Analytical Investigation of Wagon Speed and Traversed Distance during Wagon Hump Rolling under the Impact of Gravity Forces and Head Wind

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Abstract

The article gives an account of the results of construction of mathematical modeling of the wagon rolling down the hump under the impact of gravity and head wind aerodynamic drag forces that made it possible to derive analytical formulas for determining wagon speed and traversed distance during rolling down the first profile hump section. The results of the analytical research of dynamics of wagon rolling down the first high-speed hump section can be used for all the rest hump sections taking into account the specifics of braking forces on these sections. The novelty of the derived formulas of wagon speed rolling down high-speed hump section is in presenting head wind aerodynamic drag force as depending on the wagon speed rolling down hump profile and speed and direction of air flow with proper allowance for resistance forces arising during wagon movement.

Index terms— speed and direction of air flow, head wind, aerodynamic drag force, fundamental law of dynamics for wagon transportation motion, separation of variabl

1 Formulation of a Problem

nalysis of the literature **??**1 ? 5] shows that the dynamic model of the car from rolling down the hill with built correctly. The recommended formulas for determination of the distance traveled in ?????????? car with slides [1] do not take into account the motion of the car on the profile of the hills in pure rolling wheels of wheel pairs of carriages and the rolling elements of bearing on the inner and outer rings, if not believe that such a movement indirectly taken into account the concept of the main specific resistance w?, which found empirically. Estimated rate of dissolution of the car with the roller coaster is found by the formula freely falling body, which does not comply with the physical sense of the problem being solved, as it should be defined as a result of solution of a differential equation of motion of the car, how this is done in **??**6 ? 9]. The work is not yet covers the solution of a number of practical problems getting the car to the hill.

Proceeding from this, in [10, **??**1] shows accounting moments of friction (net rolling wheel on the rail thread and in the bearings ???????? nodes front and rear bogies of a wagon with their subsequent replacement of contingent slip friction. It was noted that, if the active force in the form of the projection of the force of gravity and the forces of aerodynamic drag to the direction of rolling the car, acting on the car, the more marginal the friction force at the same time with rolling it is also possible movement of wheels on the rail lines.

Thus, up to the present time out of mind researchers were missed construction of the model of the car from rolling down the hill with in strict accordance with the provisions of the classical theoretical mechanics and, accordingly, the definition of speed and covered distance from rolling on the profile of the hills in the counter and/or wind.

2 II.

3 Man-Made Assumption

Consider the General case, when the car with the sorting slides forward with a specified initial velocity v0 (usually 4÷5 km/h or 1.1 ÷ 1,38 m/s, and the maximum value of 9 km/h or 2.5 m/s). When wagon rolling down the hump with the roller coaster car will be face-the impacts are mostly external forces in the form of the force of gravity of the car with or without cargo G and the forces of aerodynamic drag (wherey r x r r F F F w w w , ? ? ?

). Distribution of the forces of gravity of the car body with a weight on the front and rear bogie depend on the technology of placing of cargo (symmetrically, or not symmetric about the axes of symmetry of the car) on the car.

Let the car roll off the sorting slides forward with portable speedw v v v e = =

(unknown value) [12].

Wind speed in relation to the top of the hill (the earth) (i. e., the absolute velocity of the particles of air)

4 + =

,

where w a v . is the absolute velocity of a particle air (wind speed);) ? cos(0 w e x x ex v v v v = = = , (**2**)

Taking into account the fact that ? (or ? 0) the tilt angle of the hill to the horizontal axis (axis Ox); We believe that the relative velocity of the particles of air (wind speed) w . r v is located on a horizontal plane H and is directed at an angle ? (or ? 0) to the horizontal axis (axis Ox), and drive speed (the speed of the car)w v v v e = =

in the vertical plane V and is directed at an angle of descent of rolling down ? (or ? 0) to the horizontal axis (axis Ox).

According to the theorem of addition of velocities at the complex motion [14,15], we write Keep in mind that if the wind direction is opposite to the direction of movement of the car (i. e. wind, see Fig. 1), the vector equation (1), in accordance with rule subtract of vectors [12,14], can be written as:) (w w r ex a v v v ? + = , where w w a ex r v v v ? = . (**3**)

Projection (3) on the axis Ox when the wind is of the form [12,14]:) ? cos( ) ? cos(w 0 . w a e x r v v v ? = , (**4**)

where ? the angle between the vector w a v (the absolute velocity of a particle air (wind speed)and longitudinal axis Ox, rad.

In accordance with the expression (4) the force of aerodynamic drag)2 0 t w w) ? cos( ) ? cos(? 5 , 0) (aw e e rwx v v A c v F ? = ? ; (5) the axis Oy ( ) .) ? sin(? 5 , 0 2 0 b w w . rw y rw v A c F = ? (5, ?)

In the last formulas [14]: c w ? dimensionless experimental coefficient resistor of air, depending on the shape of the body and how it is directed at the movement (usually take depending on the shape of the surface in the range from 0,55 up to 1.

5 Formation of a Design Model of the Car from Rolling

A simplified model of the car from rolling down the hill with, taking into account the friction rolling wheels car with a slip, take the model shown in Fig. 2, and the calculation model -in Fig. 3)) ? sin( ) ( ) ? cos( ) (50 , 0 w 50 , 0 0 r fric × ? + = e r e v F G f v F . (**8**)

where f 0 is a conditional (or) the coefficient of friction [11, 14,15]:; tq bn b 0 b r r w 0 n n k r f n r f n f r + = (8, ?)

f sl0 is the coefficient of sliding friction ridges of wheels on the rails (usually take f sl0 = 0,25); Introducing the concept of "shift" ? F sh. and "restraint" ? F res. of forces, due to the active and all jet forces, we obtain [5 -8]:) ? sin(0 sh. G G F x ? = = ; (**9**)) ( ) ? cos( ) ( ) (torm. 0 w res. e e x r e ? F v F v F + ? = .

Rewrite the last expression with the account of (**??**) and (8),) ? sin( ) ( ) ? cos( ) ? cos( ) ( ) (w sl0 0 w 0 0 0 w res. y r e x r e x e ? F f v F G f v F v F ? + ? ? ? ? ? ? ? ? ? ? + + × ? =

6 and after transformations

In (8, a), and the following designations are accepted: n w number of wheels of the cart, pieces. (n w = 8); f r the friction coefficient of friction, since this ratio is equivalent to the shoulder of a friction pair of katreatment (the wheel on a track f r = 5×10 -6 , steel hardened steel f r = 1×10 -6), r r wheel radius, equal to the freight car 0,475 m; n bn = 8 number bucks nodes in the cart, pieces.; f ?0 the friction coefficient of the rolling elements in the rings of the bearing (usually take 0,001×10 -3), m; n tq the total quantity of the rolling elements, which include the load in each of the bearing, pieces.; k is a constant factor, taken depending on the location of the rolling elements and type of bearings quality of (for the calculation shall take k = 4,6) [16]; n b number in the bearings bucks nodes in the cart, pieces.

(n b = 16); r ? outer radius of the inner ring of the bearing raceways, m (0,079 m).

7 (

).) ? cos( ) ? sin( ) ? cos( ) ( ) (sl0 0 0 0 0 0 w res. rwy e x e ? F f G f f v F v F ? + + + ? = (10)

The condition of the car from rolling on the first main area of the slide with a slope of not steeper than 50 ? at the length of this section of up to 50 m is [10]) (res. sh. e ? ? v F F » . (**11**)

Hence, the excess forces) (res. sh. ?,50 e ? ? v F F F ? = ?

. This force arises in the first specialized site of the hill and is the driving force. It causes the slide car given the force of gravity G with a velocity v e,50 (t) and the acceleration of the a 50 (t). The force depends on the foundations of the Mr. from the angle of rolling hills and, to some extent, from the coefficient of friction ridges of wheels on the rails, as well as the condition of rolling bearings in ???????? nodes carts. Therefore, in order to ensure the movement of the car at the end of the first specialized site slides with the velocity v e,50 (t) less than the speed of input v ew? (t) at the first brake position (I BP), i. e. v e (t) < v ew? (t), is sufficient selection of rational values as the basic geometric parameter slides.

IV.

8 Methods of Solution

The formation of dynamic and constructing a mathematical model of the car from rolling down the hill with is based on classical concepts and provisions of theoretical and applied mechanics (for example, the theorem of addition of velocities under complex movement, roll, slip, slide with rolling, communications, the reaction of communication, the principle clear constraints, the basic law of the dynamics of the absolute motion (the principle of the D'Alembert); basic concepts of differential and integral calculus **??**12 -14]).

V.

9 Methods of Constructing the Mathematical Model of the Car from Rolling Down the Hill with

We shall take into account, that the car rolled down the hills steadily so a portable acceleration of the where M is the mass of the car with the load, kg.

Transforming the last equation with account of (9) and (10) and the fact that G = Mg, for the first specialized site slides with a slope of not steeper than 50 ? at the length of this section of up to 50 m we obtain:( ) (

).) ? sin( ) ? cos( ) ( ) ? cos( ) ? sin(g 50 , 0 0 50 , 0 w w sl0 50 , 0 0 50 , 0 f v F F f f M dt dv M e x y r + ? ? ? ? ? ? =

Imagine the last expression in the form of:2 0 50 , 0 0 0)) ? cos((c v b F dt dv M e ? ? = , (**12**)

where F 0 is the difference famous of the largest driving forces and the forces of resistance, the attached to the system "wagon -cargo", N:( ) ,) ? cos( ) ? sin(g sl0 50 , 0 0 50 , 0 0 rwy F f f M F ? ? ? = (13)

b 0 famous for the largest constant factor in (5) for the location of the forces of aerodynamic drag x F w ? , N/(m/s) 2 : Substituting (9) and (**??**0) of the basic law of the dynamics for a portable car (or the principle of D'Alembert) in the coordinate form **??**12 -15] we have:

(

)) ? sin( ) ? cos(? 5 , 0 50 , 0 0 50 , 0 t w w 0 f A c b + = ; (**14**)

c 0 famous for the largest factor constant with the dimension of speed, m/s:) ? cos(0 aw v c = . (**15**)

Marking for the convenience of recording) ? cos(50 , 0 e v via the v and sharing both parts of (14) on the b 0 , we will have2 0 2 0 0) (c v a dt dv b M ? ? = , (**16**)

where2 0

a known constant with the dimension of speed, (m/s) 2 :0 0 2 0 / b F a = . (**17**)

VI.

10 Results of Solution a) Mathematical Modeling of The Speed of Sliding and Passed Way of the Car with the Roller Coaster

Separating the variables in the equation (12), after transformation, we obtain [14,15]:2 0 2 0 0) (c v a dv dt M b ? ? = , or, having in mind that dv c v d = ?) (0 , 2 0 2 0 0 0) ( ) (c v a c v d dt M b ? ? ? = , (**18**)

Taking the integrals of rational functions of both parts of the last equality, we have:? ? ? ? = v v c v a c v d t M b 0 2 0 2 0 0 0) ( ) (. (**19**)

The right side of the last equality is a table integral of the rational of functions of the form [17]: Hence, substituting the limits of integration v and v 0 (initial speed of the car from rolling with slides), after transformations we obtain:0 0 0 0 0 0 0 0 0 0 0 0 ln 2 1 v c a v c a v c a v c a a t M b + ? ? + ? + + ? = . (21)

Denoting the second factor of expression under the sign of natural logarithm, as the attitude of wellknown for the largest data, , 0 0 0 **??**1) in the form of:0 0 0 0 v c a v c a d + ? ? + = (22) rewrite (0 0 0 0 0 0 0 ln 2 d v c a v c a t M b a ? + + ? = .

Hence, the dependence of the time t on the speed of sliding carriage in portable moving v(t) = v e (t) has the form:0 0 0 0 0 ln 1 d v c a v c a t ? + + ? = ? . (**23**)

where ? -known in size constant, which the dimension of the 1/s:M b a 0 0 2 ? = . (**24**)

Potentiating of the expression (23) and omitting intermediate mathematical calculations, we obtain:0 0 0 0 0 0) ( ) ( ) (d c a e c a d e v t t ? ? + = + ? ?

From here after a number of transformations find the projection of the velocity v(t) for from rolling of a wagon according to the profile of slides from the impact of the projection of the force of gravity and the wind on the longitudinal axis of the Analyzing (25) note that the dependence of the speed of the car from rolling on the profile of the hills in time v e (t) exponential: the car can quickly get up to speed on the profile of the hill and continue almost uniformly, if you do not take any practical measures for its reduction, for example, do not use the brake devices. In addition, the speed of the car when rolling on the profile of the hills in the main depends on the angle of descent slides to the horizontal axis (Ox axis). The more the angle of descent 0, the less the denominator of (25) (i. e. cos(0)), and the more the v e (t). From this it is clear that the required speed of the car to the drain of the roller coaster at constant external power factors should be found by modifying the profile of the roller coaster. The second member of the right side of the last equality is a table integral containing the Exhibitor in the following form [17]:x x be a a a x be a dx ? ? + ? ? = + ? ln 1 , 0 ? a , 0 ? ? .

Introducing (26) in accordance with the last integral and substituting the limits of integration (0, t) of this ratio, we finally obtain the:. 1 ln 1 2) ( ) ? cos(1) (0 0 0 0 0 50 , 0 50 ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? + + ? ? + = ? t d e d a t a c t x t (27)

From this it is clear that the distance traveled (path) of the car depending on the time describes the dependence of (27): with the increase of the time of sliding t car distance x(t) is almost increases linearly.

In the particular case, from the expression (27) at t = 0, we have v = 0, x = 0.

Passing to the limit as t??, you can get the maximum speed of the car) ( ) ? cos(1 0 0 50 , 0 max a c v + = . (**28**)

If the first specialized site slides with a slope of not steeper than 50 ? has a length of more than 50 m, this area may consist of two core elements, and the second element can be made with a slope of 30 ÷ 35 ?. In this case the speed of the roll of the car and the path to find (25) and (27) in the form of: Thus, using the principle of the D'Alembert mechanics, method of separation of re-variables, the table integrals of rational functions and integral containing the Exhibitor, as well as linking method of solutions of piecewise-linear functions are deduced analytical formulas for determining the speed of the car from rolling on the profile of slides v e (t) and the distance x(t) with the passage of time.

11 VII.

Conclusions a) Obtained on the basis of classical provisions of theoretical mechanics, computational and mathematical models rolling off the wagon with the sorting slides under the influence of the projection of the forces of gravity and the force of resistance to the wind on the longitudinal axis of the allowed us to determine the speed of the car from rolling v e (t) and the distance x(t) on the first main area of the slide with the passage of time. In the particular case, the obtained analytical expressions of the dynamics of rolling the car will find the ultimate formula to determine the speed and distance covered, or only from the impact of the projection of the force of gravity on longitudinal axis, or from the effects of only the wind. b) Analysis of the results of analytical researches allowed to establish, that the speed of the car when ?????????? on the profile of the hills in the main depends on the angle of descent slides to the horizon. The more the angle of descent , the less cos (0) and the more than v e (t). From this it is clear that the required speed of the car to the drain of the roller coaster at constant external power factors should be found by modifying the profile of the roller coaster.

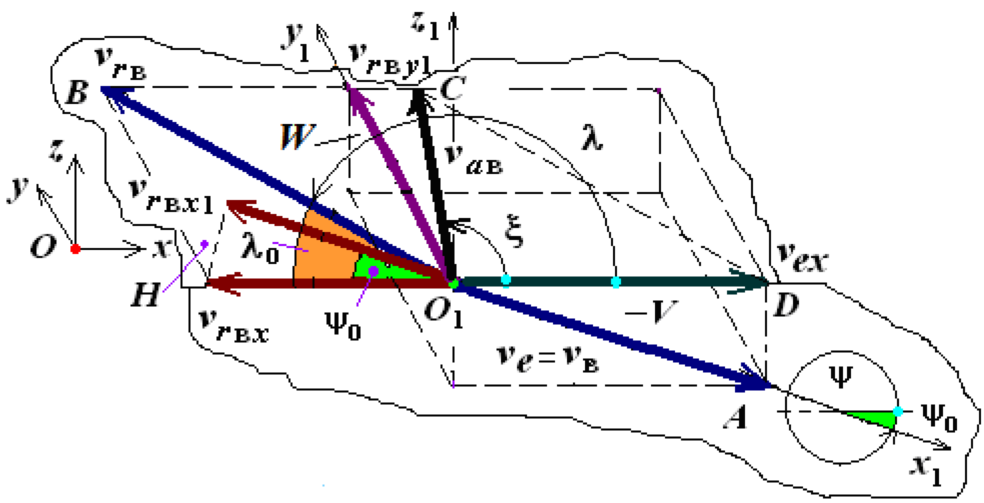
c) The results of analytical studies of the dynamics of rolling of a wagon according to the first high-speed site slides can be used for all other parts of the slide with account of peculiarities of deceleration forces on these sites.

The distinctive feature of (novelty) derived analytical formulas for the speed of the car from rolling on the high-speed road rolling down the hump is in the representation of the forces of aerodynamic resistance of the wind depending on the speed of the car from rolling ve(t) on the profile of slides and correct accounting of the forces of resistance, resulting in the movement of the car. The obtained results of the research, are available for designers sorting slides, are a step forward in solving this problem.

The advantage of (importance) of this methodology is the possibility to construct a mathematical model of the car from rolling with sorting slides taking into account the dependence of the force of aerodynamic resistance of the wind to the speed of the car from rolling ve(t) , speed and direction of air flow (?).

In the perspective of the received results of researches can be used in solving the technical problem of definition of rational geometric parameters (profile 0) slides and kinematics characteristics of the car (v(t), x(t)) at its from rolling with sorting the roller coaster. [1]

Figure 1:



1

Figure 2: Figure 1 :

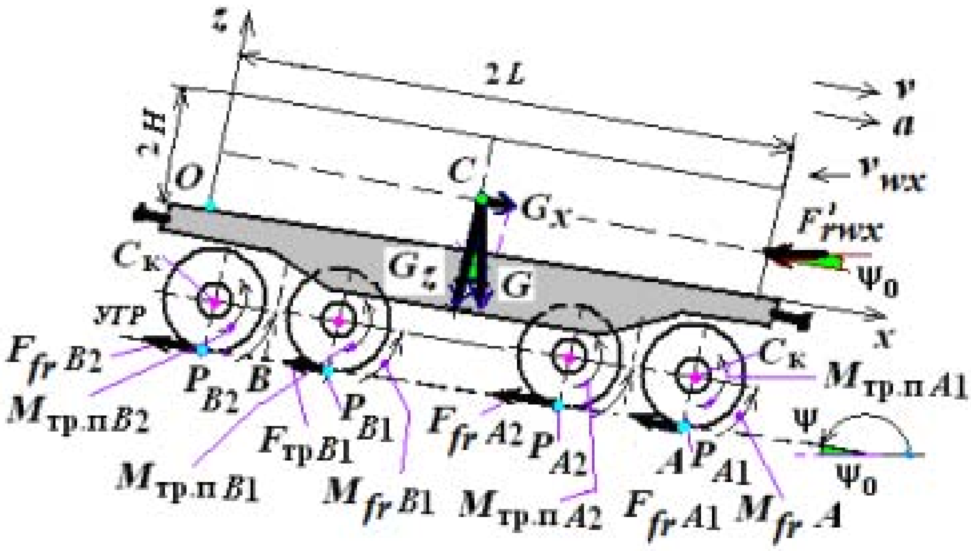


Figure 3:

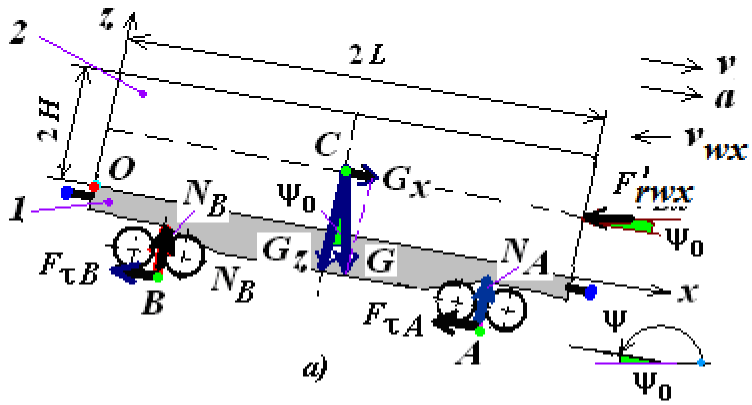


Figure 4: F

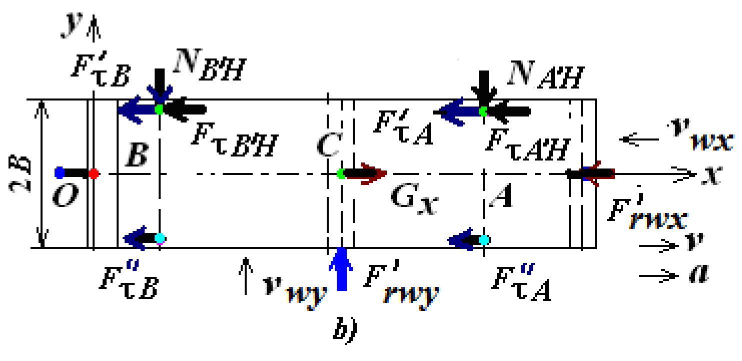


Figure 5: (

taking into account the fact that, according to the (13), **??**) taking into account the fact that v 0,30 the initial velocity of the car from rolling on the second core element of the roller coaster at t = t 1 , which is equal to the finite speed of the first core element v 0,50 for t = t 1 , v 0,30 = v 0,50 . (24, a)

Analyzing the obtained results of the research, we note that the speed of sliding cars v e,30 (t) at t = t 2 (see. (25, a)), as the speed of the input v ew? (t) at the first brake position (I BP), obtained with the method of selection of rational values of the gradient of slides and should be less than the maximum speed input on I BP, i. e. v e,30 (t) = v ew? (t) < [v t] = 8,5 m/s **??**11,18]. This must be a condition to the speed of sliding the car at the end of the second section of the v e,30 (t) at t = t 2 would be less than the speed at the end of the first section of the v e,50 (t) for t = t 1 , i. e. v e,30 (t) = v e,50 (t).

Using the method of binding decisions of the piecewise-linear equations on the basis of the dimensionless delayed a single function Heaviside [19], the speed of movement of the subway car and the path traversed by any stretch of the slides within a profile up to the moment of braking, s:

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