

Optimum Design of Autofrettaged Thick-Walled Cylinders

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Abstract

The effect of autofrettage process on thick-walled cylinders has been investigated here. It is observed that flow stress distribution along the cylinders remains same for same k values. Comparison with Zhu Yang's model has been also done in determination of optimum elasto-plastic radius, r_{opt} and optimum autofrettage pressure, p_{opt} . Equivalent von Mises stress is used as yield criterion. It is observed that percentage of reduction of maximum von Mises stress increases as value of k and working pressure increases. Maximum von Mises stress is minimum for lower working pressure at same value of k and autofrettage pressure. Autofrettage process never starts if autofrettage pressure does not exceed working pressure. It is also observed that two limits of autofrettage pressure P_{y1} P_{y2} are not appropriate. Effect of loading stages on autofrettage process is also investigated. As long as the pressures in first and last stage remains constant, there is no effect of loading stages on autofrettage process; no matters how many stages prevails between these two pressures.

Index terms— autofrettage, residual stress, von mises stress, working pressure, pressure vessel, elasto-plastic junction.

1 Introduction

In recent years, the researchers have been investigating for a long time to find out the optimum design of high pressure vessels to save materials and reduce higher cost of construction. Autofrettage is an elasto-plastic technique that increases the capacity of high pressure vessels. In autofrettage process, the cylinder is subjected to an internal pressure which is capable of causing yielding within the wall and then removed. Upon the release of this pressure, a compressive residual hoop stress is developed at certain radial depth at the bore. This compressive stresses reduce the tensile stresses developed as a result of application of working pressure, thus increasing the load bearing capacity [1]- [2]. The magnitude of applied pressure must be below the yield strength of the material. The analysis of residual stresses and deformation in an autofrettaged thick-walled cylinder has been given by Chen [3] and Franklin & Morrison [4]. Determination of optimum autofrettage pressure, p_{opt} and radius of elasto-plastic junction is the major challenge in the analysis of autofrettage process. Harvey [6] tried to give a concept of autofrettage, but the details were missing. A repeated trial calculation approach to determine optimum radius of elasto-plastic junction was proposed by Brownell & Young [7] and Yu [8]. This method was complicated and inaccurate. This method is based on first strength theory which suits brittle materials; but most of the pressure vessels are made of ductile materials which are in excellent agreement with third or fourth strength theory. The graphical method presented by Tong [9] was also inaccurate. Zhu & Yang [10] developed analytic equations for determining optimum autofrettage pressure, $p_{opt II}$.

2 ANALYTICAL APPROACH

and radius of elasto-plastic junction. Ghomi & Majzoubi [11] proposed a set of equations to determine optimum radius of elasto-plastic junction. In this work, optimum radius of elasto-plastic junction is determined using Ghomi & Majzoubi's proposed set of equations and optimum autofrettage pressure is determined using commercially available software ANSYS 11 Classic. Then total work is compared with Zhu & Yang's model.

Engineering metals exhibit a linear stress-strain response within the elastic regime, up to their initial yield stress σ_y , their post-yield stress-strain behavior is described by one of the following models: non-linear, bilinear and multi-linear. In this paper, bi-linear elasto-plastic behaviour of materials has been considered. According to fig. 1, $\sigma = \sigma_y + E_p \epsilon$ (1)

In which, σ is the effective stress, σ_y is the initial yield stress, E_p is the slope of the strain hardening segment of stress strain curve, and ϵ is the effective strain.

When a metal is strained beyond the yield point, an increasing stress is required to produce additional plastic deformation and the metal apparently becomes stronger and more difficult to deform. This effect is known as strain hardening. Strain hardening of thick-walled cylinders has been also considered in this work.

3 a) Residual Stress Pattern

Stresses that remain after the original cause of the stresses (external forces, heat gradient) has been removed are residual stresses. They remain along a cross section of the component, even without the external cause. An aluminium cylinder of internal radius, $a = 0.02$ m, and external radius, $b = 0.04$ m has been taken into consideration to find out the residual stress pattern in an autofrettaged cylinder. The material properties of aluminium are summarized below in table 1. When an internal pressure is applied to the cylinder, the wall becomes plastic up to a certain portion of the cylinder. The internal pressure is called the autofrettage pressure. When autofrettage pressure is released, there is some compressive stress left in the cylinder due to the elasto-plasticity. This compressive stress reduces maximum von Mises stress when another pressure known as working pressure is applied and hence increases the capacity of the cylinder. Ghomi & Majzoobi [11] proposed a set of equations for different regions of autofrettaged cylinder to calculate residual radial and hoop stresses. From these equations, the obtained residual stress pattern is shown in fig. 4:

4 b) Optimum Radius of Elasto-Plastic Junction

Variation of maximum von Mises stresses along the cylinder is calculated using Ghomi & Majzoobi's [11] proposed a set of equations to determine the optimum radius of elasto-plastic junction. The results are plotted in fig. 5.

For the aluminium cylinder mentioned previously, r_{opt} is 0.03114 m when working pressure, $p_w = 46$ MPa. Fourth strength theory has been used to calculate r_{opt} . From fig. 5, r_{opt} is between 0.031 m to 0.032 m.

Ghomi & Majzoobi [11] determined r_{opt} using MATLAB. There is always 3-11% deviation between Zhu & Yang and Ghomi & Majzoobi's model.

5 c) Effect of k on Optimum Radius of Elasto-Plastic Junction

To observe the effect of k on optimum radius of elasto-plastic junction fig. 6 is considered. It has been observed that MVS is lower for a specific value of k for lower working pressure and MVS decreases as k increases. This means the thicker will be the pressure vessel, the more will be the capacity due to autofrettage.

6 III.

7 NUMERICAL RESULTS

For numerical simulations and modeling, ANSYS 11Classic has been used. The element is Quad 4 Node PLANE 42. It has the capacity of elastic and plastic material modeling. A single steel cylinder of internal radius, $a = 0.1$ m & outer radius, $b = 0.2$ m has been considered. The material properties of steel are summarized below in table 2. The two pressure limits [1][2][3][4][5][6][7][8][9][10][11] of autofrettage process are: $P_{y1} = \sigma_y (1 - 1/k)$ and $P_{y2} = \sigma_y \ln(k)$

Where, P_{y1} is the pressure at which yielding starts at inner surface and P_{y2} is the pressure at which plasticity spreads throughout the cylinder.

For the considered cylinder, P_{y1} is 347 MPa and P_{y2} is 555 MPa. That means autofrettage effect will start at 347 MPa and continue affecting up to 555 MPa. Before 347 MPa, there will be no autofrettage effect as any portion of the cylinder does not go to plastic regime hence flow stress distribution throughout the cylinder remains unchanged. After 555 MPa, there will be adverse effect and capacity of cylinder will decrease.

The cylinder is subjected autofrettage pressure ranging from 250 MPa to 650 MPa for the working It is observed from the table 3 that P_{y1} & P_{y2} obtained from simulation are never equal to that obtained from the equations. There is always significant deviation. For P_{y1} , it does not vary much with the variation of working pressure for same k value. But for P_{y2} , as working pressure increases, it varies enormously for same k value. The variation may go up to 5-30% based on working pressure and k value.

In this paper, the effort is made to find out the effect of working pressure, value of k (b/a) and autofrettage stages on autofrettage process.

8 a) Effect of Working Pressure

A number of autofrettage pressures have been applied to the steel ranging from 250 MPa to 650 MPa for the working pressures of 100, 200, 300 and 400 MPa. Variation of maximum von Mises stresses has been plotted

against different autofrettage pressure for different working pressure in fig. ??, nearly P_y rather starts when autofrettage pressure exceeds 400 MPa. It means that autofrettage pressure will have to be always higher than working pressure for the beginning of yielding. The autofrettage pressure must be greater than the working pressure. If the autofrettage pressure is lower than working pressure there is no effect of autofrettage process. From graph analysis it is observed that for working pressure less than 300MPa the auto frett age effect starts when the auto frett age pressure attain a value of 350 MPa. For working pressure 400MPa it is also observed that auto frett age pressure should be more than 400MPa to initiate the auto frett age effect. The optimum point is not same for all the working pressures. As working pressure increases, p_{opt} tends to shift to higher auto frett age pressures. Zhu & Yang [10] developed equation to determine p_{opt} .

? Based on third strength theory (Tresca-yield) $P_{opt} = \frac{\sigma_y}{2} [1 - (1 - 2p/\sigma_y) \exp(2p/\sigma_y)] + p$ (6)

? Based on fourth strength theory (von mises) $P_{opt} = \frac{\sigma_y}{\sqrt{3}} [1 - (1 - \sqrt{3}p/\sigma_y) \exp(\sqrt{3}p/\sigma_y)] + p$ (7)

P is working pressure in above equations. The comparison with Zhu & Yang's P_{opt} based on fourth strength theory and simulated P_{opt} is given below in table V. From the table V, it is observed that percentage of reduction of MVS is higher at higher values of k . Though it is found that for $k=3.0$ percentage of reduction of MVS is lower than that of $k=2.5$, actually $k=3.0$ gives more reduction on higher autofrettage pressure and for $k=3.0$, autofrettage process starts later than $k=2.5$. For comparison among these three k values, we needed to consider lower autofrettage pressure as when $k=2.0$, converse effect starts after autofrettage pressure of 521 MPa. That's why we find lower reduction for $k=3.0$. But actually higher k values will give better reduction. That means the autofrettage effect is more beneficial with the increase of the thickness of the cylinder wall.

9 c) Effect of Autofrettage Stages

To observe the effect of stage loading, the cylinder was subjected to autofrettage pressure of 450 MPa and working pressure of 200 MPa in two steps. At first step, it is done in three stages and in second step; it is done in eleven stages. It is observed that the MVS in final stage in both cases is 343 MPa. So, it can be concluded if the values of pressure at first and last stage remains same, there is no effect of loading stages on autofrettage process.

IV.

10 CONCLUSION

The following decisions can be taken from the investigations mentioned in this paper: 1. In autofrettaged cylinder, maximum stress occurs in near bore region. 2. The maximum applicable pressure is limited by the yield strength of the materials. 3. Flow stress distribution remains same for same k values.

4. Optimum elasto-plastic radius is not constant for different k value. 5. Higher the working pressure; more will be the benefit from autofrettage process. 6. The limiting values of autofrettage pressure P_{y1} & P_{y2} do not follow the calculated value strictly. 7. The thicker will be the material; more will be the capacity. 8. Autofrettage pressure must have to be higher than working pressure to start yielding. 9. Loading stages has no effect on autofrettage process as long as the pressures at first & last stages remain constant. 10. Zhu & Yang's calculated optimum autofrettage pressure is always far away from the simulated result.

11 APPENDIX

1



Figure 1: Figure 1 :Figure 3 :

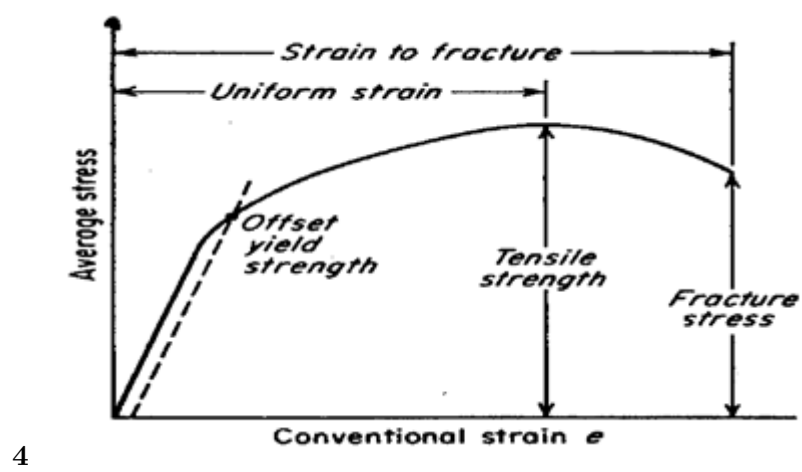


Figure 2: Figure 4 :

5

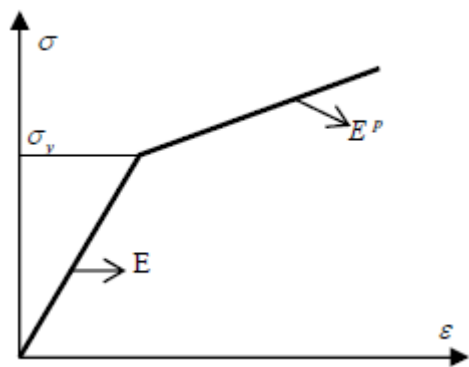


Figure 3: Figure 5 :

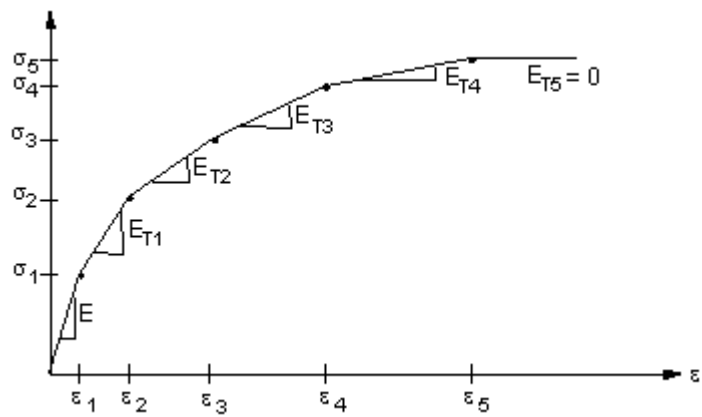


Figure 4:

1

Mat. ? y (MPa) E(GPa) E p (GPa) ?
Al.

90 72 1.75 0.33

Figure 5: Table 1 :

2

Steel 800 207 4.5 0.29

Figure 6: Table 2 :

3

k	Working pressure (MPa)	Py1according to equation (MPa)	Py1according to simulation (MPa)	Py2according to equation (MPa)	Py1according to simulation (MPa)
2.0	100	347	359	555	521
2.0	200	347	365	555	615
2.5	100	388	407	733	582
2.5	200	388	407	733	749
3.0	100	411	450	878	625
3.0	200	411	450	878	778

Figure 7: Table 3 :

4

Optimum Autofrettage Pressure And Simulated Optimum Autofrettage Pressure		
Working Pressure (MPa)	Zhu & Yang's P opt (MPa)	Approximate Simulated P opt (MPa)
100	112.52	461
200	258.00	485
300	451.94	512
400	714.76	600

Figure 8: Table 4 :

5

K (b/a)	Without autofrettage (MPa)	With Autofrettage (MPa)	% of reduction of von Mises stress
2.0	220	207	5.91
2.5	201	152	24.38
3.0	184	147	20.11

Figure 9: Table 5 :

.1 ()

A Year

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