.bA'1}, M_{fr.bA'2}\}$) and $M_{fr.bB}$ ($M_{fr.bB} \in \{M_{fr.bB1}, M_{fr.bB2}, M_{fr.bB'1}, M_{fr.bB'2}\}$) internal forces in the form of points of friction in the bearings bucks nodes front and *A* rear *B* trolley car, and $M_{fr.b} = M_{fr.bA} + M_{fr.bB}$; P_{A1}, P_{A2}, P_{B1}, P_{B2} instant centers speeds [3 – 5].

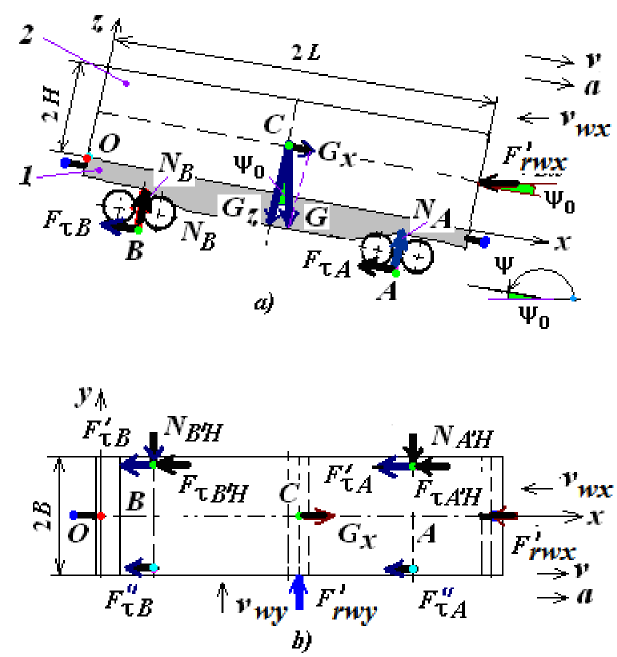

Figure 3 : Calculation model of the car from rolling down the hill with a) the view from the side; b) the view from the top

The force of friction of the rolling of a wheel with a slip $F^{\mathrm{sl}}_{\mathrm{fric}}(\overline{v}_e)$, as the force of the braking of a wagon $F_{\mathrm{torm.}}(v_e)$, aimed in the direction opposite to the from rolling car with slides, can be represented in the form of [3, 4, 9, 11, 15]:

$$F_{\mathrm{torm.}}(\overline{v}_e) = F^{\mathrm{r}}_{\mathrm{fric}}(v_e) + f_{\mathrm{sl0}}F'_{rwy}, \tag{7}$$

where $F^{\mathrm{r}}_{\mathrm{fric}}(\overline{v}_e)$ is a conventional sliding friction in pure rolling wheels and the rolling elements in the bearings bucks nodes [10]:

$$F^{\mathrm{r}}_{\mathrm{fric}}(v_e) = f_0\left(G\cos(\psi_{0,50}) + F'_{rw}(v_e)\times\sin(\psi_{0,50})\right). \quad (8)$$

where f_0 is a conditional (or) the coefficient of friction [11, 14, 15]:

$$f_0 = \frac{n_{\mathrm{w}} f_{\mathrm{r}}}{r_{\mathrm{r}}} + \frac{n_{\mathrm{b}} f_{r0}}{r_{\mathrm{b}}}\frac{k}{n_{\mathrm{bn}} n_{\mathrm{tq}}}; \quad (8,\ a)$$

f_{sl0} is the coefficient of sliding friction ridges of wheels on the rails (usually take $f_{\mathrm{sl0}} = 0{,}25$);

F'_{rwy} projection of the forces of aerodynamic drag on the transverse axis of the car (according to (5) the value of the calculated) [15].

In (8, a), and the following designations are accepted: n_{w} number of wheels of the cart, pieces. ($n_{\mathrm{w}} = 8$); f_{r} the friction coefficient of friction, since this ratio is equivalent to the shoulder of a friction pair of ka-treatment (the wheel on a track $f_{\mathrm{r}} = 5\times10^{-6}$, steel hardened steel $f_{\mathrm{r}} = 1\times10^{-6}$), r_{r} wheel radius, equal to the freight car 0,475 m; $n_{\mathrm{bn}} = 8$ number bucks nodes in the cart, pieces.; $f_{\mathrm{к0}}$ the friction coefficient of the rolling elements in the rings of the bearing (usually take $0{,}001\times10^{-3}$), m; n_{tq} the total quantity of the rolling elements, which include the load in each of the bearing, pieces.; k is a constant factor, taken depending on the location of the rolling elements and type of bearings quality of (for the calculation shall take $k = 4{,}6$) [16]; n_{b} number in the bearings bucks nodes in the cart, pieces. ($n_{\mathrm{b}} = 16$); r_{n} outer radius of the inner ring of the bearing raceways, m (0,079 m).

Introducing the concept of "shift" $F_{\mathrm{sh}.x}$ and "restraint" $F_{\mathrm{res}.x}$ of forces, due to the active and all jet forces, we obtain [5 – 8]:

$$F_{\mathrm{sh}.x} = G_x = G\sin(\psi_0); \quad (9)$$

$$F_{\mathrm{res}.x}(v_e) = F'_{rwx}(v_e)\cos(\psi_0) + F_{\mathrm{torm.}}(v_e).$$

Rewrite the last expression with the account of (7) and (8)

$$F_{\mathrm{res}.x}(v_e) = F'_{\mathrm{wx}}(v_e)\times\cos(\psi_0) + f_0\begin{pmatrix} G\cos(\psi_0) + \\ F'_{rwx}(v_e)\sin(\psi_0)\end{pmatrix} + f_{\mathrm{sl0}}F'_{rwy},$$

and after transformations

$$F_{\mathrm{res}.x}(v_e) = F'_{\mathrm{wx}}(v_e)\left(\cos(\psi_0) + f_0\sin(\psi_0)\right) + f_0 G\cos(\psi_0) + f_{\mathrm{sl0}}F'_{rwy}. \quad (10)$$

The condition of the car from rolling on the first main area of the slide with a slope of not steeper than 50 ‰ at the length of this section of up to 50 m is [10]

$$F_{\mathrm{sh}.x} >> F_{\mathrm{res}.x}(v_e). \quad (11)$$

Hence, the excess forces $\Delta F_{\mathrm{к},50} = F_{\mathrm{sh}.x} - F_{\mathrm{res}.x}(v_e)$. This force arises in the first specialized site of the hill and is the driving force. It causes the slide car given the force of gravity G with a velocity $v_{e,50}(t)$ and the acceleration of the $a_{50}(t)$. The force depends on the foundations of the Mr. from the angle of rolling hills $\psi_{0,50}$ and, to some extent, from the coefficient of friction ridges of wheels on the rails, as well as the condition of rolling bearings in буксовых nodes carts. Therefore, in order to ensure the movement of the car at the end of the first specialized site slides with the velocity $v_{e,50}(t)$ less than the speed of input $v_{ewx}(t)$ at the first brake position (I BP), i. e. $v_e(t) < v_{ewx}(t)$, is sufficient selection of rational values $\psi_{0,50}$ as the basic geometric parameter slides.

IV. Methods of Solution

The formation of dynamic and constructing a mathematical model of the car from rolling down the hill with is based on classical concepts and provisions of theoretical and applied mechanics (for example, the theorem of addition of velocities under complex movement, roll, slip, slide with rolling, communications, the reaction of communication, the principle clear constraints, the basic law of the dynamics of the absolute motion (the principle of the D'Alembert); basic concepts of differential and integral calculus [12 – 14]).

V. Methods of Constructing the Mathematical Model of the Car from Rolling Down the Hill with

We shall take into account, that the car rolled down the hills steadily so a portable acceleration of the car $\bar{a}_e = \bar{a}$ is the absolute acceleration $\bar{a}_{\mathrm{abs}} = d\bar{v}_{\mathrm{abs}}/dt$ [14, 15].

Substituting (9) and (10) of the basic law of the dynamics for a portable car (or the principle of D'Alembert) in the coordinate form [12 – 15] we have:

$$M\frac{dv}{dt}=F_{\text{sh}.x}-F_{\text{res}.x}(v_e).$$

where M is the mass of the car with the load, kg.

Transforming the last equation with account of (9) and (10) and the fact that $G = Mg$, for the first specialized site slides with a slope of not steeper than 50 ‰ at the length of this section of up to 50 m we obtain:

$$M\frac{dv}{dt}=Mg\left(\sin(\psi_{0,50})-f_0\cos(\psi_{0,50})\right)-f_{\text{sl}0}F'_{rwy}-$$
$$-F'_{\text{w}x}(v_e)\left(\cos(\psi_{0,50})+f_0\sin(\psi_{0,50})\right).$$

Imagine the last expression in the form of:

$$M\frac{dv}{dt}=F_0-b_0(v_e\cos(\psi_{0,50})-c_0)^2, \quad (12)$$

where F_0 is the difference famous of the largest driving forces and the forces of resistance, the attached to the system "wagon - cargo", N:

$$F_0=Mg\left(\sin(\psi_{0,50})-f_0\cos(\psi_{0,50})\right)-f_{\text{sl}0}F'_{rwy}, \quad (13)$$

b_0 famous for the largest constant factor in (5) for the location of the forces of aerodynamic drag $F'_{\text{w}x}$, N/(m/s)^2:

$$b_0=0{,}5c_{\text{w}}\rho_{\text{w}}A_{\text{t}}\left(\cos(\psi_{0,50})+f_0\sin(\psi_{0,50})\right); \quad (14)$$

c_0 famous for the largest factor constant with the dimension of speed, m/s:

$$c_0=v_{aw}\cos(\xi) \quad (15)$$

Marking for the convenience of recording $v_e\cos(\psi_{0,50})$ via the v and sharing both parts of (14) on the b_0, we will have

$$\frac{M}{b_0}\frac{dv}{dt}=a_0^2-(v-c_0)^2, \quad (16)$$

where a_0^2 known constant with the dimension of speed, (m/s)^2:

$$a_0^2=F_0/b_0. \quad (17)$$

VI. Results of Solution

a) Mathematical Modeling of The Speed of Sliding and Passed Way of the Car with the Roller Coaster

Separating the variables in the equation (12), after transformation, we obtain [14, 15]:

$$\frac{b_0}{M}dt=\frac{dv}{a_0^2-(v-c_0)^2},$$

or, having in mind that $d(v-c_0)=dv$,

$$\frac{b_0}{M}dt=\frac{d(v-c_0)}{a_0^2-(v-c_0)^2}, \quad (18)$$

Taking the integrals of rational functions of both parts of the last equality, we have:

$$\frac{b_0}{M}t=\int_{v_0}^{v}\frac{d(v-c_0)}{a_0^2-(v-c_0)^2}. \quad (19)$$

The right side of the last equality is a table integral of the rational of functions of the form [17]:

$$\int\frac{dx}{a^2-x^2}=\frac{1}{2a}\ln\frac{a+x}{a-x}, \text{ when } |x|<a\neq 0.$$

In accordance with this, the right-hand part of the expression (19) after the transformations are presented in the form of:

$$\frac{b_0}{M}t=\frac{1}{2a_0}\ln\left|\frac{a_0-c_0+v}{a_0+c_0-v}\right|\Bigg|_{v_0}^{v}. \quad (20)$$

Hence, substituting the limits of integration v and v_0 (initial speed of the car from rolling with slides), after transformations we obtain:

$$\frac{b_0}{M}t = \frac{1}{2a_0}\ln\left|\frac{a_0 - c_0 + v}{a_0 + c_0 - v}\frac{a_0 + c_0 - v_0}{a_0 - c_0 + v_0}\right|. \quad (21)$$

Denoting the second factor of expression under the sign of natural logarithm, as the attitude of well-known for the largest data,

$$d_0 = \frac{a_0 + c_0 - v_0}{a_0 - c_0 + v_0}, \quad (22)$$

rewrite (21) in the form of:

$$2a_0\frac{b_0}{M}t = \ln\left|\frac{a_0 - c_0 + v}{a_0 + c_0 - v}d_0\right|.$$

Hence, the dependence of the time t on the speed of sliding carriage in portable moving $v(t) = v_e(t)$ has the form:

$$t = \frac{1}{\alpha}\ln\left|\frac{a_0 - c_0 + v}{a_0 + c_0 - v}d_0\right|. \quad (23)$$

where α – known in size constant, which the dimension of the 1/s:

$$\alpha = 2a_0\frac{b_0}{M}. \quad (24)$$

Potentiating of the expression (23) and omitting intermediate mathematical calculations, we obtain:

$$v(e^{\alpha t} + d_0) = (a_0 + c_0)e^{\alpha t} - (a_0 - c_0)d_0$$

From here after a number of transformations find the projection of the velocity $v(t)$ for from rolling of a wagon according to the profile of slides from the impact of the projection of the force of gravity and the wind on the longitudinal axis of the

$$v = \frac{a_0(e^{\alpha t} - d_0) + c_0(e^{\alpha t} + d_0)}{e^{\alpha t} + d_0} = c_0 + \frac{a_0(e^{\alpha t} - d_0)}{e^{\alpha t} + d_0}.$$

Convert the last expression

$$v = c_0 + \frac{a_0e^{\alpha t} - a_0d_0 + a_0d_0 - a_0d_0}{e^{\alpha t} + d_0} = c_0 + \frac{a_0(e^{\alpha t} + d_0) - 2a_0d_0}{e^{\alpha t} + d_0}.$$

Here we finally obtain the projection of the velocity of sliding the car to the axis Ox (see Fig. 1)

$$v(t) = c_0 + a_0 - \frac{2a_0d_0}{d_0 + e^{\alpha t}}.$$

From the last formula, given that $v = v_e\cos(\psi_{0,50})$ (see equation (16)), find the speed of the car $v_e(t)$ in the direction of the slide of the hill on the axis O_1x_1

$$v_e(t)_{50} = \frac{1}{\cos(\psi_{0,50})}\left(c_0 + a_0 - \frac{2a_0d_0}{d_0 + e^{\alpha t}}\right). \quad (25)$$

Analyzing (25) note that the dependence of the speed of the car from rolling on the profile of the hills in time $v_e(t)$ exponential: the car can quickly get up to speed on the profile of the hill and continue almost uniformly, if you do not take any practical measures for its reduction, for example, do not use the brake devices. In addition, the speed of the car when rolling on the profile of the hills in the main depends on the angle of descent slides ψ_0 to the horizontal axis (Ox axis). The more the angle of descent ψ0, the less the denominator of (25) (i. e. $\cos(\psi_0)$), and the more the $v_e(t)$. From this it is clear that the required speed of the car to the drain of the roller coaster at constant external power factors should be found by modifying the profile of the roller coaster.

Again separating the variables (25) given that $v_e = \dfrac{dx_1}{dt}$, we will have

$$dx_1 = \frac{1}{\cos(\psi_{0,50})}\left((c_0 + a_0)dt - 2a_0 d_0 \frac{dt}{d_0 + e^{\alpha t}}\right).$$

Integrating the obtained expression in the range from 0 to *t*, we have:

$$x_1 = \frac{1}{\cos(\psi_{0,50})}\left((c_0 + a_0)t - 2a_0 d_0 \int_0^t \frac{dt}{d_0 + e^{\alpha t}}\right). \tag{26}$$

The second member of the right side of the last equality is a table integral containing the Exhibitor in the following form [17]:

$$\int \frac{dx}{a + be^{\alpha x}} = \frac{x}{a} - \frac{1}{\alpha a}\ln\left|a + be^{\alpha x}\right|,\ a \neq 0,\ \alpha \neq 0.$$

Introducing (26) in accordance with the last integral and substituting the limits of integration (0, *t*) of this ratio, we finally obtain the:

$$x(t)_{50} = \frac{1}{\cos(\psi_{0,50})}\left((c_0 + a_0)t - 2a_0\left(\frac{1}{\alpha}\ln\left|\frac{d_0 + e^{\alpha t}}{d_0 + 1}\right| - t\right)\right). \tag{27}$$

From this it is clear that the distance traveled (path) of the car depending on the time describes the dependence of (27): with the increase of the time of sliding *t* car distance *x*(*t*) is almost increases linearly.

In the particular case, from the expression (27) at $t = 0$, we have $v = 0$, $x = 0$.

Passing to the limit as t→∞, you can get the maximum speed of the car

$$v_{\max} = \frac{1}{\cos(\psi_{0,50})}(c_0 + a_0). \tag{28}$$

If the first specialized site slides with a slope of not steeper than 50 ‰ has a length of more than 50 m, this area may consist of two core elements, and the second element can be made with a slope of 30 ÷ 35 ‰. In this case the speed of the roll of the car and the path to find (25) and (27) in the form of:

$$v_{e,30} = \frac{1}{\cos(\psi_{0,30})}\left(c_0 + a_{0,30} - \frac{2a_{0,30}d_{0,30}}{d_{0,30} + e^{\alpha_{30}t}}\right); \tag{25, a}$$

$$x(t)_{30} = \frac{1}{\cos(\psi_{0,30})}\left((c_0 + a_{0,30})t - 2a_{0,30}\left(\frac{1}{\alpha_{30}}\ln\left|\frac{d_{0,30} + e^{\alpha_{30}t}}{d_{0,30} + 1}\right| - t\right)\right), \tag{27, a}$$

where $b_{0,30}$, $a_{0,30}$, $d_{0,30}$ and $\alpha_{0,30}$ according to (14), (17) (22) and (24) are constant, determined from the following relationships

$$b_{0,30} = 0{,}5c_w\rho_w A_t\left(\cos(\psi_{0,30}) + f_0\sin(\psi_{0,30})\right); \tag{14, a}$$

$$a_{0,30}^2 = F_{0,30} / b_{0,30} \tag{17, a}$$

taking into account the fact that, according to the (13),

$$F_{0,30} = Mg\big(\sin(\psi_{0,30}) - f_0 \cos(\psi_{0,30})\big) - f_{ск0} F'_{rwy} ; \tag{13, a}$$

$$d_{0,30} = \frac{a_{0,30} + c_0 - v_{0,30}}{a_{0,30} - c_0 + v_{0,30}}, \tag{22, a}$$

taking into account the fact that $v_{0,30}$ the initial velocity of the car from rolling on the second core element of the roller coaster at $t = t_1$, which is equal to the finite speed of the first core element $v_{0,50}$ for $t = t_1$, $v_{0,30} = v_{0,50}$.

$$\alpha_{30} = 2a_{0,30} \frac{b_{0,30}}{M} . \tag{24, a}$$

Analyzing the obtained results of the research, we note that the speed of sliding cars $v_{e,30}(t)$ at $t = t_2$ (see. (25, a)), as the speed of the input $v_{вх}(t)$ at the first brake position (I BP), obtained with the method of selection of rational values of the gradient of slides $\psi_{0,50}$ and $\psi_{0,30}$ should be less than the maximum speed input on I BP, i. e. $v_{e,30}(t) = v_{вх}(t) < [v_t] = 8{,}5$ m/s [11, 18]. This must be a condition to the speed of sliding the car at the end of the second section of the $v_{e,30}(t)$ at $t = t_2$ would be less than the speed at the end of the first section of the $v_{e,50}(t)$ for $t = t_1$, i. e. $v_{e,30}(t) = v_{e,50}(t)$.

Using the method of binding decisions of the piecewise-linear equations on the basis of the dimensionless delayed a single function Heaviside [19], the speed of movement of the subway car and the path traversed by any stretch of the slides within a profile up to the moment of braking, s:

$$v_e(t) = \frac{1}{\cos(\psi_0)}\left(c_0 + a_0 - \frac{2a_0 d_0}{d_0 + e^{\alpha t}}\right)\sigma_0(t - t_i); \tag{29}$$

$$x(t) = \frac{1}{\cos(\psi_0)}\left((c_0 + a_0)t - 2a_0\left(\frac{1}{\alpha}\ln\left|\frac{d_0 + e^{\alpha t}}{d_0 + 1}\right| - t\right)\right)\sigma_0(t - t_i), \tag{30}$$

where $\sigma_0(t - t_i)$ (where $t_i \in (t_1, t_2)$) dimensionless delayed single Heaviside function, allowing you to imagine the time t one analytical expression, fit for any value of the coordinates of the t_i in the interval of $0 \le t \le t_i$, and $\sigma_0(t - t_i) = 0$ for $t_i < t$.

Thus, using the principle of the D'Alembert mechanics, method of separation of re-variables, the table integrals of rational functions and integral containing the Exhibitor, as well as linking method of solutions of piecewise-linear functions are deduced analytical formulas for determining the speed of the car from rolling on the profile of slides $v_e(t)$ and the distance $x(t)$ with the passage of time.

VII. Conclusions

a) Obtained on the basis of classical provisions of theoretical mechanics, computational and mathematical models rolling off the wagon with the sorting slides under the influence of the projection of the forces of gravity and the force of resistance to the wind on the longitudinal axis of the allowed us to determine the speed of the car from rolling $v_e(t)$ and the distance $x(t)$ on the first main area of the slide with the passage of time. In the particular case, the obtained analytical expressions of the dynamics of rolling the car will find the ultimate formula to determine the speed and distance covered, or only from the impact of the projection of the force of gravity on longitudinal axis, or from the effects of only the wind.

b) Analysis of the results of analytical researches allowed to establish, that the speed of the car when скатывании on the profile of the hills in the main depends on the angle of descent slides ψ_0 to the horizon. The more the angle of descent ψ_0, the less cos (ψ_0) and the more than $v_e(t)$. From this it is clear that the required speed of the car to the drain of the roller coaster at constant external power factors should be found by modifying the profile of the roller coaster.

c) The results of analytical studies of the dynamics of rolling of a wagon according to the first high-speed site slides can be used for all other parts of the slide with account of peculiarities of deceleration forces on these sites.

The distinctive feature of (novelty) derived analytical formulas for the speed of the car from rolling on the high-speed road rolling down the hump is in the representation of the forces of aerodynamic resistance of the wind depending on the speed of the car from rolling ve(t) on the profile of slides and correct accounting of the forces of resistance, resulting in the movement of the car. The obtained results of the research, are available for designers sorting slides, are a step forward in solving this problem.

The advantage of (importance) of this methodology is the possibility to construct a mathematical model of the car from rolling with sorting slides taking into account the dependence of the force of aerodynamic resistance of the wind to the speed of the car from rolling ve(t) , speed and direction of air flow (ξ).

In the perspective of the received results of researches can be used in solving the technical problem of definition of rational geometric parameters (profile ψ0) slides and kinematics characteristics of the car (v(t), x(t)) at its from rolling with sorting the roller coaster.

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