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Optimum Design of Autofrettaged Thick-Walled Cylinders

By Md. Tanjin Amin, Dr. Abu Rayhan Md. Ali, Tousif Ahmed & Faisal Ahmed

University of Engineering and Technology, Bangladesh

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Md. Tanjin Amin [α], Dr. Abu Rayhan Md. Ali [σ], Tousif Ahmed [ρ] & Faisal Ahmed [Ѡ]

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I. Introduction

In recent years, the researchers have been investigating for a long time to find out the optimum design of high pressure vessels to save materials and reduce higher cost of construction. Autofrettage is an elasto-plastic technique that increases the capacity of high pressure vessels. In autofrettage process, the cylinder is subjected to an internal pressure which is capable of causing yielding within the wall and then removed. Upon the release of this pressure, a compressive residual hoop stress is developed at certain radial depth at the bore. This compressive stresses reduce the tensile stresses developed as a result of application of working pressure, thus increasing the load bearing capacity [1]-[2]. The magnitude of applied pressure must be below the yield strength of the material. The analysis of residual stresses and deformation in an autofrettaged thick-walled cylinder has been given by Chen [3] and Franklin & Morrison [4]. Determination of optimum autofrettage pressure, p_{opt} and radius of elasto-plastic junction is the major challenge in the analysis of autofrettage process. Harvey [6] tried to give a concept of autofrettage, but the details were missing. A repeated trial calculation approach to determine optimum radius of elasto-plastic junction was proposed by Brownell & Young [7] and Yu [8]. This method was complicated and inaccurate. This method is based on first strength theory which suits brittle materials; but most of the pressure vessels are made of ductile materials which are in excellent agreement with third or fourth strength theory. The graphical method presented by Kong [9] was also inaccurate. Zhu & Yang [10] developed analytic equations for determining optimum autofrettage pressure, p_{opt} and radius of elasto-plastic junction. Ghomi & Majzoobi [11] proposed a set of equations to determine optimum radius of elasto-plastic junction. In this work, optimum radius of elasto-plastic junction is determined using Ghomi & Majzoobi's proposed set of equations and optimum autofrettage pressure is determined using commercially available software ANSYS 11 Classic. Then total work is compared with Zhu & Yang's model.

Authors α σ ρ Ѡ: Department of Mechanical Engineering, Bangladesh University of Engineering and Technology, Bangladesh.
e-mails: moshia.nir@gmail.com, armali@me.buet.ac.bd, tousif.ahmed54@gmail.com, faisalme_106@yahoo.com

II. Analytical Approach

Engineering metals exhibit a linear stress-strain response within the elastic regime, up to their initial yield stress σ_y, their post-yield stress-strain behavior is described by one of the following models: non-linear, bi-linear and multi- linear.

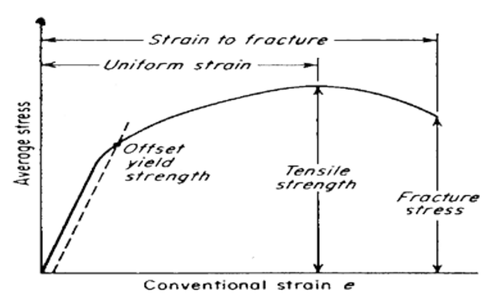

Figure 1 : Non linear Stress Strain Curve

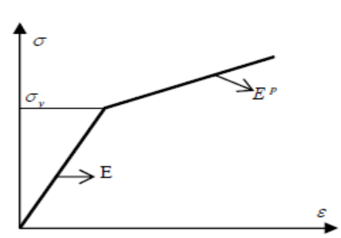

Figure 2 : Bi-linear Stress Strain Curve

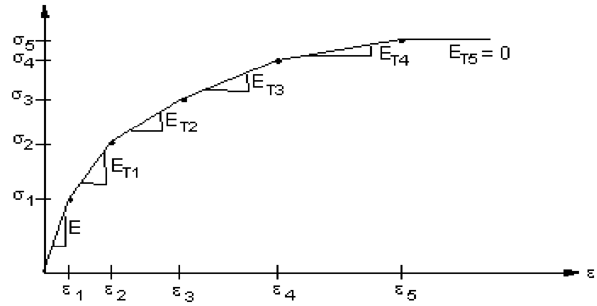

Figure 3 : Multi linear Stress Strain Curve

In this paper, bi-linear elasto-plastic behaviour of materials has been considered. According to fig. 2,

$$\sigma = \sigma_y + E^p \varepsilon \qquad (1)$$

In which, σ is the effective stress, σ_y is the initial yield stress, E^p is the slope of the strain hardening segment of stress strain curve, and ε is the effective strain.

When a metal is strained beyond the yield point, an increasing stress is required to produce additional plastic deformation and the metal apparently becomes stronger and more difficult to deform. This effect is known as strain hardening. Strain hardening of thick-walled cylinders has been also considered in this work.

a) Residual Stress Pattern

Stresses that remain after the original cause of the stresses (external forces, heat gradient) has been removed are residual stresses. They remain along a cross section of the component, even without the external cause. An aluminium cylinder of internal radius, a= 0.02 m, and external radius, b= 0.04 m has been taken into consideration to find out the residual stress pattern in an autofrettaged cylinder. The material properties of aluminium are summarized below in table I.

Table 1 : Material Properties

Mat.	σ_y(MPa)	E(GPa)	E^p(GPa)	ν
Al.	90	72	1.75	0.33

When an internal pressure is applied to the cylinder, the wall becomes plastic up to a certain portion of the cylinder. The internal pressure is called the autofrettage pressure. When autofrettage pressure is released, there is some compressive stress left in the cylinder due to the elasto-plasticity. This compressive stress reduces maximum von Mises stress when another pressure known as working pressure is applied and hence increases the capacity of the cylinder. Ghomi & Majzoobi [11] proposed a set of equations for different regions of autofrettaged cylinder to calculate residual radial and hoop stresses. From these equations, the obtained residual stress pattern is shown in fig. 4:

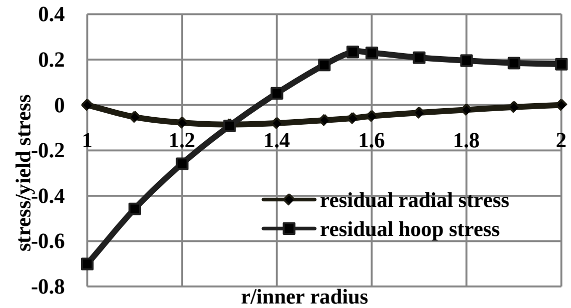

Figure 4 : Residual Stress Pattern in Autofrettaged Cylinder

From fig. 4, it is observed that residual compressive hoop stress occurs near the bore and residual tensile radial stress occurs in outer bore region. The radius up to which walls become plastic, is 0.0312 m. Rayhan, Nidul & Tanvir [5] obtained a similar figure as fig 4. with same values of stress/yield stress and r/inner radius while they examined the distribution of residual stresses with an aluminium cylinder of internal radius, a= 0.01 m, and external radius, b= 0.02 m. This proves that developed residual stresses in autofrettage process is dependent on value of k (b/a).

b) Optimum Radius of Elasto-Plastic Junction

Variation of maximum von Mises stresses along the cylinder is calculated using Ghomi & Majzoobi's [11] proposed a set of equations to determine the optimum radius of elasto-plastic junction. The results are plotted in fig. 5.

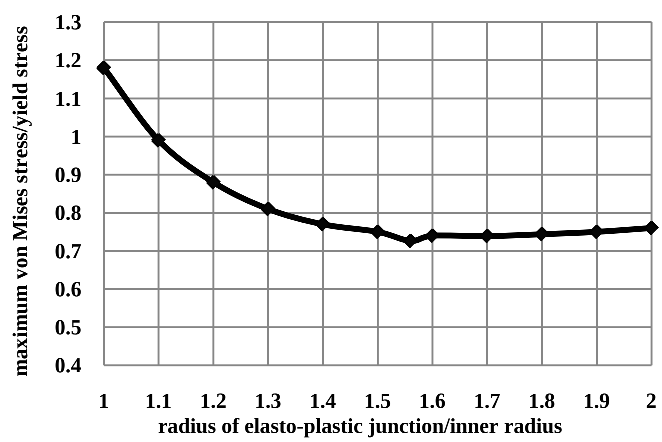

Figure 5 : Variation of Maximum von Mises Stresses at Different Elasto-plastic Radius

From fig. 5, it is observed that when working pressure is applied to the cylinder, maximum von Mises stress decreases as the radius of elasto-plastic junction increases. It decreases to a certain value and then again starts to increase. The point where maximum von Mises stress is minimum, is the optimum radius of elasto-plastic junction [5].

Zhu & Yang [10] developed an equation to determine the optimum radius of elasto-plastic junction.

a) Based on third strength theory (Tresca-yield)

$$r_{opt} = a \exp(p_w / \sigma_y) \qquad (2)$$

b) Based on fourth strength theory (von Mises)

$$r_{opt} = a \exp(\sqrt{3}p_w / 2\sigma_y) \qquad (3)$$

For the aluminium cylinder mentioned previously, r_{opt} is 0.03114 m when working pressure, p_w= 46 MPa. Fourth strength theory has been used to calculate r_{opt}. From fig. 5, r_{opt} is between 0.031 m to 0.032 m.

Ghomi & Majzoobi [11] determined r_{opt} using MATLAB. There is always 3-11% deviation between Zhu & Yang and Ghomi & Majzoobi's model.

c) Effect of k on Optimum Radius of Elasto-Plastic Junction

To observe the effect of k on optimum radius of elasto-plastic junction fig. 6 is considered.

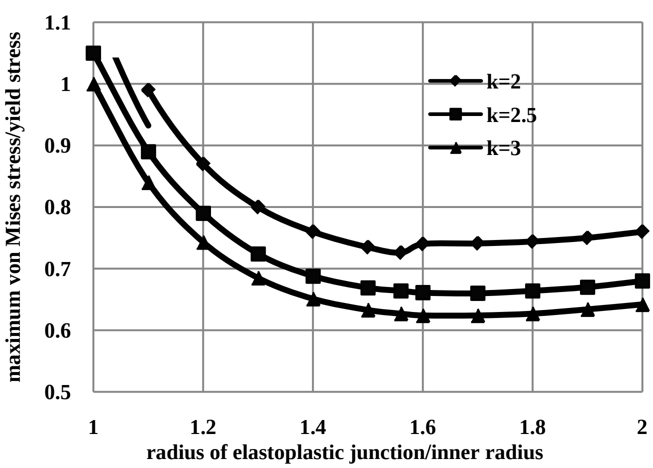

Figure 6 : Variation of Maximum von Mises Stresses at Different Elasto-plastic Radius for Different k values

According to Zhu & Yang, r_{opt} is dependent on inner radius, a for constant value of p_w and σ_y. No matter what is the value of k, r_{opt} will be constant as long as a, p_w and σ_y are constant. But from fig. 6, it is observed that the values of r_{opt} are not same for different values of k though a, p_w and σ_y are constant. As the value of k increases, r_{opt} tends to be increased from Zhu & Yang's calculated r_{opt}. Value of k has significance on optimum elasto-plastic radius.

d) Effect of k on MVS

To observe the effect of k on variation of maximum von Mises stress, maximum von Mises stresses correspond to optimum elasto-plastic radius for different k values at working pressure of 46 MPa & 55 MPa have been potted in fig. 7.

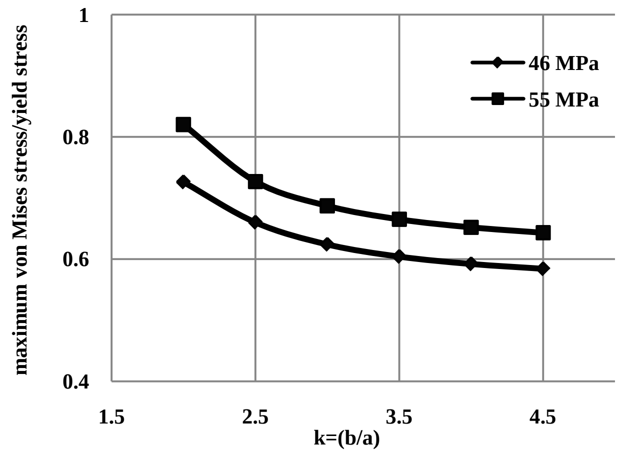

Figure 7 : Variation of Maximum von Mises Stresses Correspond to Optimum Elasto-plastic Radius for Different k values

It has been observed that MVS is lower for a specific value of k for lower working pressure and MVS decreases as k increases. This means the thicker will be the pressure vessel, the more will be the capacity due to autofrettage.

III. Numerical Results

For numerical simulations and modeling, ANSYS 11Classic has been used. The element is Quad 4 Node PLANE 42. It has the capacity of elastic and plastic material modeling. A single steel cylinder of internal radius, a= 0.1 m & outer radius, b= 0.2 m has been considered. The material properties of steel are summarized below in table II.

Table 2 : Material Properties

Mat.	σ_y(MPa)	E(GPa)	E^p(GPa)	ν
Steel	800	207	4.5	0.29

The two pressure limits [1-11] of autofrettage process are:

$$P_y1 = \sigma_y (1-1/k^2)/\sqrt{3} \qquad (4)$$

$$P_y2 = \sigma_y \ln(k) \qquad (5)$$

Where, P_y1 is the pressure at which yielding starts at inner surface and P_y2 is the pressure at which plasticity spreads throughout the cylinder.

For the considered cylinder, P_y1 is 347 MPa and P_y2 is 555 MPa. That means autofrettage effect will start at 347 MPa and continue affecting up to 555 MPa. Before 347 MPa, there will be no autofrettage effect as any portion of the cylinder does not go to plastic regime hence flow stress distribution throughout the cylinder remains unchanged. After 555 MPa, there will be adverse effect and capacity of cylinder will decrease.

The cylinder is subjected autofrettage pressure ranging from 250 MPa to 650 MPa for the working pressure of 100 MPa. Variation of MVS with autofrettage pressure is shown in fig. 8.

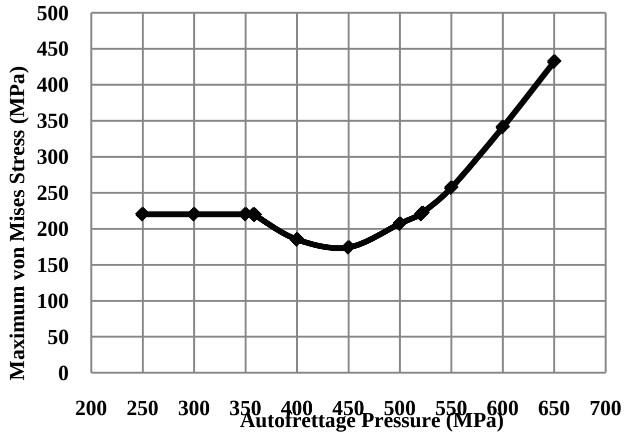

Figure 8 : Variation of Maximum von Mises Stresses at Different Autofrettage Pressure

From fig. 8, it is observed that at first maximum von Mises stress decreases as autofrettage pressure increases. MVS decreases up to a certain minimum value. After that MVS starts to increase again. This minimum value gives the optimum point. That means the autofrettage pressure corresponds to which maximum von Mises stress is minimum, is the optimum autofrettage pressure, p_{opt}. From fig. 8, it is observed that this optimum autofrettage pressure is around 460 MPa. But one interesting observation is that autofrettage starts at 359 MPa and converse effect starts after 521 MPa. The limiting values should be 347 & 555 MPa. To make sure that there is some deviations in simulated value from the value obtained using equation (4) & (5), three single steel cylinders of different k value are internally pressurized at 100 & 200 MPa working pressure. The results are summarized in table III.

Table 3 : P_y1 & P_y2 Values From Equation & Simulation For Different K Values And Working Pressure

k	Working pressure (MPa)	Py1according to equation (MPa)	Py1according to simulation (MPa)	Py2according to equation (MPa)	Py1according to simulation (MPa)
2.0	100	347	359	555	521
2.0	200	347	365	555	615
2.5	100	388	407	733	582
2.5	200	388	407	733	749
3.0	100	411	450	878	625
3.0	200	411	450	878	778

It is observed from the table III that Py1& Py2 obtained from simulation are never equal to that obtained from the equations. There is always significant deviation. For Py1, it does not vary much with the variation of working pressure for same k value. But for Py2, as working pressure increases, it varies enormously for same k value. The variation may go up to 5-30% based on working pressure and k value.

In this paper, the effort is made to find out the effect of working pressure, value of k (b/a) and autofrettage stages on autofrettage process.

a) Effect of Working Pressure

A number of autofrettage pressures have been applied to the steel ranging from 250 MPa to 650 MPa for the working pressures of 100, 200, 300 and 400 MPa. Variation of maximum von Mises stresses has been plotted against different autofrettage pressure for different working pressure in fig. 9.

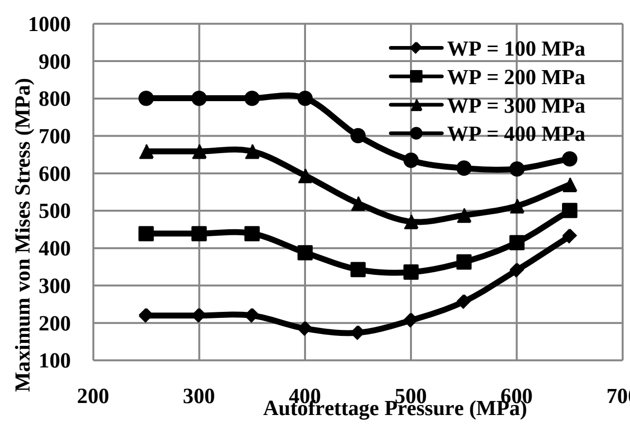

Figure 9 : Variation of Maximum von Mises Stresses at Different Autofrettage Pressure for Different Working Pressure

It has been observed from fig. 9 that MVS remains unaffected nearly P_y1, then it starts to decrease to a certain value. After obtaining this lowest value i.e. p_{opt}, MVS starts to increase. Increasing to a value nearly P_y2 depending on working pressure and k value, MVS experiences converse effect and plasticity spreads throughout the cylinder. For working pressure 100, 200 and 300 MPa, autofrettage starts nearly P_y1. But for working pressure 400 MPa, autofrettage never starts nearly P_y1 rather starts when autofrettage pressure exceeds 400 MPa. It means that autofrettage pressure will have to be always higher than working pressure for the beginning of yielding. The autofrettage pressure must be greater than the working pressure. If the autofrettage pressure is lower than working pressure there is no effect of autofrettage process. From graph analysis it is observed that for working pressure less than 300MPa the auto frett age effect starts when the auto frett age pressure attain a value of 350 MPa. For working pressure 400MPa it is also observed that auto frett age pressure should be more than 400MPa to initiate the auto frett age effect. The optimum point is not same for all the working pressures. As working pressure increases, p_{opt} tends to shift to higher auto frett age pressures. Zhu & Yang [10] developed equation to determine p_{opt}.

- Based on third strength theory (Tresca-yield)

$$P_{opt} = \sigma_y/2[1-(1-2p/\sigma_y) \exp(2p/\sigma_y)] + p \qquad (6)$$

- Based on fourth strength theory (von mises)

$$P_{opt} = \sigma_y/\sqrt{3}\,[1-(1-\sqrt{3}p/\sigma_y) \exp(\sqrt{3}p/\sigma_y)] + p \qquad (7)$$

P is working pressure in above equations. The comparison with Zhu & Yang's P_{opt} based on fourth strength theory and simulated P_{opt} is given below in tableIV.

Table 4 : Comparison Between Zhu & Yang's Calculated Optimum Autofrettage Pressure And Simulated Optimum Autofrettage Pressure

Working Pressure (MPa)	Zhu & Yang's P_{opt} (MPa)	Approximate Simulated P_{opt} (MPa)
100	112.52	461
200	258.00	485
300	451.94	512
400	714.76	600

It is observed from table IV that Zhu & Yang's calculated P_{opt} is far away from simulated P_{opt} for lower working pressure. But as working pressure increases, both values tend to be closer. Further increase in pressure creates significant deviations again.

b) Effect of value of k

The cylinder is subjected to different autofrettage pressures ranging from 250 MPa to 650 MPa for different values of k at 100 MPa working pressure. Inner radius, a is kept constant for all values of k. The results are shown in fig. 10.

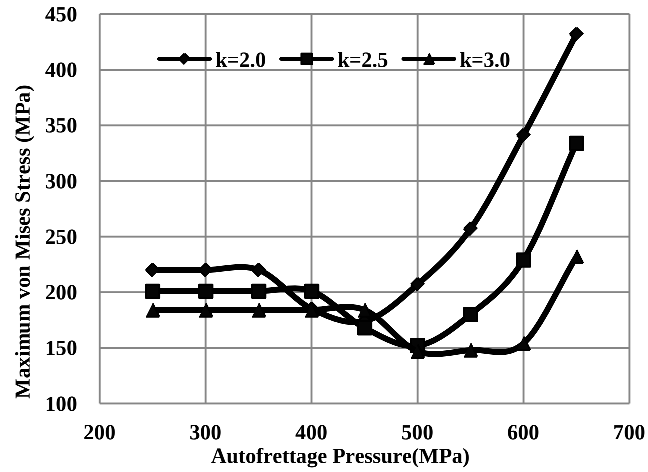

Figure 10 : Variation of Maximum von Mises Stresses at Different Autofrettage Pressure for Different Values of k at 100 MPa Working Pressure keeping inner radius constant

It is observed from fig. 10 that optimum autofrettage pressure increases with the increase of value of k.

Again different autofrettage pressures are applied to the cylinder ranging from 250 MPa to 650 MPa for different values of k at 100 MPa working pressure. This time outer radius, b is kept constant for all values of k. The results are shown below in fig. 11.

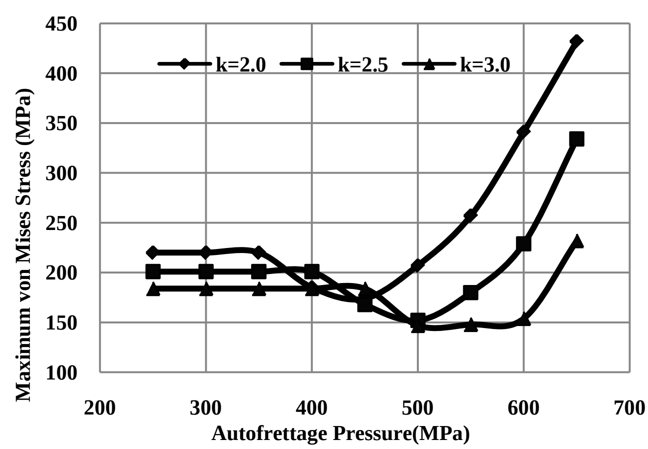

Figure 11 : Variation of Maximum von Mises Stresses at Different Autofrettage Pressure for Different Values of k at 100 MPa Working Pressure keeping outer radius constant

From fig. 11, it is observed that the flow stress pattern remains same though inner radius and outer radius are changed from the previous case. Fig. 10 & fig. 11 suggest that p_{opt} is only dependent on k value. For same k values, p_{opt} is always same.

The percentage of reduction of MVS due to different k values for autofrettage pressure of 500 MPa and working pressure of 100 MPa is summarized in table V.

Table 5 : Effect of K Value on Mvs Reduction

K (b/a)	Without autofrettage (MPa)	With Autofrettage (MPa)	% of reduction of von Mises stress
2.0	220	207	5.91
2.5	201	152	24.38
3.0	184	147	20.11

From the table V, it is observed that percentage of reduction of MVS is higher at higher values of k. Though it is found that for k=3.0 percentage of reduction of MVS is lower than that of k=2.5, actually k=3.0 gives more reduction on higher autofrettage pressure and for k=3.0, autofrettage process starts later than k=2.5. For comparison among these three k values, we needed to consider lower autofrettage pressure as when k=2.0, converse effect starts after autofrettage pressure of 521 MPa. That's why we find lower reduction for k=3.0. But actually higher k values will give better reduction. That means the autofrettage effect is more beneficial with the increase of the thickness of the cylinder wall.

c) Effect of Autofrettage Stages

To observe the effect of stage loading, the cylinder was subjected to autofrettage pressure of 450 MPa and working pressure of 200 MPa in two steps. At first step, it is done in three stages and in second step; it is done in eleven stages.

3 Stage Autofrettage

Stage 1 (MPa)	Stage 2 (MPa)	Stage 3 (MPa)
450	0	250

11 Stage Autofrettage

Stage 1 (MPa)	Stage 2 (MPa)	Stage 3 (MPa)	Stage 4 (MPa)	Stage 5 (MPa)	Stage 6 (MPa)
450	0	400	0	350	0

Stage 7 (MPa)	Stage 8 (MPa)	Stage 9 (MPa)	Stage 10 (MPa)	Stage 11 (MPa)
300	0	250	0	250

It is observed that the MVS in final stage in both cases is 343 MPa. So, it can be concluded if the values of pressure at first and last stage remains same, there is no effect of loading stages on autofrettage process.

IV. Conclusion

The following decisions can be taken from the investigations mentioned in this paper:

1. In autofrettaged cylinder, maximum stress occurs in near bore region.
2. The maximum applicable pressure is limited by the yield strength of the materials.
3. Flow stress distribution remains same for same k values.
4. Optimum elasto-plastic radius is not constant for different k value.
5. Higher the working pressure; more will be the benefit from autofrettage process.
6. The limiting values of autofrettage pressure Py1 & Py2 do not follow the calculated value strictly.
7. The thicker will be the material; more will be the capacity.
8. Autofrettage pressure must have to be higher than working pressure to start yielding.
9. Loading stages has no effect on autofrettage process as long as the pressures at first & last stages remain constant.
10. Zhu & Yang's calculated optimum autofrettage pressure is always far away from the simulated result.

Appendix

Table : Nomen Clature

Symbol	Meaning	Unit
a	Internal radius	(m)
b	External radius	(m)
$\sigma_{\theta r}$	Residual hoop stress	(MPa)
σ_{rr}	Residual radial stress	(MPa)
P_w	Working pressure	(MPa)

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