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Resonances of Elastic Spheroidal Bodies

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I. Introduction

In the paper are investigated resonances of prolate and oblate spheroidal bodies (entire and in the form of shells) by the three-dimensional and axis-symmetrical irradiation. By the three-dimensional irradiation for the solution of the problem of the diffraction are used Debye's potentials. To resonances of elastic spheroidal bodies are devoted publications [1 – 9].

II. The First Part of the Article Investigat the Solution of the Three-Dimensional Problem of the Diffraction at the Elastic Spheroidal Body with Help of Debye's Potentials

Debye first proposed expanding the vector potential $\vec{A}$ in the scalar potentials U and V in his publication [10] devoted to studying the behavior of light waves near the local point or local line. Later, this approach was used in solving diffraction problems for cases of the electromagnetic wave diffraction of a sphere, a circular disk and a paraboloid of a revolution [11 – 16], as well as for the diffraction of longitudinal and transverse waves by spheroidal bodies [7, 17].

As applied to problems based on the dynamic elasticity theory, the introduction of Debye's potentials occurs as follows.The displacement vector $\vec{u}$ of an elastic isotropic medium obeys the Lame equation:

$$(\lambda+\mu) grad div \vec{u} - \mu curl curl \vec{u} = -\rho\omega^2 \vec{u}, \qquad (1)$$

where λ and μ are Lame constans, ρ is the density of the isotropic medium and ω is the circular frequency of harmonic vibrations. According to the Helmholtz theorem, the displacement vector $\vec{u}$ is expressed through scalar Φ and vector $\vec{\Psi}$ potentials:

$$\vec{u} = -grad\Phi + curl\vec{\Psi} \qquad (2)$$

Substituting Eg. (2) in Eg. (1), we obtain two Helmholtz equations, which include one scalar equation for Φ and one vector equation for $\vec{\Psi}$:

$$\Delta\Phi + h^2\Phi = 0, \qquad (3)$$

$$\Delta\vec{\Psi} + k_2^2\vec{\Psi} = 0. \qquad (4)$$

Here $h = \omega / c_1$ is the wavenumber of the longitudinal elastic wave, c_1 is the velocity of this wave, $k_2 = \omega / c_2$ is the wavenumber of the transverse elastic wave and c_2 is the velocity of the transverse wave.

In the three-dimensional case, variables involved in scalar equation (3) can be separated into 11 coordinate systems. As for Eq. (4), in the three-dimensional problem, this equation yields three independent equations for each of components of the vector function $\vec{\Psi}$ in Cartesian coordinate system alone. To overcome this difficulty, one can use Debye's potentials U and V, which obey the Helmholtz scalar equation

$$\Delta V + k_2^2 V = 0; \ \Delta U + k_2^2 U = 0. \qquad (5)$$

Vector potential $\vec{\Psi}$ (according to Debye) is expanded in potentials V and U as

$$\vec{\Psi} = curl curl(\vec{R}U) + ik_2 curl(\vec{R}V), \qquad (6)$$

where $\vec{R}$ is the radius vector of a point of the elastic body or the elastic medium.

Let us demonstrate the efficiency of using Debye's potentials in solving the three-dimensional diffraction problem for the case of diffraction by an elastic spheroidal shell. The advantage of the representation (6) becomes evident, if we take into account that potentials V and U obey the Helmholtz scalar equation. It is convenient to represent components of $\vec{\Psi}$ in the spherical coordinate system by expressing them through U, V and $\vec{R}$ and then, using formulas of the vector analysis, to change to spheroidal components. The expressions for spherical components of the vector function $\vec{\Psi}(\Psi_R, \Psi_\theta, \Psi_\varphi)$ in terms of Debye's potentials have the form [7]:

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$$\Psi_R = (\partial\xi/\partial R)^2(\partial^2 B/\partial\xi^2) + 2\ \partial\xi(/\partial R)(\partial\eta/\partial R)(\partial^2 B/\partial\xi\partial\eta) + (\partial\eta/\partial R)^2(\partial^2 B/\partial\eta^2) + (\partial^2\xi/\partial R^2)(\partial B/\partial\xi) + (\partial^2\eta/\partial R^2)(\partial B/\partial\eta) + k_2^2 B, \quad (7)$$

$$\Psi_\theta = [h_0(\xi^2 - 1 + \eta^2)]^{-1}[(\partial\xi/\partial\theta)(\partial\xi/\partial R)(\partial^2 B/\partial\xi^2) + (\partial\xi/\partial\theta)(\partial\eta/\partial R)(\partial^2 B/\partial\xi\partial\eta) + (\partial\xi/\partial R)(\partial\eta/\partial\theta)(\partial^2 B/\partial\xi\partial\eta) + (\partial\eta/\partial R)(\partial\eta/\partial\theta)(\partial^2 B/\partial\eta^2) + (\partial B/\partial\xi)(\partial^2\xi/\partial R\partial\theta) + (\partial B/\partial\eta)(\partial^2\eta/\partial R\partial\theta)] + ik_2(\sin\theta)^{-1}(\partial V/\partial\varphi), \quad (8)$$

$$\Psi_\varphi = [h_0(\xi^2 - 1 + \eta^2)^{1/2}\sin\theta]^{-1}[\partial\xi/\partial R)(\partial^2 B/\partial\xi\partial\varphi) + (\partial\eta/\partial R)(\partial^2 B/\partial\eta\partial\varphi) - ik_2 \times [(\partial\xi/\partial\theta)(\partial V/\partial\xi) + (\partial\eta/\partial\theta)(\partial V/\partial\eta)], \quad (9)$$

where:

$$B = h_0(\xi^2 - 1 + \eta^2)^{1/2}U; -1 \le \eta \le +1; 1 \le \xi \le +\infty.$$

Spheroidal components of the function $\overrightarrow{\Psi}(\Psi_\xi, \Psi_\eta, \Psi_\varphi)$ are expressed as follows [7]:

$$\Psi_\xi = \Psi_R(h_0/h_\xi)\xi(\xi^2 - 1 + \eta^2)^{-1/2} + \Psi_\theta(h_0/h_\xi)(\xi^2 - 1 + \eta^2)^{1/2}(\partial\theta/\partial\xi), \quad (10)$$

$$\Psi_\eta = \Psi_R(h_0/h_\eta)\eta(\xi^2 - 1 + \eta^2)^{-1/2} + \Psi_\theta(h_0/h_\eta)(\xi^2 - 1 + \eta^2)^{1/2}(\partial\theta/\partial\eta), \quad (11)$$

$$\Psi_\varphi \equiv \Psi_\varphi, \quad (12)$$

where:

$$h_\xi = h_0(\xi^2 - \eta^2)^{1/2}(\xi^2 - 1)^{1/2}; \ h_\eta = (\xi^2 - \eta^2)^{1/2}(1 - \eta^2)^{1/2}.$$

Let us consider in the form of an isotropic elastic spheroidal shell (Fig. 1). All potentials, including the plane wave potential Φ_0, the scattered wave potential Φ_1, the scalar shell poten-tial Φ_2, Debye's potentials U and V and potential Φ_3 of the gas filling the shell, can be ex-panded in spheroidal functions:

$$\Phi_0 = 2\sum_{m=0}^{\infty}\sum_{n\ge m}^{\infty} i^{-n}\varepsilon_m \overline{S}_{m,n}(C_1,\eta_0)\overline{S}_{m,n}(C_1,\eta)R_{m,n}^{(1)}(C_1,\xi)\cos m\varphi \quad (13)$$

$$\Phi_1 = 2\sum_{m=0}^{\infty}\sum_{n\ge m}^{\infty} B_{m,n}\overline{S}_{m,n}(C_1,\eta)R_{m,n}^{(3)}(C_1,\xi)\cos m\varphi \quad (14)$$

$$\Phi_2 = 2\sum_{m=0}^{\infty}\sum_{n\ge m}^{\infty}[C_{m,n}R_{m,n}^{(1)}(C_l,\xi) + D_{m,n}R_{m,n}^{(2)}(C_l,\xi)]\overline{S}_{m,n}(C_l,\xi)\cos m\varphi \quad (15)$$

$$\Phi_3 = 2\sum_{m=0}^{\infty}\sum_{n\ge m}^{\infty} E_{m,n}R_{m,n}^{(1)}(C_2,\xi)\overline{S}_{m,n}(C_2,\eta)\cos m\varphi \quad (16)$$

$$U = 2\sum_{m=1}^{\infty}\sum_{n\ge m}^{\infty}[F_{m,n}R_{m,n}^{(1)}(C_t,\xi) + G_{m,n}R_{m,n}^{(2)}(C_t,\xi)]\overline{S}_{m,n}(C_t,\eta)\sin m\varphi; \quad (17)$$

$$V = 2\sum_{m=0}^{\infty}\sum_{n\ge m}^{\infty}[H_{m,n}R_{m,n}^{(1)}(C_t,\xi) + I_{m,n}R_{m,n}^{(2)}(C_t,\xi)]\overline{S}_{m,n}(C_t,\eta)\cos m\varphi \quad (18)$$

where:

$\overline{S}_{m,n}(C_1,\eta) -$ the angular spheroidal function; $R_{m,n}^{(1)}(C_1,\xi)$, $R_{m,n}^{(2)}(C_1,\xi)$ and $R_{m,n}^{(3)}(C_1,\xi) -$ radial spheroidal functions of first, second and third genders; $C_l = hh_0$; $C_t = k_2h_0$; $C_1 = kh_0$, $k -$ is the wavenumber of the sound wave in the liquid; $C_2 = k_1h_0$, $k_1 -$ is the wavenumber of the sound wave in the gas filling the shell; $h_o -$ the half – focal distance; $B_{m,n}, C_{m,n}, D_{m,n}, E_{m,n}, F_{m,n}, G_{m\ n}, H_{m\ n}, I_{m\ n} -$ are unknown expansion coefficients.

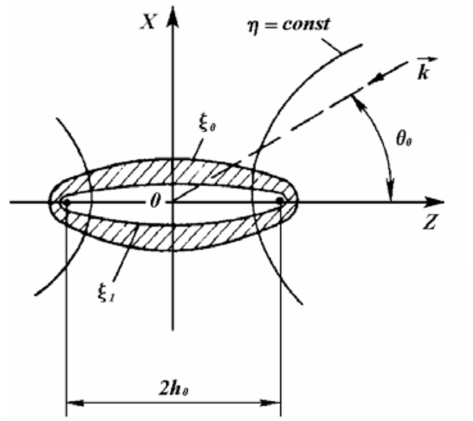

Figure 1 : Elastic spheroidal shell in a plane harmonic wave field

Expansion coefficients are determined from physical boundary conditions preset at two surfaces of the shell (ξ_0 and ξ_1, see Fig. 1) [7]:

1. the continuity of the normal displacement component at both of the boundaries ξ_0 and ξ_1;
2. the identity between the normal stress in the elastic shell and the sound pressure in the liquid (ξ_0) or in the gas (ξ_1);
3. the absence of tangential stresses at both of the shell boundaries, ξ_0 and ξ_1.

The corresponding expressions for boundary conditions have the form [7]:

$$(h_\xi)^{-1}(\partial/\partial\xi)(\Phi_0+\Phi_1)=(h_\xi)^{-1}(\partial\Phi_2/\partial\xi)+(h_\eta h_\varphi)^{-1}[(\partial/\partial\eta)(h_\varphi\Psi_\varphi)-(\partial/\partial\varphi)(h_\eta\Psi_\eta)]_{\xi=\xi_0}; \quad (19)$$

$$(h_\xi)^{-1}(\partial\Phi_1/\partial\xi)=(h_\xi)^{-1}(\partial\Phi_2/\partial\xi)+(h_\eta h_\varphi)^{-1}[(\partial/\partial\eta)(h_\varphi\Psi_\varphi)-(\partial/\partial\varphi)(h_\eta\Psi_\eta)]_{\xi=\xi_1}; \quad (20)$$

$$-\lambda_0 k^2(\Phi_0+\Phi_1)=-\lambda h^2\Phi_2+2\mu[(h_\xi h_\eta)^{-1}(\partial h_\xi/\partial\eta)u_\eta+(h_\xi)^{-1}(\partial u_\xi/\partial\xi)]_{\xi=\xi_0}; \quad (21)$$

$$-\lambda_1 k_1^2\Phi_3=-\lambda h^2\Phi_2+2\mu[(h_\xi h_\eta)^{-1}(\partial h_\xi/\partial\eta)u_\eta+(h_\xi)^{-1}(\partial u_\xi/\partial\xi)]_{\xi=\xi_1}; \quad (22)$$

$$0=(h_\eta/h_\xi)(\partial/\partial\xi)(u_\eta/h_\eta)+(h_\xi/h_\eta)(\partial/\partial\eta)(u_\xi/h_\xi)_{\xi=\xi_0;\xi=\xi_1}; \quad (23)$$

$$0=(h_\varphi/h_\xi)(\partial/\partial\xi)(u_\varphi/h_\varphi)+(h_\xi/h_\varphi)(\partial/\partial\varphi)(u_\xi/h_\xi)_{\xi=\xi_0;\xi=\xi_1}, \quad (24)$$

where:

$h_\varphi=h_0(\xi^2-1)^{1/2}(1-\eta^2)^{1/2}$; λ_0- is the bulk compression coefficient of the liquid; λ_1- is the bulk compression coefficient of the gas filling the shell;

$$u_\xi=(h_\xi)^{-1}(\partial\Phi_2/\partial\xi)+(h_\eta h_\varphi)^{-1}[(\partial/\partial\eta)(h_\varphi\Psi_\varphi)-(\partial/\partial\varphi)(h_\eta\Psi_\eta)];$$

$$u_\eta=(h_\eta)^{-1}(\partial\Phi_2/\partial\eta)+(h_\xi h_\varphi)^{-1}[(\partial/\partial\varphi)(h_\xi\Psi_\xi)-(\partial/\partial\xi)(h_\varphi\Psi_\varphi)];$$

$$u_\varphi=(h_\varphi)^{-1}(\partial\Phi_2/\partial\varphi)+(h_\xi h_\eta)^{-1}[(\partial/\partial\xi)(h_\eta\Psi_\eta)-(\partial/\partial\eta)(h_\xi\Psi_\xi)].$$

The substitution of series (13) – (18) in boundary conditions (19) – (24) yields an infinite system of equations for the determining of desired coefficients. Because of the ortogonality of trigonometric functions $\cos m\varphi$ and $\sin m\varphi$, the infinite system of equations breaks into infinite subsystems with fixed numbers m Each of subsystems is solved by the truncation method. The number of retained terms of expansions (13) – (18) is the greater the wave size for the given potential.The solution of the axissymmetrical problem of the diffraction at elastic spheroidal bodies was presented in [1, 2, 7 – 9].

III. The Second Part of the Article Investigates Results of Numerical Experiment for Determination of low Frequency Resonances of Elastic Spheroidal Bodies

Characteristics of the prolate gas – filled shell were calculated for two angles of the irradiation $\theta_0 = 0^0$ and $\theta_0 = 90^0$. At the Fig. 2 are presented in the different scale moduluses of angular characteristics of the scattering $|D(\theta)|$ of the steel prolate gas – filled spheroidal shell (curve 1), of the soft prolate spheroid (curve 2) and of the hard spheroid (curve 3) by $\theta_0 = 0^0$ and $C_1 = 1,0$.

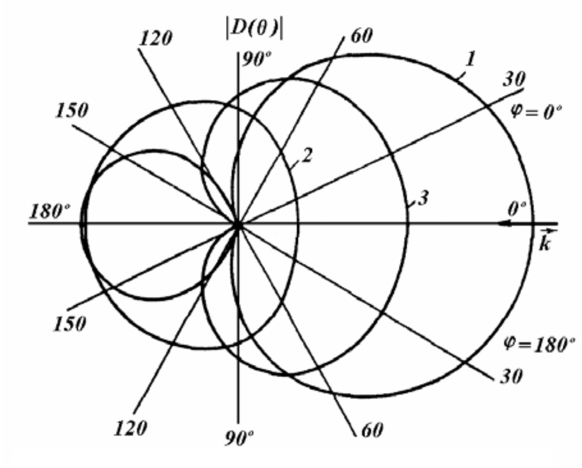

Figure 2 : Moduluses of angular characteristics of scattering of spheroidal scatterers

Same angular distributions, but by $C_1 = 3,1$ (the elastic shell, $C_1 = 3,0 -$ for ideal sphe-roids) and $C_1 = 10,0$ according are presented at Fig. 3 and 4. Notations of curves at all three Fig. identical. The analysis of presented results shows, what by the angle of the irradiation $\theta_0 = 0^0$ and the wave dimension $C_1 = 1,0$ (see Fig. 2) the angular characteristic of the elastic shell is similarly at the characteristic of the hard spheroid. By $C_1 = 3,1$ and by the angle of the irradiation $\theta_0 = 0^0$ the situation becomes indeterminated: the angular characteristic of the shell has dipole character as and by the hard spheroid (see Fig. 3). By the increase of the wave dimension C_1 the character of the sound scattering by the shell remains complicated (see Fig. 4): in the lit region the characteristic $|D(\theta)|$ of the hard spheroid, but in the shade region it is nearer to the shade lobe of the soft spheroid.

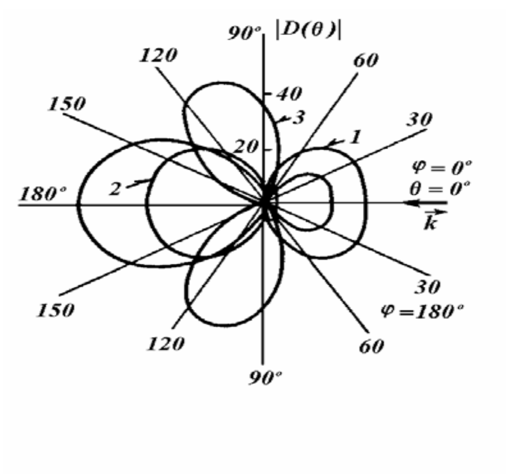

Figure 3 : Moduluses of angular characteristics of spheroidal scatterers

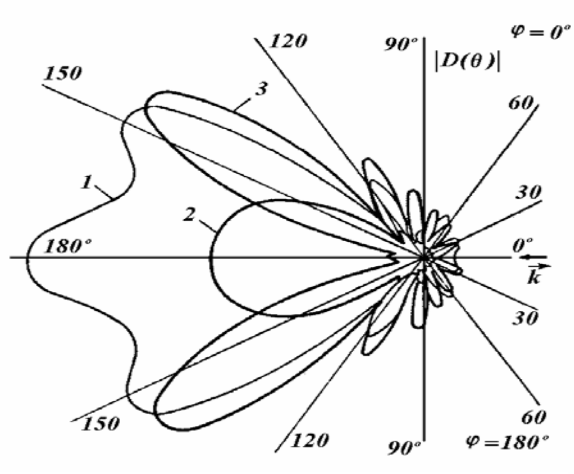

Figure 4 : Moduluses of angular chararacteristics of spheroidal scatterers

Over known angular characteristics of the scattering $D(\theta,\varphi)$ can be calculated back- scattering cross sections σ_0 of elastic spheroidal bodies [7]. At Fig. 5 are presented meanings of relative backscattering cross sections σ_0 of prolate spheroids with a correlation of semi – axises 1 : 10 ($(\xi_0 = 1{,}005)$ by the axially symmrtric irradiation $(\theta_0 = 0^0)$. The continuous elastic sphe-roid over the its conduct is very near to the ideal hard scatterer. This was seen by the compare-son of angular characteristics $D(\theta,\varphi)$ of steel and ideal spheroids. A coincidence is observed every where with the exception of a resonant point $C = 7{,}4$. Thies resonance is called by the surface wave of the "type of the Rayleigh wave" [5]. By the wave dimension $C =$ 7,4 on the surface along a contour of the steel continuous prolate spheroid is gone 2,5 λ_R, where λ_R is a length of the wave of the wave of the "type Rayleigh wave". A velocity this wave c_R is equal 2889 m/s, but on the plane boundary steel – vacuum a velocity of the Rayleigh wave is equal 2980 m/s.

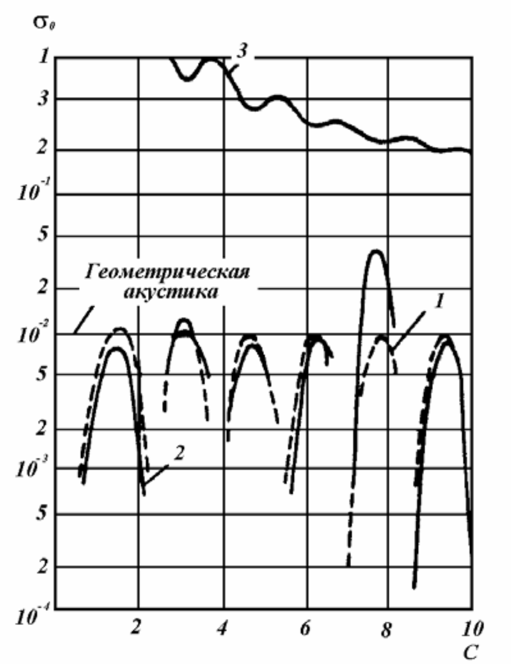

Figure 5 : Relative backscattering cross sections of prolate spheroids

On the Fig. 6 are presented relative back-scattering cross sections σ_0 of oblate spheroids with the correlation of the semi – axises 1:10 $(\xi_0 = 0{,}1005)$ by the axially symmetric irradiation $\theta_0 = 0^0$, the notations coincide with the Fig. 5. Until the resonance of rhe zero antisymmetrical-

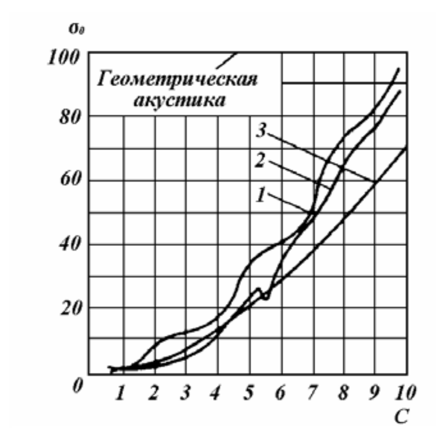

Figure 6 : Relative backscattering cross sections of oblate spheroids

flexural wave $(C \approx$ 5,3) σ_0 of the steel oblate spheroid over a level nearer to σ of the soft

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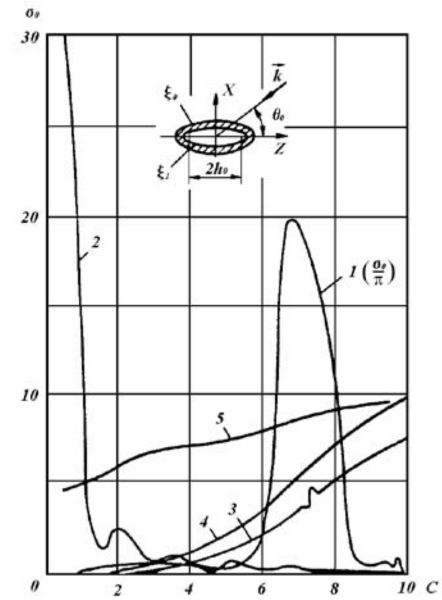

Figure 7 : Relative backscatterings cross sections of prolate spheroidal scatterers

spheroid, but by $C > 5,3$ draws near to σ_0 of the hard spheroid, at least the angular characteristic $D(\theta)$ of the elastic spheroid by $\theta_0 = 0^0$ and by all meanings of the wave dimension C is near to the angular characteristic $D(\theta)$ of the hard spheroid. On the Fig. 7 are presented sections σ_0 of the prolate spheroidal scatterers. The steel prolate spheroid and by $\theta_0 = 90^0$ has the resonance of the surface wave by same meaning $C =$ 7,4 (see curve 2, Fig. 5) [7]. Itself section of the scattering σ_0 of t5he steel continuous spheroid (curve 3) by $\theta_0 = 90^0$ is visibley nearer to σ_0 of the hard spheroid (curve 4) over the comparison with σ_0 of the soft spheroid (curve 5). This nearness of the scattering properties of continuous elastic and hard spheroids was shown too in the angular characteristic $D(\theta,\varphi)$. A frequency dependence of the relative section σ_0 of the prolate spheroidal shell (curve 1) by $\theta_0 = 0^0$ shows a prwesente of the considerable resonan- ce by $C = 6,75$ [1, 7 – 9]. On a Fig. 8 are shown moduluses of angular characteristics $|D(\theta)|$ of prolate spheroidal scatterers. A curve 1 concerns to the steel gas – filled shell by the wave dimension $C = 6,75$ corresponding its resonance, the curve 2 concerns to a soft spheroid, a curve 3 concerns to a hard spheroid, for ideal spheroids a wave dimension C is equal 10,0. From the

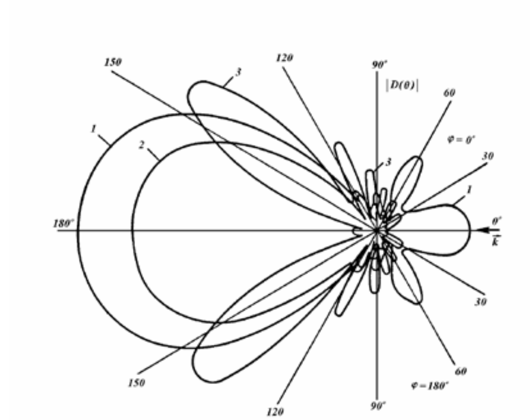

Figure 8 : Moduluses of angular characteristics of prolate spheroidal bodies

comparison of three curves we see, what a shade lobe of the angular characteristic of the shell shows at "the soft background", but the lobe of the backscattering shows at "the hard background". A relative.

Table 1

Wave dimension, C	σ_0 by $\theta_0 = 90^\circ$		
	Spheroidal gas – filled shell ($\xi_0 = 1{,}005075;\ \xi_1 = 1{,}005$)	Hard spheroid ($\xi_0 = 1{,}005$)	Soft spheroid ($\xi_0 = 1{,}005$)
0,5	$0{,}3012 \cdot 10^{-3}$	$0{,}2452 \cdot 10^{-3}$	4,506
1,0	$0{,}4748 \cdot 10^{-2}$	$0{,}3908 \cdot 10^{-2}$	4,760
1,5	$0{,}2365 \cdot 10^{-1}$	$0{,}1965 \cdot 10^{-1}$	5,194
2,0	$0{,}7354 \cdot 10^{-1}$	$0{,}6147 \cdot 10^{-1}$	5,748
2,5	0,1751	0,1479	6,300
3,0	0,3470	0,3006	6,754
3,5	0,6068	0,5418	7,094
4,0	0,9736	0,8911	7,358
4,5	1,447	1,362	7,592
5,0	2,014	1,960	7,815
5,5	2,599	2,680	8,029

backscattering cross section σ_0 of a spheroidal shell by $\theta_0 = 90^0$ was calculated until a wave dimension $C = 5,5$. Meanings σ_0 of a ashell are very near to σ_0 of a hard spheroid, what was shown worth while compare these sections with sections of t spheroid in a table form. As we see from a table 1 by this angle of a irradiation until $C = 5,5$ is shown "a hard background" o9f a scattering, what we see and from comparison of angular characteristics of a scattering $D(\theta,\varphi)$. A full scattering cross section σ [7] is determined through a square of a modulus of a angular characteristic of a sound scattering

$$D(\theta,\varphi) : \sigma = \int_0^{\pi}\int_0^{2\pi} |D(\theta,\varphi)|^2 \sin\theta d\theta d\varphi.$$

A relative scattering cross section σ_r, by a way, is equal

$$\sigma_r = \sigma / 2A_0,$$

where A_0 is an area of a geometrical shade of a scatterer.

With a help of an optical theorem a scattering cross section σ cab be found through a meaning of an imaginaty part of of an angular characteristic in a direction of a falling wave (a scattering "forward") $\operatorname{Im} D(180^0 - \theta_0; 180^0)$ [7]:

$$\sigma = (4\pi / k) \operatorname{Im} D(180^0 - \theta_0; 180^0),$$

where θ_0 is an angle of a fall; $\varphi_0 = 0^0$.

At an analogy with the scattering cross section σ can introduce an idea of a section σ_{rad} of an elastic or liquid body under an action of a point source [7]:

$$\sigma_{rad} = \int_0^{\pi}\int_0^{2\pi} |F(\theta,\varphi)|^2 \sin\theta d\theta d\varphi,$$

where $F(\theta,\varphi)$ is an angular characteristic of a sound radiation of a body under an action of a point source.

At a basis of presented formulas was made an account of full σ and relative σ_r scattering cross sections and a radiation cross section σ_{rad} of spheroidal (prolate and oblate) bodies. On a Fig.9 are presented relative sections of a scattering σ_r of an ideal hard oblate spheroid (curve 1), of a steel oblate spheroid (curve 2) and of an ideal soft oblate spheroid (xcurve 3). In all three ca-ses a relation of a semi – axises $a/b = 1:10(\xi_0 = 0{,}1005)$, but an angle of an irradiation $\theta_0 = 0^0$. A relative section σ_r of an elastic spheroid shows a r5esonance of a coincidence as this was and in a relative backscattering σ_0 (see Fig. 6), but a point of a maximum was by $C = 5,25$, for σ_r is by $C - 5,35$. With an increase C a curve 2 draws near to a meaning $\sigma_r = 1,0$ corresp0onding a geometrical acoustics. Calculations show, what by $C = 15,0$ for an elastic oblate spheroid $\sigma_r = 0,866$, but by $C = 20,0 \to \sigma_r = 0,941$. On a Fig. 10 are presented relative sections of a sections of a scattering σ_r (curves 1 and 2) and a section of a radiation σ_{rad} (curve 3) of prolate spheroidal bodies. A curve 1 shows a frequency dependence $\sigma_r(C)$ of an ideal soft prolate spheroid $[a/b = 1:10(\xi_0 = 1{,}005)]$, a curve 2 corresponds $\sigma_r(C)$ of steel gas – filled prolate spheroi-

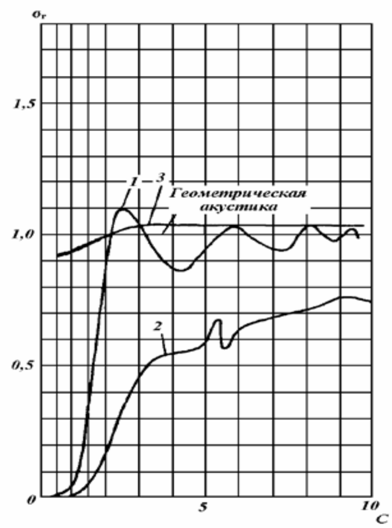

Figure 9 : Relative scattering cross sections of oblate spheroids

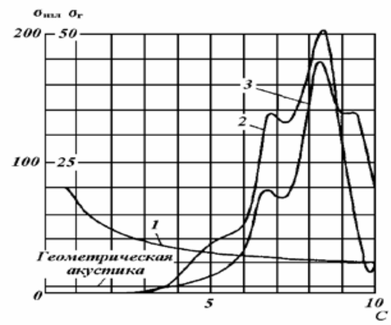

Figure 10 : Relative scattering cross sections and the section of radiating of prolate spheroidal bodies

dal shell $(\xi_0 = 1,005075; \xi_1 = 1,005).$ Both curves correspond $\theta_0 = 0^0$ (an axially symmetric problem). A curve 2 for an elastic shell unlike from an its relative backscattering section (curve 1 on a Fig. 7) has two maximums (two resonances). A first from theirs is observed by $C = 6,7$ (unlike from $C = 6,75$ for σ_0), a second resonance is observed by $C \approx 8,25$ and corresponds $L = 1,5\Lambda,$ where L is a length of a contour of a neutral surface of a shell, Λ is a length of a longitudinal wave (of a zero symmetrical Lamb's wave) spreading with a velocity $c_1 \approx 5420 \quad m/s.$ A curve 1 for an ideal soft spheroid aspires asymptotical to a meaning of a geometrical acoustics $\sigma_r = 1,0):$ $\sigma_r(15,0) = 4,16; \sigma_r(65,0) = 2,23; \sigma_r(100) = 1,93.$ A curve 3 characterises a radiating faculty of a same shell, if it is exited from an outside by a point source by $\theta_0 = 0^0(h_0 = 50 \quad m).$ A section of a radiation σ_{rad} has an extremums in those points, what and a relative section of a scattering $\sigma_{r.}$ A comparison of curves 2 and 3 presented on a Fig. 10 with curve 1 of a Fig. 7 shows, what a relative backscattering section does not give sometimes of a full information about a resonant properties of elastic scattering.

IV. Conclusions

With the help of the numerical experiment are found low frequency resonances of elastic spheroidal bodies (entire and in the form of shells) both prolate and oblate by the three – dimensional and axissymmetrical irradiation.

V. Acknowledgments

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