A Comparison Test for Net Sensitivity

By John J. Flaig

California Southern University, United States

Abstract- A common problem in supplier management involves being able to tell if the difference in quality performance between suppliers is significant. Net sensitivity is a process capability measure of the nonconformance risk associated with a supplier's product or service performance. This paper provides a two-sample confidence interval test that will allow the practitioner to determine if there is a significant risk difference between two suppliers with respect to their net sensitivities.

ACOMPARISONTESTFORNETSENSITIVITY

Strictly as per the compliance and regulations of :

A Comparison Test for Net Sensitivity

John J. Flaig

Abstract - A common problem in supplier management involves being able to tell if the difference in quality performance between suppliers is significant. Net sensitivity is a process capability measure of the nonconformance risk associated with a supplier's product or service performance. This paper provides a two-sample confidence interval test that will allow the practitioner to determine if there is a significant risk difference between two suppliers with respect to their net sensitivities.

Keywords: *process capability analysis, sensitivity analysis, net sensitivity.*

I. Introduction

Individuals involved in supplier management need to be able to determine if the quality performance of one supplier is significantly different from another. This information can be used in supplier selection, allocation of the amount of product purchased from each supplier, supplier process improvement programs, and the decision to terminate the purchasing relationship. "The goal of having a good performance metric is to allow the purchaser to assess supplier related performance risk and to take appropriate action" [Bernstein, 1996].

Given a quality characteristic and specified requirements for conformance, there are several statistics that are commonly used to measure supplier process capability. These include the traditional fraction nonconforming (i.e., p or NC), and the modern capability indices C_p, C_{pk}, C_{pm}, C_{pmk}, etc. [Kotz, 1993]. The proper application of these modern indices assumes that the process distribution is stable and approximately Normal. To get around this Normality requirement, several authors have offered alternate solutions [Chou, 1998], [Somerville, 1997]. Or alternately, a Box-Cox power transform can be used to Normalize the observed non-normal data distribution.

Statistical two-sample comparison tests procedures have been developed for all of the common capability indices. However, another measure of potential process risk is net sensitivity (NS) [Flaig, 1999]. Net Sensitivity is a measure of the robustness of the process to potential changes in the mean, and/or variance, and/or specification limits. More specifically, Net Sensitivity is the instantaneous rate of change of the combined areas under the distribution curve above the USL and below the LSL given a change in parameters or specifications. It basically measures the potential effect on the nonconformance rate of changes in the distribution mean, standard deviation, USL, or LSL. This is a useful process performance measure, but it is relatively new and until now there was no two-sample comparison test procedure for practitioners to use to compare net sensitivity results.

A reasonable approach to evaluating the differences in supplier performance for the purchasing department might be to measure the nonconformance rate and the net sensitivity for each supplier and then test to see if any observed differences are significant. Since tests for differences in nonconformance rates exist, the only remaining thing to develop is a test for differences in net sensitivity. This is the goal of the next section.

a) Methodology

It is assumed that theproduct performance distributions for both suppliers' are stable,mound shaped, that the specification limits are near the tails of each distribution for the quality characteristic of interest, and that the observed distributions can be adequately approximated by Johnson distribution curves. This is illustrated in Figure 1.
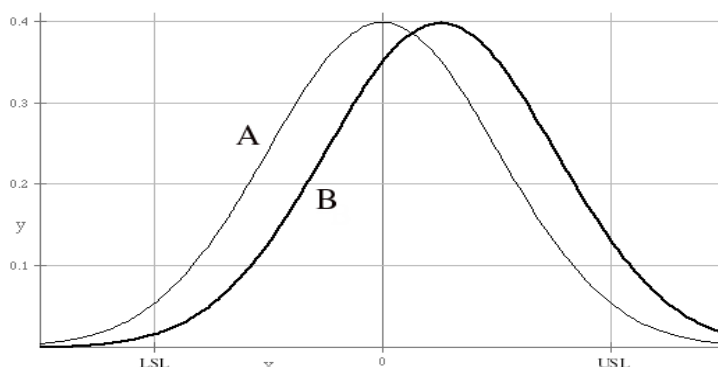

Figure 1 : The Performance Distribution of Supplier A and B

Author : *California Southern University, 1237 Clark Way California San Jose United States. e-mail: johnflaig@yahoo.com*

8

The distribution shift sensitivities are measures of the average effect on the nonconformance rate of a change in the distribution parameters. In other words, they are measures of $\Delta F/\Delta x$, where F is the cumulative density function (cdf) of f(x). Therefore, the left and right sensitivity measures could be defined as follows:

Let A_L be the area under the Johnson curve to the left of the LSL, then

$$A_L = \int_{-\infty}^{LSL} j(x)dx = \int_{-\infty}^{Z_L} f(z)\,dz = F(z_L) - F(-\infty)$$

where j(x) is the Johnson curve and $f(z) = \dfrac{1}{\sqrt{2\pi}}e^{-.5z^2}$ is the standard Normal curve. Then the Sensitivity on the Left (SL) is:

$$NS = \left[f(z_L) - f(z_U)\right]\frac{dz}{dx} = \frac{1}{\sqrt{2\pi}}\left(e^{-.5z_L^2} - e^{-.5z_U^2}\right)\frac{dz}{dx}, \text{ where f(x), z, and } \frac{dz}{dx} \text{ are given by:}$$

$$f(z) = \frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/2\sigma^2}, \ -\infty < \mu < \infty, \ \sigma > 0, \ -\infty < x < \infty$$

$$\frac{dz}{dx} = \frac{\eta}{\lambda}\left(\left[\frac{x-\varepsilon}{\lambda}\right]^2 + 1\right)^{-1/2} \text{ for SU type distributions (i.e., unbounded)}$$

$$\frac{dz}{dx} = \eta\left|\frac{\lambda}{(x-\varepsilon)(\lambda+\varepsilon-x)}\right| \text{ for SB type distributions (i.e., bounded)}$$

$$\frac{dz}{dx} = \frac{1}{\sigma} \text{ for Normal distributions}$$

where $z = \gamma + \eta k_i(x, \lambda, \varepsilon)$ and where the k_i are given by:

$$k_1(x, \lambda, \varepsilon) = \sinh^{-1}\left(\frac{x-\varepsilon}{\lambda}\right) \text{ for SU type distributions}$$

$$k_2(x, \lambda, \varepsilon) = \ln\left(\frac{x-\varepsilon}{\lambda+\varepsilon-x}\right) \text{ for SB type distributions}$$

x_L = LSL and z_L is the transformed value

x_U = USL and z_U is the transformed value

The formula used to compute the constants η, γ, λ, and ε for the SU and SB distributions are given by Farnum [Farnum, 1996].

If the observed distribution is approximately Normal or can be transformed into an approximately Normal distribution, then the Net Sensitivity (NS) can be approximated by:

$$SL = \frac{dA_L}{dx} = \frac{d}{dz}(F(z_L) - F(-\infty))\frac{dz}{dx} = f(z_L)\frac{dz}{dx}$$

Let A_U be the area under the curve to the right of the USL, then

$$A_U = 1 - \int_{-\infty}^{Z_U} f(z)dz$$

and the Sensitivity on the Right (SR) is

$$SR = \frac{dA_U}{dx} = \frac{d}{dz}(F(-\infty) - F(z_U))\frac{dz}{dx} = -f(z_U)\frac{dz}{dx}$$

Since the observed distribution can be satisfactorily modeled by a Johnson curve, then the Net Sensitivity of the combined left tail (L) and the right tail (U) is given by:

$$NS = \left[f(z_L) - f(z_U)\right]\frac{dz}{dx} = \frac{1}{\sigma\sqrt{2\pi}}(e^{-.5z_L^2} - e^{-.5z_U^2})$$

$$\hat{NS} = \frac{1}{s\sqrt{2\pi}}(e^{-.5(LSL-m)^2/s^2} - e^{-.5(USL-m)^2/s^2}) \tag{1}$$

Net Sensitivity is an estimate of the instantaneous rate of change in the fraction nonconforming (i.e., the area under the approximating curve to the left of the LSL combined with the area to the right of the USL) given a change in the mean or standard deviation, or the specification limits of the process.

The variability of NS is determined by the random variables in equation (1). The fixed variables in equation (1) are the specification limits (i.e., USL and LSL) and the random variables are the mean and standard deviation (i.e., m and s). The two distributions making up NS are Normal and the mean and standard deviation are independent for Normal distributions, so the variability of NS follows from the sampling distribution of the random variables m and s. The standard error of the mean is $SEm = s/\sqrt{n}$, and the standard error of the standard deviation is $SEs = s/\sqrt{2n}$ respectively.

II. Example

Let the following supplier management scenario, product specifications, and performance results for two suppliers form the basis for the comparison. The supply base manager would like to know if the nonconformance risk performance difference between the two suppliers is significant at a 95% confidence level. The procedure for answering this question might go as follows:

1. The engineer selects Net Sensitivity as the metric to assess the processes nonconformance risk.
2. Assume $H_0: NS_A = NS_B$ and $H_1: NS_A \neq NS_B$
3. The product has characteristic performance requirements of LSL = -1, and USL = 2
4. The characteristic performance distributions for each supplier are approximately Normal
5. The sample characteristic performance statistics from each supplier are:

Supplier A: n = 100, m = 0, s = 1

Supplier B: n = 100, m = 1.7, s = 1

$$NS(+,+) = \frac{1}{(s + t(\alpha/2, df)*SEs)\sqrt{2\pi}} e^{-.5(LSL-(m+t(\alpha/2,df)*SEm)^2/(s+t(\alpha/2,df)*SEs)}$$

$$- \frac{1}{(s + t(\alpha/2, df)*SEs)\sqrt{2\pi}} e^{-.5(USL-(m+t(\alpha/2,df)*SEm)^2/(s+t(\alpha/2,df)*SEs)}$$

where $t(\alpha/2, df) = 1.98$, $SEm = s/\sqrt{n} = .1000$, and $SEs = s/\sqrt{2n} = .0707$. Evaluation of the four cases yields:

NS(+,+) = 101,000 DPM/unit x, where DPM is Defects Per Million

NS(+,-) = 124,000 DPM/unit x

NS(-,+) = 219,000 DPM/unit x

$$NS_L = \min\{NS(+,+), NS(+,-), NS(-,+), NS(-,-)\} = 101,000 \text{ DPM/unit x,}$$

and

$$NS_U = \max\{NS(+,+), NS(+,-), NS(-,+), NS(-,-)\} = 283,000 \text{ DPM/unit x}$$

The result is the 100*(1 - α) % confidence internal for NS, i.e., $NS_L < NS < NS_U$.

$$CI_A = (101,000 < NS_A < 283,000)$$

$$Min(Abs(NS_A)) = 101,000 \text{ DPM/unit x}$$

Similarly, the confidence interval for the net sensitivity of supplier B's performance (NS_B) is:

$$CI_B = (-459,000 < NS_B < -286,000)$$

$$Min(Abs(NS_B)) = 286,000 \text{ DPM/unit x}$$

The objective in robust process design is to have the value of Net Sensitivity (NS) as close to zero as possible. So in this case, there is sufficient evidence to reject the Null Hypothesis and conclude that the nonconformance risk of supplier A is significantly smaller than that of supplier B with 95% confidence.

The practitioner needs exercise care when applying equation (1) to non-Normal data as it can lead to significant errors because NS is quite sensitive to the distribution shape. Hence when computing the confidence interval for non-Normal data the practitioner must apply the correct dz/dx formula for the type of Johnson curve that is being used to approximate the

Then the 95% confidence interval (CI) for the net sensitivity of supplier A's performance (NS_A) is:

$$CI_A = (101,000 < NS_A < 283,000)$$

The two sided 100*(1- α)% confidence limits for the upper and lower limits are found by evaluating equation (1) using the t-distribution $t(\alpha/2, df)$ for the four combinations of the mean $\pm t(\alpha/2, df)*SEm$, and the standard deviation $\pm t(\alpha/2, df)*SEs$. For example,

NS(-,-) = 283,000 DPM/unit x

Given the sample estimate of the supplier's net sensitivity, then the confidence interval for the population value of NS is denoted:

$$NS_L < NS < NS_U$$

The lower and upper values of the confidence interval for NS are computed as follows:

observed data distribution, or alternately use the Box-Cox transformation to Normalize the observed data distribution.

III. SUMMARY

Sensitivity analysis provides a way of assessing the robustness of a process to the possible impact of changes in the process distribution parameters or specification limits on process capability. So it is important to be able to determine if the net sensitivity of one supplier is significantly different from another. However, this test should be combined with a test for the difference in fraction nonconforming to get a more complete picture of the similarities and differences between suppliers. In some sense, the nonconformance test is a test of expected performance and the net sensitivity is a test of the potential variance of performance. Applying both tests provides a rigorous decision making tool for supplier management.

REFERENCES RÉFÉRENCES REFERENCIAS

1. Bernstein, P. L. (1996). The New Religion of Risk Management. Harvard Business Review, March-April 1996.

2. Chou, Y-M., Polansky, A. M. and Mason, R. L. (1998). Transforming Non-Normal Data to Normality in Statistical Process Control. Journal of Quality Technology, Vol. 30, No. 2, pp. 133-141.
3. Flaig, J. J. (1999). Process Capability Sensitivity Analysis. Quality Engineering, Marcel Dekker, Vol. 11, No. 4, 1999, pp. 587-592.
4. Flaig, J. J. (2000). Process Capability Sensitivity Analysis.Quality Control and Applied Statistics, Marcel Dekker, Vol. 45, No. 3, 2000
5. Farnum, N. R. (1996). Using Johnson Curves to Describe Non-normal Process Data. Quality Engineering, Marcel Dekker, Vol. 9, No. 2, pp. 329-336.
6. Kotz, S. and Johnson, N. L. (1993). Process Capability Indices. New York, Chapman and Hall, 1993.
7. Somerville, S. E. and Montgomery, D. C. (1997). Process Capability Indices and Non-normal Distributions. Quality Engineering, Marcel Dekker, Vol. 9, No. 2, pp. 305-316.

10