Resonances of Elastic Spheroidal Bodies

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Abstract

At the basis of the dynamic elasticity theory with the use of Debye?s potentials are found resonances of elastic spheroidal bodies (prolate and oblate) as entire so and in form of shells. In addition to analytic solutions, computer calculations are performed of moduluses of the angular characteristics of the scattering and sections of the scattering of spheroidal bodies.

Index terms— diffraction, debye?s potential, elastic shell, boundary conditions.

1 Introduction

n the paper are investigated resonances of prolate and oblate spheroidal bodies (entire and in the form of shells) by the three-dimensional and axissymmetrical irradiation. By the three-dimensional irradiation for the solution of the problem of the diffraction are used Debye's potentials. To resonances of elastic spheroidal bodies are devoted publications **??**1 -9].

Debye first proposed expanding the vector potential A

2 ??

in the scalar potentials U andV in his publication [10] devoted to studying the behavior of light waves near the local point or local line. Later, this approach was used in solving diffraction problems for cases of the electromagnetic wave diffraction of a sphere, a circular disk and a paraboloid of a revolution **??**11 -16], as well as for the diffraction of longitudinal and transverse waves by spheroidal bodies [7,17].

As applied to problems based on the dynamic elasticity theory, the introduction of Debye's potentials occurs as follows.The displacement vector u ? of an elastic isotropic medium obeys the Lame equation:

where ? and µ are Lame constans, ? is the density of the isotropic medium and ? is the circular frequency of harmonic vibrations. According to the Author : Saint -Petersburg State Navy Technical University, Russia, 190008, Saint -Petersburg, Lotsmanskaya St., 3. e-mail: alexalex -2@yandex.ru Helmholtz theorem, the displacement vector u ? is expressed through scalar ? and vector ? ??? potentials:u grad curl = ? ? + ? ? ?? ?**(2)**

Substituting Eg. (2) in Eg. (1), we obtain two Helmholtz equations, which include one scalar equation for ? and one vector equation for ???? : 2 0, h ?? + ? = (3) 2 2 0. k ?? + ? = ?? ? ?? ?**(4)**

Here1 / h c ? =

is the wavenumber of the longitudinal elastic wave, 1 c is the velocity of this wave,2 2 / k c ? =

is the wavenumber of the transverse elastic wave and 2 c is the velocity of the transverse wave.

In the three-dimensional case, variables involved in scalar equation (3) can be separated into 11 coordinate systems. As for Eq. (4), in the threedimensional problem, this equation yields three independent equations for each of components of the vector function ? ??? in Cartesian coordinate system alone. To overcome this difficulty, one can use Debye's potentials U and V , which obey the Helmholtz scalar equation2 2 0; V k V ? + = 2 2 0. U k U ? + = (5) Vector potential ? ??? (according to Debye) is expanded in potentials V and U as 2 ( ) ( ), curlcurl RU ik curl RV ? = + ?? ? ?? ??**(6)**

where R ?? is the radius vector of a point of the elastic body or the elastic medium.

Let us demonstrate the efficiency of using Debye's potentials in solving the three-dimensional diffraction problem for the case of diffraction by an elastic spheroidal shell. The advantage of the representation (6) becomes evident, if we take into account that potentials V and U obey the Helmholtz J 2 2 2 2 2 2 2 (/) (/) 2 (/)(/)(/) (/) (/) R R B R R B R B ? ? ? ? ? ? ? ? ? ? = ? ? ? ? + ? ? ? ? ? ? ? ? + ? ? ? ? + 2 2 2 2 2 2 (/)(/) (/)(/) , R B R B k B ? ? ? ? ? ? ? ? ? + ? ? ? ? + (7) 2 2 1 2 2 2 0 [(1)] [(/)(/)(/) (/)(/)(/) h R B R B ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? = ? + ? ? ? ? ? ? + ? ? ? ? ? ? ? + 2 2 2 2 (/)(/)(/) (/)(/)(/) (/)(/) R B R B B R ? + ? ? ? ? ? ? + ? ? ? ? ? + 2 1 2 (/)(/)] (sin) (/), B R ik V ? ? ? ? ? ? ? ? ? ? ? ? + ? ? (8) 2 2 1/2 1 2 2 0 2 [(1) sin] [/)(/) (/)(/) h R B R B ik ? ? ? ? ? ? ? ? ? ? ? ? = ? + ? ? ? ? ? + ? ? ? ? ? ? × [(/)(/) (/)(/)], V V ? ? ? ? ? ? ? ? ? ? + ? ? ? ?**(9)**

where:2 2 1/2 0 (1) ; 1 1;1 . B h U ? ? ? ? = ? + ? ? ? + ? ? +?

Spheroidal components of the function (, ,)? ? ? ? ? ? ? ?? ?

are expressed as follows [7]:2 2 1/2 2 2 1/2 0 0 (/) (1) (/)(1) (/), R h h h h ? ? ? ? ? ? ? ? ? ? ? ? ? =? ? + + ? ? + ? ? (10) 2 2 1/2 2 2 1/2 0 0 (/) (1) (/)(1) (/), R h h h h ? ? ? ? ? ? ? ? ? ? ? ? ? =? ? + + ? ? + ? ?**(11)**

,? ? ? ? ?**(12)**

where:2 2 1/2 2 1/2 0 ( ) (1) ; h h ? ? ? ? = ? ? 2 2 1/2 2 1/2 ( )**(1)**

.h ? ? ? ? =? ?

Let us consider in the form of an isotropic elastic spheroidal shell (Fig. 1). All potentials, including the plane wave potential 0 , ? the scattered wave potential 1 , ? the scalar shell poten-tial 2 , ? Debye's potentials U and V and potential 3 ? of the gas filling the shell, can be ex-panded in spheroidal functions:(1) , , 0 1 0 1 , 1 0 2 (,) (,) (,)co n m n m n m m n m n m i S C S C R C m ? ? ? ? ? ? ? ? ? = ? ? = ? ? (**13**) (3) , 1 , 1 , 1 0 2 (,) (,)co m n m n m n m n m B S C R C m ? ? ? ? ? = ? ? = ? ? (14)**(1) (2) , 2 , , , , 0 2** [(,) (,)] (,)co m n m n m n l m n m n l l m n m C R C D R C S C m ? ? ? ? ? ? = ? ? = + ? ? (**15**) (1) , 3 , , 2 2 0 2 (,) (,)co m n m n m n m n m E R C S C m ? ? ? ? ? = ? ? = ? ? (16) (1)**(2) , , , , , 1 2** [(,) (,)] (,)sin ; m n m n m n t m n m n t t m n m U F R C G R C S C m ? ? ? ? ? ? = ? = + ?? (**17**) (1) (2) , , , , , 0 2 [(,) (,)] (,)co m n m n m n t m n m n t t m n m V H R C I R C S C m ? ? ? ? ? ? = ? = + ? ?**(18)**

where:

, 1

3 (,)

m n S C ? ? the angular spheroidal function;

(1) , 1

(,),m n R C ? (2) , 1 (,) m n R C ? and**(3) , 1 (,)**

m n R C ? ? radial spheroidal functions of first, second and third genders;0 ; l C hh = 2 0 ; t C k h = 1 0 , C kh = k ? is the wavenumber of the sound wave in the liquid; 2 1 0 , C k h = 1

k ? is the wavenumber of the sound wave in the gas filling the shell;o h ? the half -focal distance; , , ,, **, , , , , ,**m n m n m n m n m n B C D E F , , ,

, , The corresponding expressions for boundary conditions have the form [7]:m n m n m n G H I ? are unknown expansion coefficients.0 1 1 1 0 1 2 ( ) (/)( ) ( ) (/) ( ) [(/)( ) (/)( )] ; h h h h h h ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? = ? ? ? + ? = ?? ? + ? ? ? ? ? ? ? ? (19) 1 1 1 1 1 2 ( ) (/) ( ) (/) ( ) [(/)( ) (/)( )] ; h h h h h h ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? = ?? ? = ?? ? + ? ? ? ? ? ? ? ? (20) 0 2 2 1 1 0 0 1 2 ( ) 2 [( ) (/) ( ) (/)] ; k h h h h u h u ? ? ? ? ? ? ? ? ? ? µ ? ? ? ? = ? ? + ? = ? ? + ? ? + ? ? (21) 1 2 2 1 1 1 1 3 2 2 [( ) (/) ( ) (/)] ; k h h h h u h u ? ? ? ? ? ? ? ? ? ? µ ? ? ? ? = ? ? =? ? + ? ? + ? ? (22) 0 1 ; 0 (/)(/)(/) (/)(/)(/) ; h h u h h h u h ? ? ? ? ? ? ? ? ? ? ? ? ? ? = = = ? ? + ? ? (23)**0 1**

;0 (/)(/)(/) (/)(/)(/) , h h u h h h u h ? ? ? ? ? ? ? ? ? ? ? ? ? ? = = = ? ? + ? ?**(24)**

where:2 1/2 2 1/2 0 (1) (1) ; h h ? ? ? = ? ? 0 ? ? is the bulk compression coefficient of the liquid; 1 ? ? is the bulk

compression coefficient of the gas filling the shell;1 1 2 ( ) (/) ( ) [(/)( ) (/)( )]; u h h h h h ? ? ? ? ? ? ? ? ? ? ? ? ? = ?? ? + ? ? ? ? ? ? ? ? 1 1 2 ( ) (/) ( ) [(/)( ) (/)( )]; u h h h h h ? ? ? ? ? ? ? ? ? ? ? ? ? = ?? ? + ? ? ? ? ? ? ? ? 1 1 2 ( ) (/) ( ) [(/)( ) (/)( )]. u h h h h h ? ? ? ? ? ? ? ? ? ? ? ? ? = ?? ? + ? ? ? ? ? ? ?

The substitution of series (13) -(18) in boundary conditions (**??**9) -(24) yields an infinite system of equations for the determining of desired coefficients. Because of the ortogonality of trigonometric functions cos m? and sin m? , the infinite system of equations breaks into infinite subsystems with fixed numbers m Each of subsystems is solved by the truncation method. The number of retained terms of expansions (13) -(18) is the greater the wave size for the given potential.The solution of the axissymmetrical problem of the diffraction at elastic spheroidal bodies was presented in **??**

4 ? =

The continuous elastic sphe-roid over the its conduct is very near to the ideal hard scatterer. This was seen by the compare-son of angular characteristics

5 ? =

A relative section r ? of an elastic spheroid shows a r5esonance of a coincidence as this was and in a relative backscattering 0 ? (see Fig. **??**), but a point of a maximum was by). m A section of a radiation rad ? has an extremums in those points, what and a relative section of a scattering . r ? A comparison of curves 2 and 3 presented on a Fig. 10 with curve 1 of a Fig. **??** shows, what a relative backscattering section does not give sometimes of a full information about a resonant properties of elastic scattering.

6 IV.

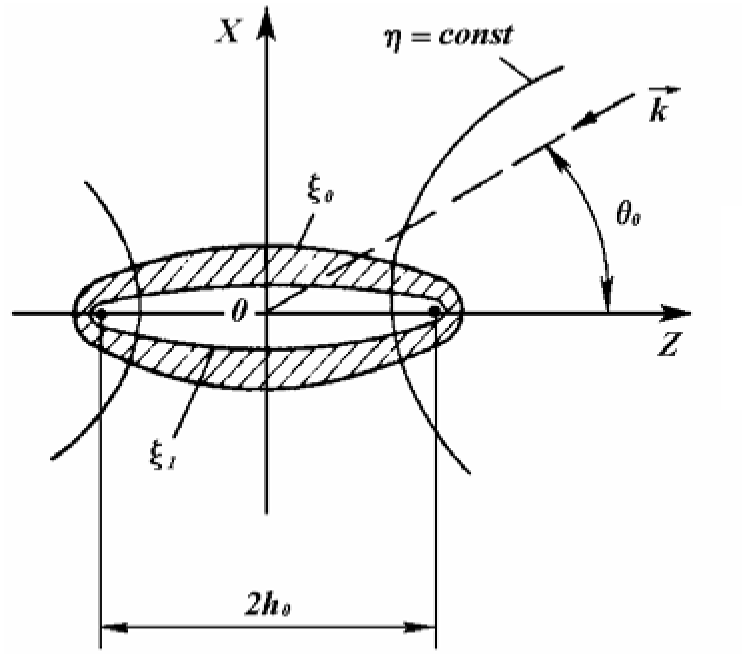
7 Conclusions

With the help of the numerical experiment are found low frequency resonances of elastic spheroidal bodies (entire and in the form of shells) both prolate and oblate by the three -dimensional and axissymmetrical irradiation.

2

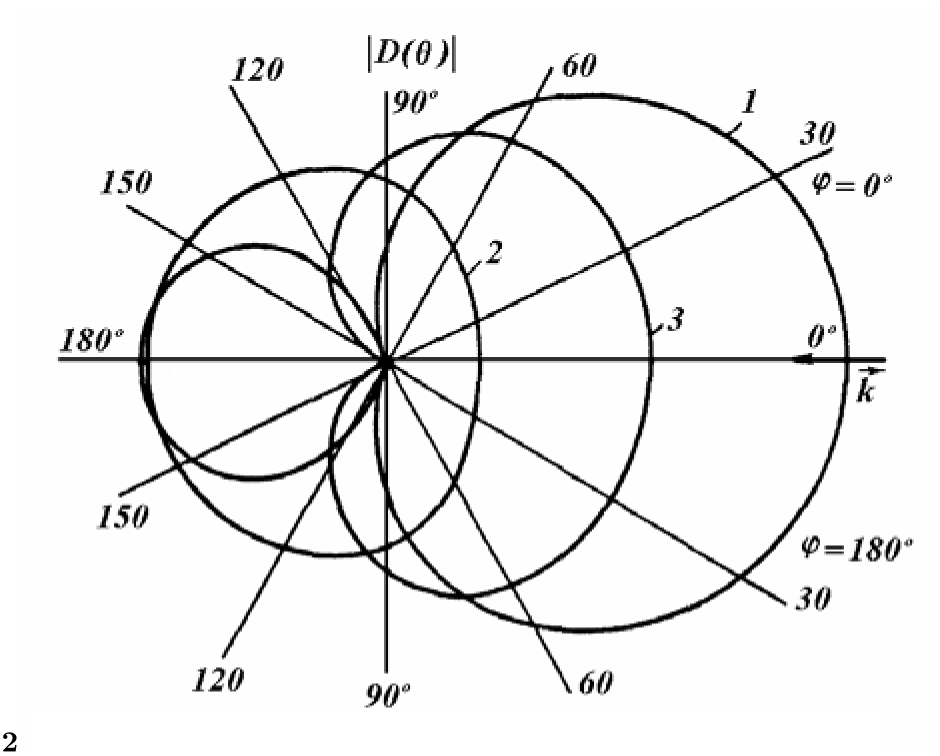
Figure 1: 2 Global

[1]



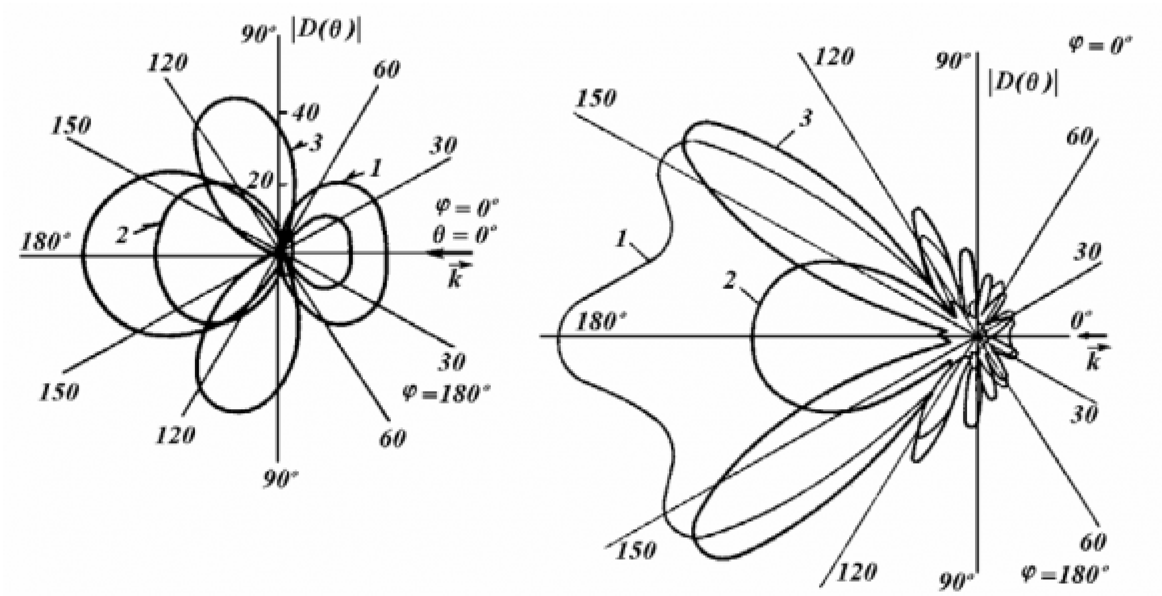
1

Figure 2: Figure 1 :



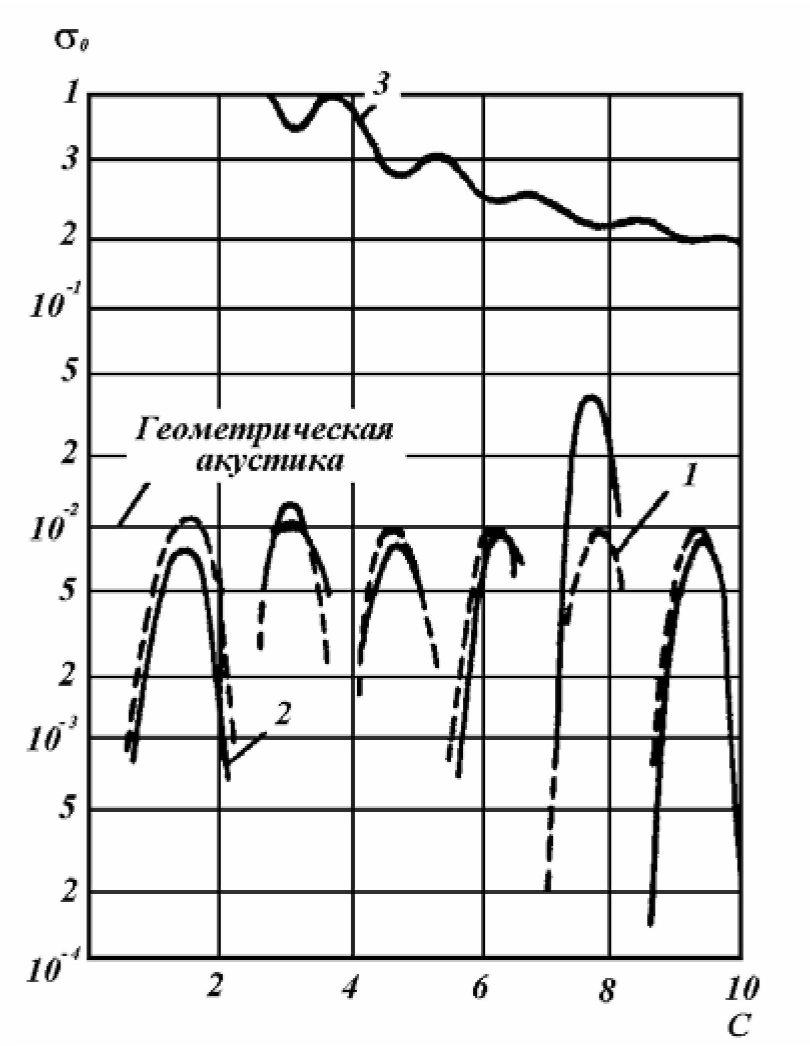
2

Figure 3: Figure 2 :



32

Figure 4: Figure 3 : 2 Global



56272

Figure 5: DFigure 5 :Figure 6 : 2 GlobalFigure 7 : 2 Global

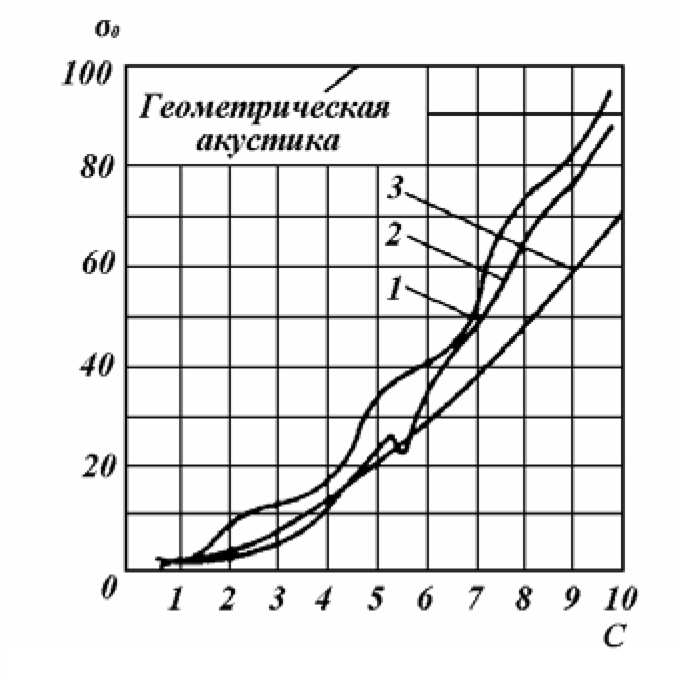

σ₀
100
Геометрическая
акустика
80
60
40
20
0
3
2
1
1 2 3 4 5 6 7 8 9 10
C

Figure 6: D

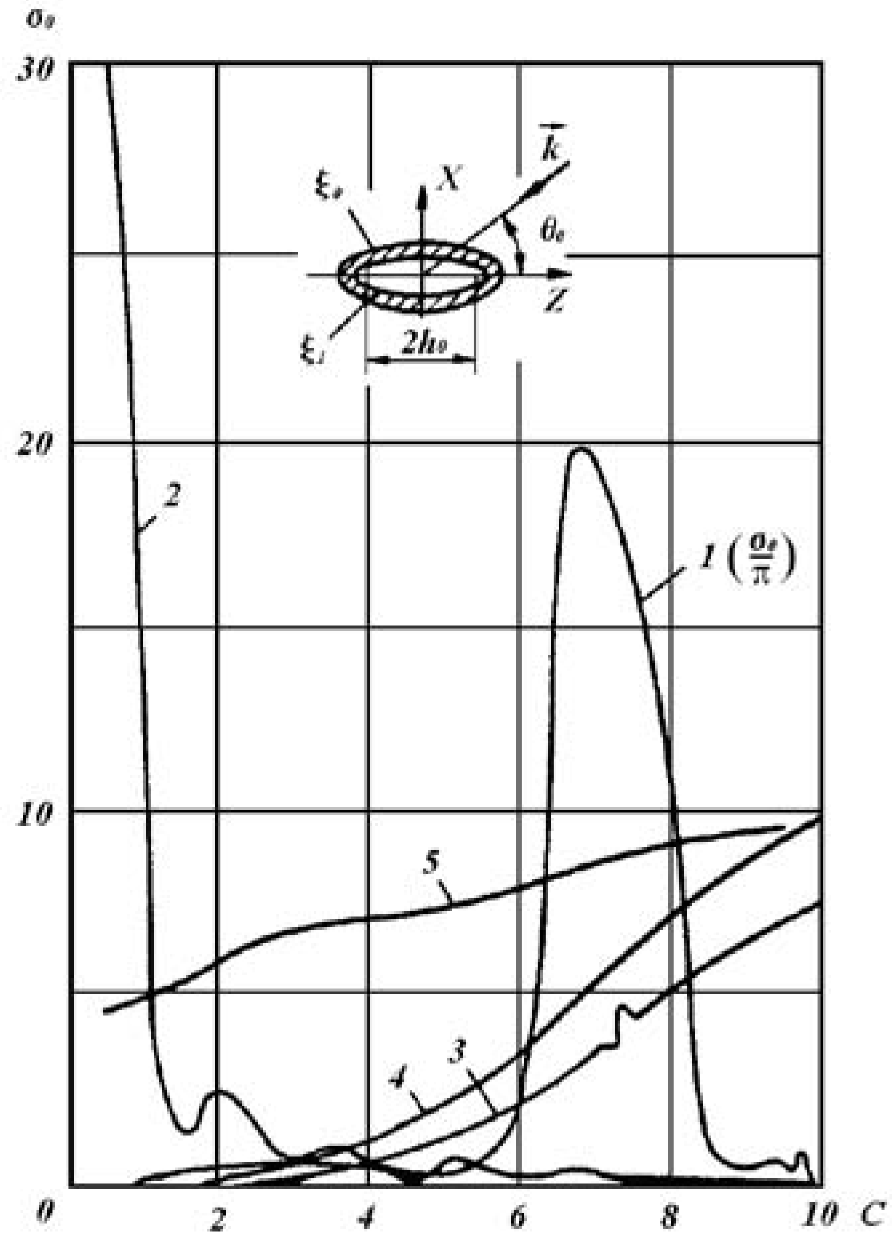


Figure 7: A

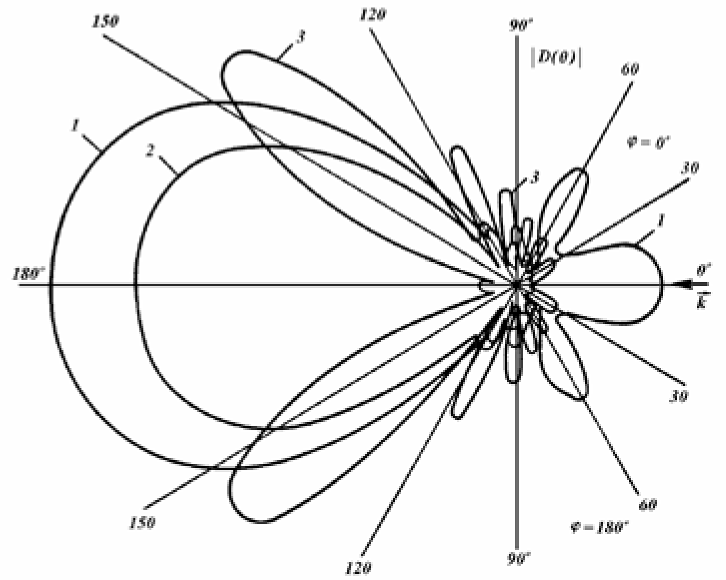


Figure 8: F

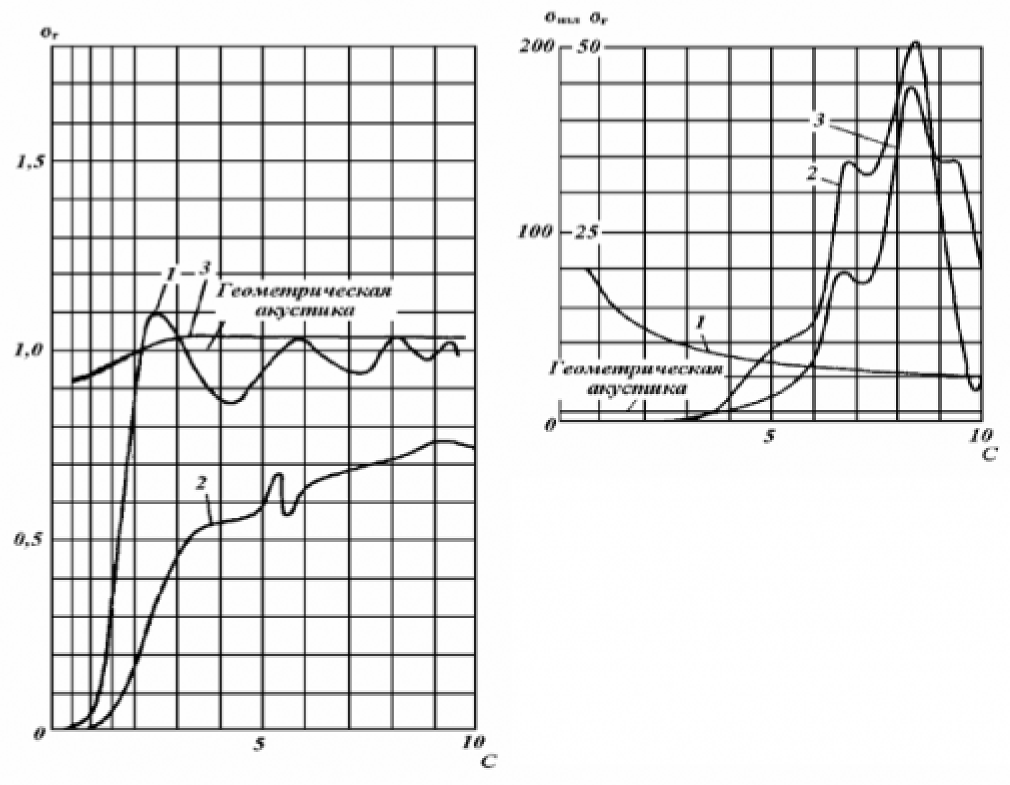


Figure 9:

.1 Acknowledgments

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[Relative scattering cross sections Figure 10 : Relative scattering cross sections and of oblate spheroids the section of radiating of 'Relative scattering cross sections Figure 10 : Relative scattering cross sections and of oblate spheroids the section of radiating of prolate spheroidal bodies 8. A. A. Kleshchev. With Reference to Low Frequency Resonances of Elastic Spheroidal Bodies'. *J.Techn.Ac* 1995. 9.

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