Resonances of Elastic Spheroidal Bodies

By A. Kleshchev
Saint Petersburg State Navy Technical University, Russia

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Resonances of Elastic Spheroidal Bodies

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Abstract - At the basis of the dynamic elasticity theory with the use of Debye’s potentials are found resonances of elastic spheroidal bodies (prolate and oblate) as entire so and in form of shells. In addition to analytic solutions, computer calculations are performed of moduluses of the angular characteristics of the scattering and sections of the scattering of spheroidal bodies.

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I. Introduction

In the paper are investigated the resonances of spheroidal bodies (prolate and oblate) in the three-dimensional and axis-symmetrical radiation. By the three-dimensional radiation for the solution of the problem of the diffraction are used Debye’s potentials. To resonances of spheroidal bodies are devoted publications [1 – 9].

II. The First Part of the Article Investigating the Solution of the Three-Dimensional Problem of the Diffraction at the Elastic Spheroidal Body with the Help of Debye’s Potentials

Debye first proposed expanding the vector potential $\vec{A}$ in the scalar potentials $U$ and $V$ in his publication [10] devoted to studying the behavior of light waves near the local point or local line. Later, this approach was used in solving diffraction problems for the cases of the electromagnetic wave diffraction of a sphere, a circular disk and a paraboloid of a revolution [11 – 16], as well as for the diffraction of longitudinal and transverse waves by spheroidal bodies [7, 17].

As applied to problems based on the dynamic elasticity theory, the introduction of Debye’s potentials occurs as follows. The displacement vector $\vec{u}$ of an elastic isotropic medium obeys the Lame equation:

$$(\lambda + \mu)\nabla^2 \vec{u} - \mu \nabla (\nabla \cdot \vec{u}) = -\rho \omega^2 \vec{u},$$

where $\lambda$ and $\mu$ are Lame constants, $\rho$ is the density of the isotropic medium and $\omega$ is the circular frequency of harmonic vibrations. According to the

Helmholtz theorem, the displacement vector $\vec{u}$ is expressed through scalar $\Phi$ and vector $\vec{\Psi}$ potentials:

$$\vec{u} = -\nabla \Phi + \nabla \times \vec{\Psi}$$

Substituting Eq. (2) in Eq. (1), we obtain two Helmholtz equations, which include one scalar equation for $\Phi$ and one vector equation for $\vec{\Psi}$:

$$\Delta \Phi + h^2 \Phi = 0,$$

$$\Delta \vec{\Psi} + k_z^2 \vec{\Psi} = 0.$$  

Here $h = \omega / c_1$ is the wavenumber of the longitudinal elastic wave, $c_1$ is the velocity of this wave, $k_z = \omega / c_2$ is the wavenumber of the transverse elastic wave and $c_2$ is the velocity of the transverse wave.

In the three-dimensional case, variables involved in the scalar equation (3) can be separated into 11 coordinate systems. As for Eq. (4), in the three-dimensional problem, this equation yields three independent equations for each of components of the vector function $\vec{\Psi}$ in Cartesian coordinate system alone. To overcome this difficulty, one can use Debye’s potentials $U$ and $V$, which obey the Helmholtz scalar equation

$$\Delta V + k_z^2 V = 0; \Delta U + k_z^2 U = 0.$$  

Vector potential $\vec{\Psi}$ (according to Debye) is expanded in potentials $V$ and $U$ as

$$\vec{\Psi} = \nabla \times (\nabla \times \vec{u}^*) + ik_z \nabla \times (\nabla \times \vec{u}^*),$$

where $\vec{R}$ is the radius vector of a point of the elastic body or the elastic medium.

Let us demonstrate the efficiency of using Debye’s potentials in solving the three-dimensional diffraction problem for the case of diffraction by an elastic spheroidal shell. The advantage of the representation (6) becomes evident, if we take into account that potentials $V$ and $U$ obey the Helmholtz scalar equation. It is convenient to represent components of $\vec{\Psi}$ in the spherical coordinate system by expressing them through $U$, $V$ and $\vec{R}$ and then, using formulas of the vector analysis, to change to spheroidal components. The expressions for spherical components of the vector function $\vec{\Psi}(\Psi_\rho, \Psi_\theta, \Psi_\varphi)$ in terms of Debye’s potentials have the form [7]:
\[
\Psi_{\varphi} = (\partial / \partial R)^2 (\partial^2 B / \partial \xi^2) + 2 (\partial / \partial R)(\partial / \partial R)(\partial^2 B / \partial \xi \partial \eta) + (\partial / \partial R)^2 (\partial^2 B / \partial \eta^2) + (\partial^2 / \partial \xi \partial \eta)(\partial B / \partial \eta) + k_2^2 B,
\]

\[
\Psi_{\varphi} = [h_0(\xi^2 - 1 + \eta^2)]^{-1}[(\partial / \partial \varphi)(\partial / \partial R)(\partial^2 B / \partial \xi \partial \eta) + (\partial / \partial \varphi)(\partial / \partial R)(\partial^2 B / \partial \eta \partial \varphi) - ik_2 \times
\]

where:

\[
B = h_0(\xi^2 - 1 + \eta^2)^{1/2} U; -1 \leq \eta \leq +1; 1 \leq \xi \leq +\infty.
\]

Spheroidal components of the function \(\Psi(\Psi_\xi, \Psi_\eta, \Psi_\varphi)\) are expressed as follows [7]:

\[
\Psi_\xi = \Psi_R(h_0 / h_\xi)(\xi^2 - 1 + \eta^2)^{-1/2} + \Psi_\varphi(h_0 / h_\xi)(\xi^2 - 1 + \eta^2)^{1/2}(\partial \theta / \partial \xi),
\]

\[
\Psi_\eta = \Psi_R(h_0 / h_\eta)(\eta^2 - 1 + \eta^2)^{-1/2} + \Psi_\varphi(h_0 / h_\eta)(\xi^2 - 1 + \eta^2)^{1/2}(\partial \theta / \partial \eta),
\]

\[
\Psi_\varphi = \Psi_\varphi,
\]

where:

\[
h_\xi = h_0(\xi^2 - 1 + \eta^2)^{1/2}(\xi^2 - 1)^{1/2}; h_\eta = (\xi^2 - 1\eta^2)^{1/2}(1 - \eta^2)^{1/2}.
\]

Let us consider in the form of an isotropic elastic spheroidal shell (Fig. 1). All potentials, including the plane wave potential \(\Phi_0\), the scattered wave potential \(\Phi_1\), the scalar shell potential \(\Phi_2\), Debye's potentials \(U\) and \(V\), and potential \(\Phi_3\) of the gas filling the shell, can be expressed in spheroidal functions:

\[
\Phi_0 = 2 \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{m,n} \xi \eta \bar{S}_{m,n}(C_1, \eta_0) \bar{S}_{m,n}(C_1, \eta) R_{m,n}^{(i)}(C_1, \xi) \cos m\varphi
\]

\[
\Phi_1 = 2 \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} B_{m,n} \bar{S}_{m,n}(C_1, \eta) \bar{S}_{m,n}(C_1, \xi) \cos m\varphi
\]

\[
\Phi_2 = 2 \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} [C_{m,n} R_{m,n}^{(2)}(C_1, \xi) + D_{m,n} R_{m,n}^{(2)}(C_1, \xi)] \bar{S}_{m,n}(C_1, \xi) \cos m\varphi
\]

\[
\Phi_3 = 2 \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} E_{m,n} R_{m,n}^{(3)}(C_2, \xi) \bar{S}_{m,n}(C_2, \eta) \cos m\varphi
\]

\[
U = 2 \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{m,n} [F_{m,n} R_{m,n}^{(1)}(C_1, \xi) + G_{m,n} R_{m,n}^{(2)}(C_1, \xi)] \bar{S}_{m,n}(C_1, \eta) \sin m\varphi
\]

\[
V = 2 \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{m,n} [H_{m,n} R_{m,n}^{(1)}(C_1, \xi) + I_{m,n} R_{m,n}^{(2)}(C_1, \xi)] \bar{S}_{m,n}(C_1, \eta) \cos m\varphi
\]

where:

\[
\bar{S}_{m,n}(C_1, \eta) - \text{the angular spheroidal function}; \ R_{m,n}^{(1)}(C_1, \xi), \ R_{m,n}^{(2)}(C_1, \xi) \ \text{and} \ R_{m,n}^{(3)}(C_1, \xi) - \text{radial spheroidal functions of first, second and third genders;} \ C_i = hh_0; \ C_t = k_2 h_0; \ C_1 = kh_0; \ k - \text{is the wavenumber of the sound wave in the liquid}; \ C_2 = k_1 h_0, \ k_1 - \text{is the wavenumber of the sound wave in the gas filling the shell}; \ h_\varphi - \text{the half focal distance}; \ B_{m,n}, C_{m,n}, D_{m,n}, E_{m,n}, F_{m,n}, G_{m,n}, H_{m,n}, I_{m,n} - \text{are unknown expansion coefficients.}
\]
Figure 1: Elastic spheroidal shell in a plane harmonic wave field

Expansion coefficients are determined from physical boundary conditions preset at two surfaces of the shell (ξ₀ and ξ₁, see Fig. 1) [7]:

1. the continuity of the normal displacement component at both of the boundaries ξ₀ and ξ₁;
2. the identity between the normal stress in the elastic shell and the sound pressure in the liquid (ξ₀) or in the gas (ξ₁);
3. the absence of tangential stresses at both of the shell boundaries, ξ₀ and ξ₁.

The corresponding expressions for boundary conditions have the form [7]:

\[
(h^2_{ξ})^{-1}\left(\frac{\partial}{\partial ξ}(Φ₀ + Φ₁) = (h^2_{ξ})^{-1}\left(\frac{\partial Φ₂}{\partial ξ} + (h_ξ h_ρ)\right)\right)\left(\frac{\partial Φ₂}{\partial ξ} + (h_ξ h_ρ)\right) - (\frac{\partial}{\partial ξ}(h_ρ Ψ_ρ) - (\frac{\partial}{\partial ξ}(h_ρ Ψ_ρ))\right)_{ξ=ξ₀};
\]

\[
(h^2_{ξ})^{-1}\left(\frac{\partial Φ₁}{\partial ξ} = (h^2_{ξ})^{-1}\left(\frac{\partial Φ₂}{\partial ξ} + (h_ξ h_ρ)\right)\right)\left(\frac{\partial Φ₂}{\partial ξ} + (h_ξ h_ρ)\right) - (\frac{\partial}{\partial ξ}(h_ρ Ψ_ρ) - (\frac{\partial}{\partial ξ}(h_ρ Ψ_ρ))\right)_{ξ=ξ₁};
\]

\[
-λ_0k^2(Φ₀ + Φ₁) = -λh^2Φ₂ + 2μ[(h_ξ h_ρ)\left(\frac{\partial Φ₂}{\partial ξ} + (h_ξ h_ρ)\right) + (h_ξ h_ρ)\left(\frac{\partial u_ξ}{\partial ξ}\right)]_{ξ=ξ₀};
\]

\[
-λk^2(Φ₁) = -λh^2Φ₂ + 2μ[(h_ξ h_ρ)\left(\frac{\partial Φ₂}{\partial ξ} + (h_ξ h_ρ)\right) + (h_ξ h_ρ)\left(\frac{\partial u_ξ}{\partial ξ}\right)]_{ξ=ξ₁};
\]

\[
0 = (h_ξ/ h_ξ)\left(\frac{\partial}{\partial ξ}(u_ξ/ h_ξ) + (h_ξ/ h_ξ)\left(\frac{\partial}{\partial ξ}(u_ξ/ h_ξ)\right)\right)_{ξ=ξ₀};
\]

\[
0 = (h_ρ/ h_ρ)\left(\frac{\partial}{\partial ξ}(u_ρ/ h_ξ) + (h_ρ/ h_ρ)\left(\frac{\partial}{\partial ξ}(u_ρ/ h_ξ)\right)\right)_{ξ=ξ₁};
\]

where:

\[
h_ρ = h_ρ(\xi^2 - 1)^{1/2}(1 - \eta^2)^{1/2}; \quad λ_0 \quad \text{is the bulk compression coefficient of the liquid; } \quad λ_1 \quad \text{is the bulk compression coefficient of the gas filling the shell;}
\]

\[
u_ξ = (h_ξ)^{-1}\left(\frac{\partial Φ₂}{\partial ξ} + (h_ξ h_ρ)\right)\left(\frac{\partial Φ₂}{\partial ξ} + (h_ξ h_ρ)\right) - (\frac{\partial}{\partial ξ}(h_ρ Ψ_ρ) - (\frac{\partial}{\partial ξ}(h_ρ Ψ_ρ));
\]

\[
u_η = (h_η)^{-1}\left(\frac{\partial Φ₂}{\partial η} + (h_η h_ρ)\right)\left(\frac{\partial Φ₂}{\partial η} + (h_η h_ρ)\right) - (\frac{\partial}{\partial η}(h_ρ Ψ_ρ) - (\frac{\partial}{\partial η}(h_ρ Ψ_ρ));
\]

\[
u_φ = (h_φ)^{-1}\left(\frac{\partial Φ₂}{\partial φ} + (h_φ h_ρ)\right)\left(\frac{\partial Φ₂}{\partial φ} + (h_φ h_ρ)\right) - (\frac{\partial}{\partial φ}(h_ρ Ψ_ρ) - (\frac{\partial}{\partial φ}(h_ρ Ψ_ρ)).
\]

The substitution of series (13) – (18) in boundary conditions (19) – (24) yields an infinite system of equations for the determining of desired coefficients. Because of the orthogonality of trigonometric functions \(\cos m\phi\) and \(\sin m\phi\), the infinite system of equations breaks into infinite subsystems with fixed numbers \(m\) Each of subsystems is solved by the truncation method. The number of retained terms of expansions (13) – (18) is the greater the wave size for the given potential. The solution of the axisymmetrical problem of the diffraction at elastic spheroidal bodies was presented in [1, 2, 7 – 9].
III. THE SECOND PART OF THE ARTICLE
INVESTIGATES RESULTS OF NUMERICAL
EXPERIMENT FOR DETERMINATION OF
LOW FREQUENCY RESONANCES OF ELASTIC
SPHEROIDAL BODIES

Characteristics of the prolate gas – filled shell were calculated for two angles of the irradiation \( \theta_0 = 0^0 \) and \( \theta_0 = 90^0 \). At the Fig. 2 are presented in the different scale moduluses of angular characteristics of the scattering \( |D(\theta)| \) of the steel prolate gas – filled spheroidal shell (curve 1), of the soft prolate spheroid (curve 2) and of the hard spheroid (curve 3) by \( \theta_0 = 0^0 \) and \( C_1 = 1,0 \).

![Figure 2: Moduluses of angular characteristics of scattering of spheroidal scatterers](image)

Same angular distributions, but by \( C_1 = 3,1 \) (the elastic shell, \( C_1 = 3,0 \) – for ideal spheroids) and \( C_1 = 10,0 \) accordingly are presented at Fig. 3 and 4. Notations of curves at all three Fig. identical. The analysis of presented results shows, what by the angle of the irradiation \( \theta_0 = 0^0 \) and the wave dimension \( C_1 = 1,0 \) (see Fig. 2) the angular characteristic of the elastic shell is similarly at the characteristic of the hard spheroid. By \( C_1 = 3,1 \) and by the angle of the irradiation \( \theta_0 = 0^0 \) the situation becomes indeteminated: the angular characteristic of the shell has dipole character as and by the hard spheroid (see Fig. 3). By the increase of the wave dimension \( C_1 \) the character of the sound scattering by the shell remains complicated (see Fig. 4): in the lit region the characteristic \( |D(\theta)| \) of the hard spheroid, but in the shade region it is nearer to the shade lobe of the soft spheroid.

![Figure 3: Moduluses of angular characteristics of spheroidal scatterers](image)

![Figure 4: Moduluses of angular characteristics of spheroidal scatterers](image)
Over known angular characteristics of the scattering $D(\theta, \varphi)$ can be calculated back-scattering cross sections $\sigma_0$ of elastic spheroidal bodies [7]. At Fig. 5 are presented meanings of relative backscattering cross sections $\sigma_0$ of prolate spheroids with a correlation of semi - axes $1:10 (\xi = 1,005)$ by the axially symmetric irradiation ($\theta_0 = 0^\circ$). The continuous elastic spheroid over its conduct is very near to the ideal hard scatterer. This was seen by the comparison of angular characteristics $D(\theta, \varphi)$ of steel and ideal spheroids. A coincidence is observed everywhere with the exception of a resonant point $C = 7,4$. This resonance is called by the surface wave of the “type of the Rayleigh wave” [5]. By the wave dimension $C = 7,4$ on the surface along a contour of the steel continuous prolate spheroid is gone $2,5 \lambda_R$, where $\lambda_R$ is a length of the wave of the wave of the “type Rayleigh wave”. A velocity this wave $c_R$ is equal $2889 \, m/s$, but on the plane boundary steel – vacuum a velocity of the Rayleigh wave is equal $2980 \, m/s$.

**Figure 5**: Relative backscattering cross sections of prolate spheroids

On the Fig. 6 are presented relative backscattering cross sections $\sigma_0$ of oblate spheroids with the correlation of the semi - axes $1:10 (\xi = 0,1005)$ by the axially symmetric irradiation $\theta_0 = 0^\circ$, the notations coincide with the Fig. 5. Until the resonance of the zero antisymmetrical-

**Figure 6**: Relative backscattering cross sections of oblate spheroids

flexural wave ($C \approx 5,3$) $\sigma_0$ of the steel oblate spheroid over a level nearer to $\sigma$ of the soft
spheroid, but by $C > 5,3$ draws near to $\sigma_0$ of the hard spheroid, at least the angular characteristic $D(\theta)$ of the elastic spheroid by $\theta_0 = 0^\circ$ and by all meanings of the wave dimension $C$ is near to the angular characteristic $D(\theta)$ of the hard spheroid. On the Fig. 7 are presented sections $\sigma_0$ of the prolate spheroidal scatterers. The steel prolate spheroid and by $\theta_0 = 90^\circ$ has the resonance of the surface wave by same meaning $C = 7,4$ (see curve 2, Fig. 5) [7]. Its own section of the scattering $\sigma_0$ of the steel continuous spheroid (curve 3) by $\theta_0 = 90^\circ$ is visibly nearer to $\sigma_0$ of the hard spheroid (curve 4) over the comparison with $\sigma_0$ of the soft spheroid (curve 5). This nearness of the scattering properties of continuous elastic and hard spheroids was shown too in the angular characteristic $D(\theta, \varphi)$. A frequency dependence of the relative section $\sigma_0$ of the prolate spheroidal shell (curve 1) by $\theta_0 = 0^\circ$ shows a presence of the considerable resonance by $C = 6,75$ [1, 7 – 9]. On a Fig. 8 are shown moduluses of angular characteristics $|D(\theta)|$ of prolate spheroidal scatterers. A curve 1 concerns to the steel gas – filled shell by the wave dimension $C = 6,75$ corresponding its resonance, the curve 2 concerns to a soft spheroid, a curve 3 concerns to a hard spheroid, for ideal spheroids a wave dimension $C$ is equal 10,0. From the

**Figure 7**: Relative backscatterings cross sections of prolate spheroidal scatterers

**Figure 8**: Moduluses of angular characteristics of prolate spheroidal bodies
backscattering cross section $\sigma_0$ of a spheroidal shell by $\theta_0 = 90^\circ$ was calculated until a wave dimension $C = 5.5$. Meanings $\sigma_0$ of a ashell are very near to $\sigma_0$ of a hard spheroid, what was shown worth while compare these sections with sections of t spheroid in a table form. As we see from a table 1 by this angle of a fall; $\sigma_0$ (curve 2 draws near to a hard spheroid (curve 1), of a steel oblate spheroid (curve 2) and of an ideal soft oblate spheroid (xcurve 3). In all three ca-ses a relation of a semi – axises $a / b = 1:10(\xi_0 = 0,1005)$, but an angle of an irradiation $\theta_0 = 0^\circ$. A relative section $\sigma_r$ of an elastic spheroid shows a $\xi$sonance of a coincidence as this was in a relative backscattering $\sigma_0$ (see Fig. 6), but a point of a maximum was by $C = 5.25$, for $\sigma_r$ is by $C = 5.35$. With an increase $C$ a curve 2 draws near to a meaning $\sigma_r = 1.0$ corresponding a geometrical acoustics. Calculations show, what by $C = 15.0$ for an elastic oblate spheroid $\sigma_r = 0.866$, but by $C = 20.0 \rightarrow \sigma_r = 0.941$. On a Fig. 10 are presented relative sections of a scattering $\sigma_r$ of an elastic oblate spheroid (curves 1 and 2) and a section of a radiation $\sigma_{rad}$ of spheroidal bodies. A curve 1 shows a frequency dependence $\sigma_r(C)$ of an ideal soft prolate spheroid $[a / b = 1:10(\xi_0 = 1.005)]$, a curve 2 corresponds $\sigma_r(C)$ of steel gas – filled prolate spheroid-

<table>
<thead>
<tr>
<th>Wave dimension, C</th>
<th>Spheroidal gas – filled shell $(\xi_0 = 1.00575; \xi_f = 1.005)$</th>
<th>Hard spheroid $(\xi_0 = 1.005)$</th>
<th>Soft spheroid $(\xi_0 = 1.005)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.3012 $10^{-3}$</td>
<td>0.2452 $10^{-3}$</td>
<td>4.506</td>
</tr>
<tr>
<td>1.0</td>
<td>0.4748 $10^{-2}$</td>
<td>0.3908 $10^{-2}$</td>
<td>4.760</td>
</tr>
<tr>
<td>1.5</td>
<td>0.2365 $10^{-1}$</td>
<td>0.1965 $10^{-1}$</td>
<td>5.194</td>
</tr>
<tr>
<td>2.0</td>
<td>0.7354 $10^{-1}$</td>
<td>0.6147 $10^{-1}$</td>
<td>5.748</td>
</tr>
<tr>
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<td>0.1751</td>
<td>0.1479</td>
<td>6.300</td>
</tr>
<tr>
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<td>0.3470</td>
<td>0.3066</td>
<td>6.754</td>
</tr>
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<td>0.5413</td>
<td>7.094</td>
</tr>
<tr>
<td>4.0</td>
<td>0.94730</td>
<td>0.8911</td>
<td>7.358</td>
</tr>
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<td>1.447</td>
<td>1.362</td>
<td>7.592</td>
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</tr>
<tr>
<td>5.5</td>
<td>2.599</td>
<td>2.680</td>
<td>8.029</td>
</tr>
</tbody>
</table>

where $F(\theta, \varphi)$ is an angular characteristic of a sound radiation of a body under an action of a point source.

At a basis of presented formulas was made an account of full $\sigma$ and relative $\sigma_r$ scattering cross sections and a radiation cross section $\sigma_{rad}$ of spheroidal (prolate and oblate) bodies. On a Fig.9 are presented relative sections of a scattering $\sigma_r$ of an ideal hard oblate spheroid (curve 1), of a steel oblate spheroid (curve 2) and of an ideal soft oblate spheroid (xcurve 3). In all three ca-ses a relation of a semi – axises $a / b = 1:10(\xi_0 = 0,1005)$, but an angle of an irradiation $\theta_0 = 0^\circ$. A relative section $\sigma_r$ of an elastic spheroid shows a $\xi$sonance of a coincidence as this was and in a relative backscattering $\sigma_0$ (see Fig. 6), but a point of a maximum was by $C = 5.25$, for $\sigma_r$ is by $C = 5.35$. With an increase $C$ a curve 2 draws near to a meaning $\sigma_r = 1.0$ corresponding a geometrical acoustics. Calculations show, what by $C = 15.0$ for an elastic oblate spheroid $\sigma_r = 0.866$, but by $C = 20.0 \rightarrow \sigma_r = 0.941$. On a Fig. 10 are presented relative sections of a scattering $\sigma_r$ (curves 1 and 2) and a section of a radiation $\sigma_{rad}$ (curve 3) of prolate spheroidal bodies. A curve 1 shows a frequency dependence $\sigma_r(C)$ of an ideal soft prolate spheroid $[a / b = 1:10(\xi_0 = 1.005)]$, a curve 2 corresponds $\sigma_r(C)$ of steel gas – filled prolate spheroid-

$D(\theta, \varphi) : \sigma = \int_0^{2\pi} \int_0^\pi |D(\theta, \varphi)|^2 \sin \theta d\theta d\varphi.$

A relative scattering cross section $\sigma_r$, by a way, is equal

$$\sigma_r = \sigma / 2 A_0,$$

where $A_0$ is an area of a geometrical shade of a scatterer.

With a help of an optical theorem a scattering cross section $\sigma$ can be found through a meaning of an imaginary part of of an angular characteristic in a direction of a falling wave (a scattering “forward”) $\text{Im} D(180^\circ - \theta_0; 180^\circ)$ [7]:

$$\sigma = (4\pi / k) \text{Im} D(180^\circ - \theta_0; 180^\circ),$$

where $\theta_0$ is an angle of a fall; $\varphi_0 = 0^\circ$.

At an analogy with the scattering cross section $\sigma$ can introduce an idea of a section $\sigma_{rad}$ of an elastic or liquid body under an action of a point source [7]:

$$\sigma_{rad} = \int_0^{2\pi} \int_0^\pi |F(\theta, \varphi)|^2 \sin \theta d\theta d\varphi,$$
Both curves correspond \( \theta_0 = 0^\circ \) (an axially symmetric problem). A first from theirs is observed by \( C = 6,7 \) (unlike from \( C = 6,75 \) for \( \sigma_0 \)), a second resonance is observed by \( C \approx 8,25 \) and corresponds \( L = 1,5 \Lambda \), where \( L \) is a length of a contour of a neutral surface of a shell, \( \Lambda \) is a length of a longitudinal wave (of a zero symmetrical Lamb’s wave) spreading with a velocity \( c_1 \approx 5420 \ m/s \). A curve 1 for an ideal soft spheroid aspires asymptotical to a meaning of a geometrical acoustics \( \sigma_r = 1,0 \) : \( \sigma_r(15,0) = 4,16 ; \sigma_r(65,0) = 2,23 ; \sigma_r(100) = 1,93 \). A curve 3 characterises a radiating faculty of a same shell, if it is exited from an outside by a point source by \( \theta_0 = 0^\circ (h_0 = 50 \ m) \). A section of a radiation \( \sigma_{rad} \) has extremums in those points, what and a relative section of a scattering \( \sigma_r \). A comparison of curves 2 and 3 presented on a Fig. 10 with curve 1 of a Fig. 7 shows, what a relative backscattering section does not give sometimes of a full information about a resonant properties of elastic scattering.

**IV. Conclusions**

With the help of the numerical experiment are found low frequency resonances of elastic spheroidal bodies (entire and in the form of shells) both prolate and oblate by the three – dimensional and axissymmetrical irradiation.

**V. Acknowledgments**

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**References Références Referencias**

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