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Optimum Design of Compound Pressure Vessel

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OPTIMUMDESIGNOFCOMPOUNDPRESSUREVESSEL

Strictly as per the compliance and regulations of:

Optimum Design of Compound Pressure Vessel

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I. Introduction

Compound cylinders are widely used in the field of high pressure technology such as hydraulic presses, forging presses, power plants, gas storages, chemical and nuclear plants, military applications etc. To enhance load bearing capacity and life of multilayer pressure vessels, different processes such as shrink fit and autofrettage are usually employed. Shrink fit increases load capacity. Many researchers have focused on methods to extend lifetimes of vessels. Majzoobi et al. have proposed the optimization of bi-metal compound cylinders and minimized the weight of compound cylinder for a specific pressure [1]. The variables were shrinkage radius and shrinkage tolerance. Patil S. A. has introduced optimum design of two layer compound cylinder and optimized intermediate, outer diameter and shrinkage tolerance to get minimum volume of two layer compound cylinders [2]-[3]. Hamid Jahed et al. have investigated the optimum design of a three-layered vessel for maximum fatigue life expectancy under the combined effects of autofrettage and shrink fit [4]. Miraje Ayub A. & Patil Sunil A. have found minimum volume of three-layer open type compound cylinder considering plane stress hypothesis [5]. Yang Qiu-Ming et al. have presented a simple and visual tool to calculate the residual stress and describe the distribution of residual stress for both the elastic-perfectly plastic model and the strain-hardening mode [6].

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II. Analytical Approach

As stated earlier, an elasto-plastic behavior has been designated in this work.

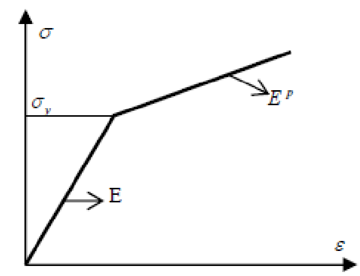

Figure 1 : Bi-linear Stress Strain Curve

The model, shown in fig. 1 is described as follows:

$$\sigma = \sigma_y + E^P \varepsilon \tag{1}$$

In which, σ is the effective stress, σ_y is the initial yield stress, E^p is the slope of the strain hardening segment of stress strain curve, and ε is the effective strain.

a) Tangential & Radial Stress Pattern

To observe the stress pattern in a compound cylinder, a sample cylinder with internal diameter a = 0.1m, interference radius c = 0.15m, and external radius b = 0.20m has been considered. Material Properties of this cylinder summarized in table I

Table 1 : Material Properties

Material	σ_y(MPa)	E(GPa)	υ
Steel	800	207	0.29

An outer cylinder with the internal diameter slightly smaller than the outer diameter of the main cylinder is heated and fitted onto the main cylinder. When the assembly cools down to room temperature a compound cylinder isobtained. In this process the main cylinder is subjected to an external pressure leading to a compressive radial stress at the interface. The outer cylinder or the jacket is subjected to an internal pressure leading to a tensile circumferential stress at the inner wall. Under this condition as the internal pressure increases the compression in the inner cylinder is first released and then only the cylinder begins to act in tension.

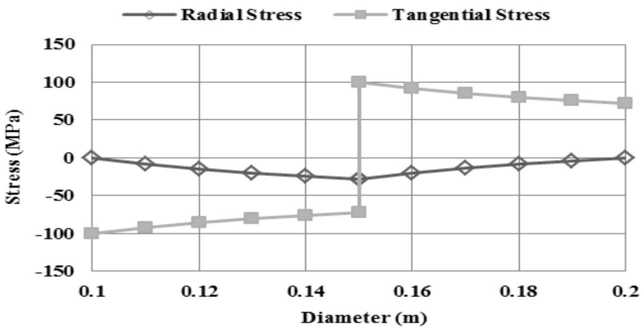

Figure 2 : Radial & Tangential Stress Distribution through the Walls of a Compound Cylinder

Though outer cylinder is fitted on the top of the inner cylinder, a compressive stress is developed in the near bore region. Here in the fig. 2, compressive stress is developed in the inner diameter while tensile stress occurs at the outer portion. Here the initial stress is compressive so if internal pressure is applied, then a tensile stress will be developed & this tensile stress even quite high, but the resultant stress not that high because of this compressive stress.

b) Comparison of Stress between Single cylinder & Compound Cylinder

By using Lame's equation for thick-walled cylinder, the stress pattern is obtained for both single cylinder & compound cylinder. If these two cylinders undergoes same internal pressure then the overall stress pattern will change, which is shown in Fig. 3, here the internal pressure is 250 MPa.

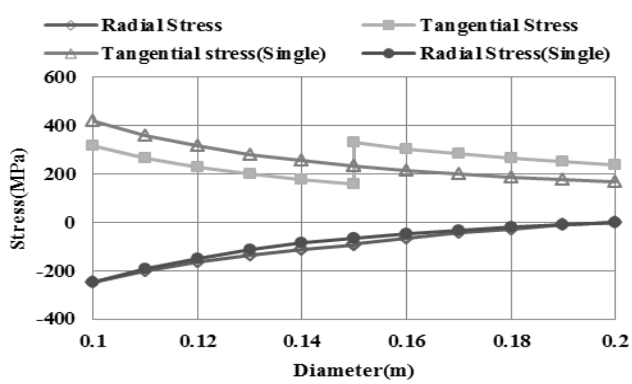

Figure 3 : Comparison of Stresses between Single & Compound Cylinder

Because of the compressive tangential stress at inner bore, the resultant tangential stress becomes lower in the compound cylinder than the original tangential stress developed at the single cylinder. Radial stress doesn't vary significantly after compounding. Radial stress doesn't vary significantly after compounding. Maximum stress occurs at the junction point of inner & outer cylinder rather than inner bore.

c) Comparison of Overall von Mises Stresses

According to the third strength theory, the equivalent stress is as follows [12]-[13]:

$$\sigma_{eq}^{III} = \sigma_t - \sigma_r \quad (2)$$

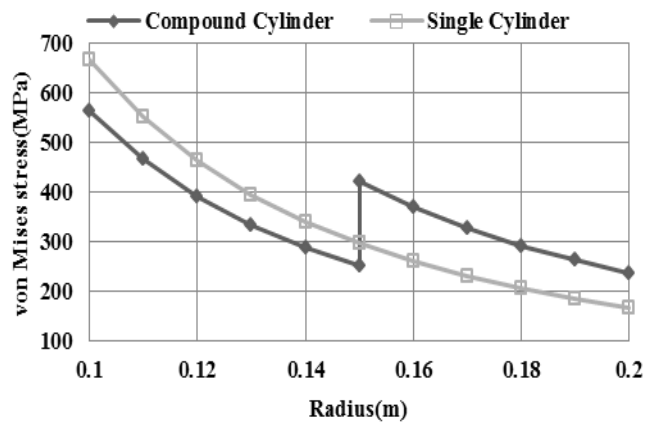

Figure 4 : Comparison of von Mises stresses between Single & Compound cylinder according to the third strength theory

The equivalent stress based on the fourth strength theory is as follows [12[-[13]:

$$\sigma_{eq}^{IV} = \sqrt{\frac{1}{2}\left[(\sigma_t - \sigma_r)^2 + (\sigma_r - \sigma_z)^2 + (\sigma_z - \sigma_t)^2\right]} \quad (3)$$

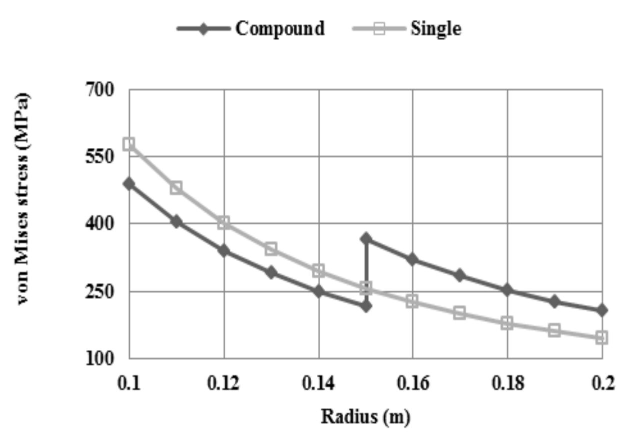

Figure 5 : Comparison of von Mises stresses between Single & Compound cylinder according to the third strength theory

Substituting $\sigma_z = (\sigma_t + \sigma_r)/2$ (Lame formula) Eq.(2), we obtain:

$$\sigma_{eq}^{IV} = \frac{\sqrt{3}}{2}(\sigma_t - \sigma_r) = \frac{\sqrt{3}}{2}\sigma_{eq}^{III} \quad (4)$$

Eq. (3) s the relation between the third & the fourth strength theory for a cylinder and also the relation between the Tresca's & Mises yield form.

As the fig. 4 & 5 suggest, maximum von Mises stress starts to decreases from inner bore to junction of two cylinders. Stress suddenly increases because of the contact pressure between two mating surface. It again decreases from junction point to outer portion.

i. *Sample calculation*

- In this case study, a=0.1m, b=0.2m, c=.15m, working pressure, Pworking = 250MPa.
- From third strength theory, MVS = 666 MPa
- From fourth strength theory, MVS =577 MPa
- From these two theories, it is observed that MVS for both is very similar. Indeed there is no significant variation between these two methods.
- Fourth strength theory is considered for all comparison.

III. Numerical Result

The compounding process may be simulated by Finite Element Method, making use of elastic-plastic analysis. It is possible to model the compound cylinder by applying pressure to the inner surface of the model, and then analyzing it for different interference radius & different shrinkage tolerance.

Using a 3D axisymmetric element available in *ANSYS V11* SP1*,* a finite element mesh of a cylinder with an inside radius 100 mm and outside radius of 200 mm was generated. An internal pressure of 250 MPa was applied. The stress distributions were evaluated in the compound cylinder. The von Mises equivalent stress was used in the subsequent analysis.

Structural Steel of flexible stiffness behavior has been used. Compound cylinder with the dimension; a = 0.1 m, b = 0.2 m, an elastic plastic material's model with σ = 800 MPa; Modulus of elasticity E = 207 GPa; ν = 0.29; were used for numerical modeling.

Here, the effects of following factors are considered in the compounding process. The considered factors are:

- Interference radius; 2. Value of k (b/a); 3. Different material at inner cylinder.

a) Change of Interference Radius

The cylinders were subjected to different internal pressures of 100, 200 and 300 MPa. From the numerical simulation with ANSYS software, the curve of the von Mises stress distribution was obtained for each working pressure. From the curve, the values of maximum von Mises stresses (MVS) were extracted. This stresses were then plotted against interference radius for each working pressure. The results are shown in fig. 6.

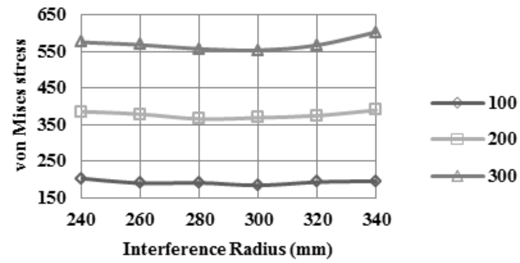

Figure 6 : Variation of MVS verses interference radius at different working pressure

It is observed that with the increase of interference radius, the MVS decreases slightly. Because at small interference radius, lower contact pressure is developed at the junction of the two cylinders. When the radius increases then the contact pressure increases and there is much compressive stress at the inner bore region resulting lower MVS. With the increase of interference radius much stress is developed at the outer cylinder; resulting a higher overall stress distribution.

The value of MVS is calculated for different interference radius for different working pressures.

Table 2 : Von Mises Stresses at Different Interference Radius for Different Working Pressure

Interference Radius (mm)	Pressure 100 (MPa)	Pressure 200 (MPa)	Pressure 300 (MPa)
240	203	386	576
260	190	379	569
280	191	366	558
300	184	369	554
320	193	375	568
340	195	390	604

From the table II, it is observed that the minimum values of MVS are at 280 mm & 300 mm. This means in those radius tangential stresses at inner wall of inner cylinder and inner wall of outer cylinder almost same.

For a constant value of interference radius (280 mm) the percent reduction of MVS is calculated for different working pressure.

Table 3 : Effect of Working Pressure at Maximum Von Mises Stress

Working Pressure (MPa)	MVS of single cylinder (MPa)	MVS of compound cylinder (MPa)	% reduction of MVS
100	233	191	18.03
200	467	366	21.63
300	701	558	20.40

From the table III, it is observed that the percent reduction of MVS is higher at moderate & higher working pressures.

b) Change of Shrinkage Tolerance

In the preceding case minimum MVS was found in between 280 mm & 300 mm of interference radius. Now the cylinder is subjected to a working pressure 200 MPa and interference radius is considered as 280 mm. From the curve, the values of maximum von Mises stress (MVS) are extracted. This stresses are then plotted against the shrinkage tolerance. The results are shown in fig. 8.

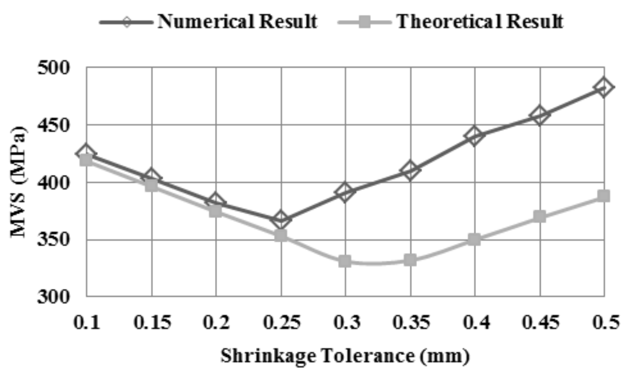

Figure 8 : Variation of MVS versus different shrinkage tolerance

It is observed that the developed MVS starts to decrease with the increase of the shrinkage radius. With the increase of the shrinkage radius, the compressive stress in the inner bore region increases & so that MVS decreases. But after a certain point with the increase of the shrinkage radius, MVS increases because then the stress of the inner bore of the outer cylinder increases. And also there is a close relation between the numerical result & theoretical result shown in the fig. 8.

c) Values of k

- Constant inner diameter

For constant inner radius (a = 0.1m) of the inner cylinder & constant interference radius, c = 280 mm cylinder with different k values (k=2, 2.5, 3) are subjected to same working pressure.

Here the working pressure remains constant at $P_{working}$ = 200 MPa

From numerical simulation, the value & the position of MVS are extracted. This MVS is then plotted against the diametral interference for each k value. The results are shown in fig. 9

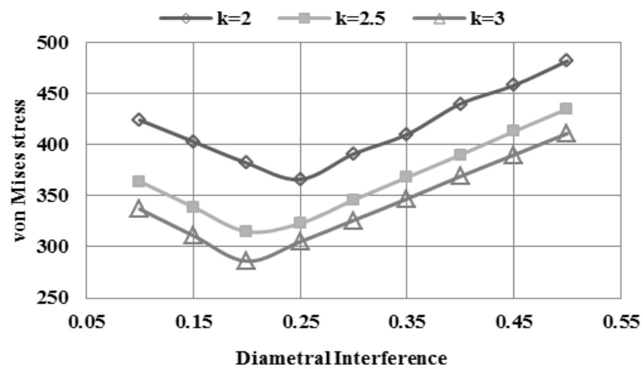

Figure 9 : Effect of k (b/a) on the diametral interference (constant inner radius = 0.1m)

As in fig. 9, suggests for different values of k, the minimum MVS obtains a higher value of thicker cylinder at lower diametral interference.

For a constant working pressure (Pworking = 200 MPa), percent reduction of MVS is calculated for different k values.

Table 4 : Effect Of K = (B/A) On Mvs At Constant Inner Radius

k	MVS of single cylinder (MPa)	MVS of compound cylinder (MPa)	% reduction of MVS
2.0	467	366	21.63
2.5	414	315	23.91
3.0	392	286	27.04

From table IV, it is observed that, percent reduction of MVS is higher for higher value of k. So, the process is more beneficial with the increase of the thickness of the cylinder.

- Constant outer diameter

In the preceding case the inner radius of the inner cylinder was kept constant. Now by assuming the outer radius (b= 0.2m) constant, cylinder with three k values of 2.0, 2.5, 3.0m are considered. From the numerical simulation, the curve of MVS distribution is obtained for each value. The developed MVS are then plotted against diametreal interference at fig. 10.

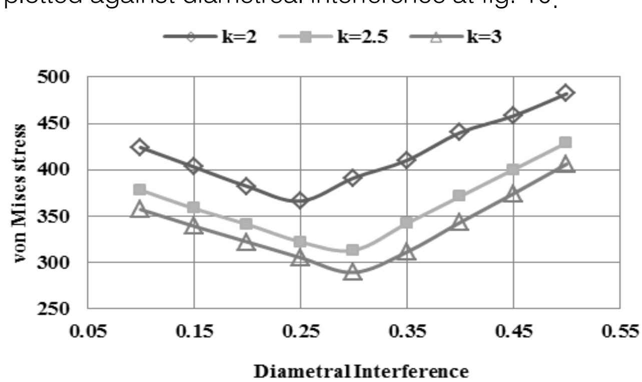

Figure 10 : Effect of k (b/a) on the diametral interference (constant outer radius = 0.2m)

As in fig. 10, suggest for different values of k, the minimum MVS obtains a higher value of thicker cylinder at higher diametral interference.

For a constant working pressure (Pworking = 200 MPa), percent reduction of MVS is calculated for different k values.

Table 5 : Effect Of K = (B/A) On Mvs at Constant Outer Radius

k	MVS of single cylinder (MPa)	MVS of compound cylinder (MPa)	% reduction of MVS
2.0	467	366	21.63
2.5	415	313	24.57
3.0	392	289	26.28

From the table V, it is noticed that for same k value, developed MVS is almost same though the value of inner & outer radius have been changed. So the optimum diametral interference depends on k value only. If the inner & outer radius are changed keeping the k value constant, then there will be no such change in diametral interference.

From above two graphs it can be said that, constant inner radius of inner cylinder is more effective than other one. Because in inner radius constant process the minimum von Mises stress found in lower diametral interference.

d) Different Material as inner cylinder

Now inner cylinder of the compound cylinder is replaced by cast iron cylinder and copper cylinder respectively. Here the working pressure remains constant at $P_{working}$ = 200 MPa From the numerical simulations, the curve of von Mises stress distribution is obtained for different material model. From the graph, the value & the position of the maximum von Mises stress are extracted and plotted against diametral interference.

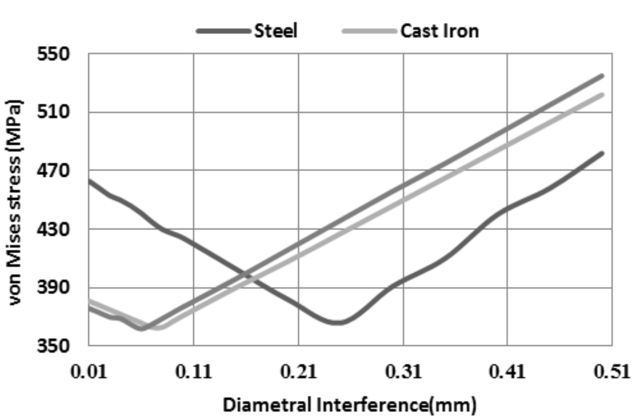

Figure 11 : Effect of different material on diametral interference

From the fig. 11, it is observed that cast iron & copper alloy shows minimum von mises stress at small diametral interference. This means if material of low tensile stress is compounded by a high tensile stress material, then at lower diamteral interference they show the desired effect.

Here cast iron & copper alloy are separately shown in fig. 12.

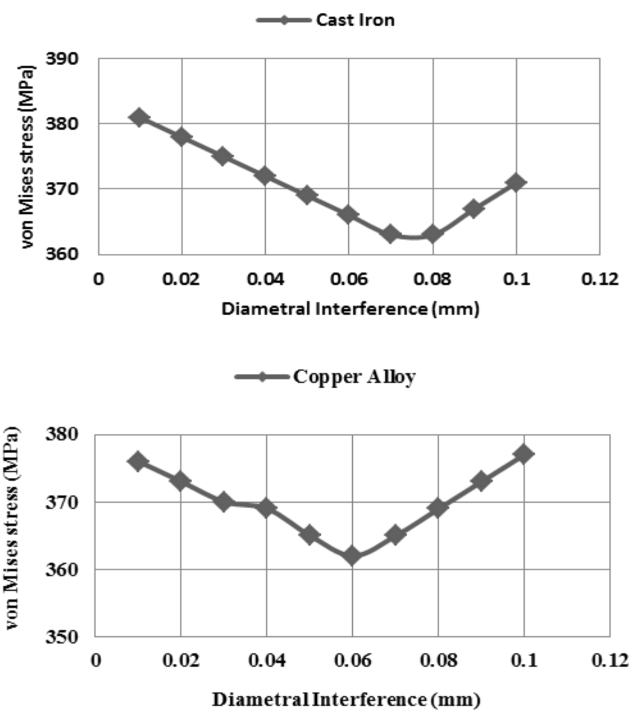

Figure 12 : Effect of different material on diametral interference (Cast iron & Copper alloy)

From fig. 12, it is noticed that, these two metals jacketed by steel work better at small diametral interference.

IV. Conclusion

The following decisions can be taken from the investigations mentioned in this paper:

1. In compound cylinder, maximum von Mises stress does not occur at smaller interference radius instead of that, it occurs at midway section.
2. Maximum stress depends on the interference radius as well as diametral interference.
3. The optimum diametral interference depends on k value only rather than inner & outer radius.
4. The thicker the cylinder the more will be the reduction in MVS.
5. Two element cylinder of different material reduces diametral interference more than two element cylinder of same material.

6. The size of a compound cylinder could be reduced by significant amount with respect to its equivalent single cylinder for the same working pressure.
7. The difference between a compound cylinder and its equivalent single cylinder becomes more significant at higher working pressures.

Appendix

Table : Nomenclature

Symbol	Meaning	unit
a	Inner radi	(m)
b	Outer radius	(m)
c	Interference ı	(m)
σ_y	Yield stress	(MPa)
σ_θ	Tangential s	(MPa)
σ_r	Radial stress	(MPa)
σ_z	Longitudinal	(MPa)

V. Acknowledgment

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