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# Deflection of Functionally Gradient Material Plate under Mechanical, Thermal and Thermomechanical Loading

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**Abstract** - Functionally gradient materials are one of the most widely used materials. The objective of this research work is to perform a thermo-mechanical analysis of functionally gradient material square laminated plate made of Aluminum / Zirconia and compare with pure metal and ceramic. The plates are assumed to have isotropic, two-constituent material distribution through the thickness, and the modulus of elasticity of the plate is assumed to vary according to a power-law distribution in terms of the volume fractions of the constituents. To achieve this objective, we shall use first shear deformation theory of plates and numerical analysis will be accomplished using finite element model prepared in ANSYS software. The laminated Functionally Gradient Material plate is divided in to layers and their associated properties are then layered together to establish the through-the-thickness variation of material properties. The displacement fields for functionally gradient material plate structures under mechanical, thermal and thermo mechanical loads are analyzed under simply supported boundary condition.

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## I. INTRODUCTION

History is often marked by the materials and technology that reflect human capability and understanding. Many times scales begins with the stone age, which led to the Bronze, Iron, Steel, Aluminum and Alloy ages as improvements in refining, smelting took place and science made all these possible to move towards finding more advance materials possible. It has become possible to develop new composite materials with improved physical and mechanical properties. Functionally gradient materials (FGM) are a class of composites that have a gradual variation of material properties from one surface to another. These novel materials were proposed by the Japanese in 1984 and are projected as thermal barrier materials for applications in space planes, space structures and nuclear reactors, to name only a few. In general, all the multi-phase materials, in which the material properties are varied gradually in a predetermined manner, fall into the category of

functionally gradient materials. The gradients can be continuous on a microscopic level, or they can be laminates comprised of gradients of metals, ceramics, polymers, or variations of porosity/density as shown in figure 1.

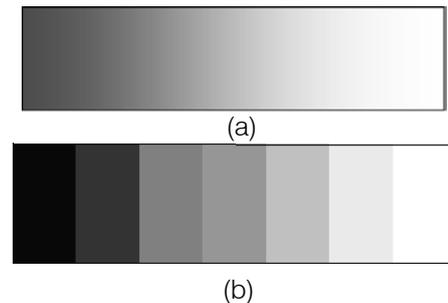


Figure 1 : Gradient of FGMs; (a) continuously graded and (b) discretely layered FGMs

## II. LITERATURE REVIEW

A huge amount of published literature observed for evaluation of thermomechanical behavior of functionally gradient material plate using finite element techniques. It includes both linearity and non linearity in various areas. Few of published literature highlight the importance of topic. A laminated theory for a desired degree of approximation of the displacements through the laminate thickness, allowing for piecewise approximation of the inplane deformation through individual laminae reported by reddy [1]. S. Suresh and A. Mortensen (1997) focus a review of the processing of functionally graded metal-ceramic composites and their thermo mechanical behavior. They discussed various approximations for determination of properties and their limitations are highlighted. They have focused on various issues related to functionally gradient material manufacturing [2]. G. N. Praveen and reddy (1997) reported the static and dynamic response of the functionally graded material plates by varying the volume fraction of the ceramic and metallic constituents using a simple power law distribution. [3]. J. N. Reddy (1998) reported theoretical formulations and finite element analyses of the thermomechanical, transient response of functionally graded cylinders and plates with Nonlinearity. [4]. J. N. Reddy (2000) gives Navier's solutions of rectangular plates, and Finite element models based on the third-order shear deformation

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plate theory for functionally graded plates. [5]. J.N. Reddy et. al. (2001) reported three-dimensional thermomechanical deformations of simply supported, functionally graded rectangular plates. The temperature, displacements and stresses of the plate are computed for different volume fractions of the ceramic and metallic constituents. [6]. Bhavani V. Sankar (2002) solved the thermoelastic equilibrium equations for a functionally graded beam in closed-form [7]. Senthil S. Vel and R.C. Batra (2003) calculate an analytical solution for three-dimensional thermomechanical deformations of a simply supported functionally graded rectangular plate subjected to time-dependent thermal loads [8]. M. Tahani<sup>1</sup>, M. A. Torabizadeh and A. Fereidoon (2006), have reported analytical method is developed to analyze analytically displacements and stresses in a functionally graded composite beam subjected to transverse load and the results obtained from this method are compared with the finite element solution done by ANSYS [9]. Ki-Hoon Shin (2006) suggests that the Finite Element Analysis (FEA) is an important step for the design of structures or components formed by heterogeneous objects such as multi-materials, Functionally Graded Materials (FGMs), etc [10]. Fatemeh Farhatnia, Gholam-Ali Sharifi and Saeid Rasouli (2009), determined the thermo-mechanical stress distribution has been determined for a three layered composite beam having a middle layer of functionally graded material (FGM), by analytical and numerical methods. They found that there is no practically considerable difference, between stress profiles obtained analytically and from FEM model and ANSYS [11]. M.K. Singha, T.Prakash and M.Ganapathi (2011) reported The nonlinear behaviors of functionally graded material (FGM) plates under transverse distributed load. [12].

D.K. Jha, Tarun Kant and R.K. Singh (2012) reported a critical review of the reported studies in the area of thermo-elastic and vibration analyses of functionally graded (FG) plates since 1998. They have presented various areas of work for FGM and their application. [13]. Srinivas.G and Shiva Prasad.U focused on analysis of FGM flat plates under pressure i.e. mechanical loading in order to understand the effect variation of material properties has on structural response. [14].

### III. FGM PLATE MODELLING

#### a) Modeling Introduction

With the advent of powerful computers and robust software, computational modeling has emerged as a very informative and cost effective tool for materials design and analysis. Modeling often can both eliminate costly experiments and provide more information than can be obtained experimentally. A wide variety of software, for e.g. ABAQUS, ANSYS etc., are commercially available and can be used to model and

analyze FGM's. In this report Ansys 13.0 is used as a tool for analysis and the element SHELL 181 is used.

#### b) Material Properties

Volume fraction and material properties of FGM's may vary in the thickness direction or in the plane of a plate. The FGM modeled usually is done with one side of the material as ceramic and the other side as metal. A mixture of the two materials composes the through-the-thickness characteristics. This material variation is dictated by a parameter,  $n$ . At  $n = 0$  the plate is a fully ceramic plate while at  $n = \infty$  the plate is fully metal. Material properties are dependent on the  $n$  value and the position in the plate and vary according to a power law.

Here we assume that the material property gradation is through the thickness and we represent the profile for volume fraction variation by the expression of power law, i.e.

$$P(z) = (P_t - P_b)V + P_b$$

$$V = \left(\frac{z}{h}\right)^n$$

For the material index ( $n$ )=2;

**At bottom layer**, ( $z/h$ )=0 and so  $V=0$

$$\text{hence } P(z) = P_b$$

**At top layer**, ( $z/h$ )=1 and so  $V=1$

$$\text{hence } P(z) = P_t$$

where  $P$  denotes a generic material property like modulus,  $P_t$  and  $P_b$  denote the property of the top and bottom faces of the plate, respectively,  $h$  is the total thickness of the plate, and  $n$  is a parameter that dictates the material variation profile through the thickness.

Table 1 : Material Property

S.No.	Property	Aluminum	Zirconia
1	Young's modulus	70 GPa	151 GPa
2	Poisson's ratio	0.3	0.3
3	Thermal conductivity	204 W/mK	2.09 W/mK
4	Thermal expansion	$23 \times 10^{-6} / ^\circ\text{C}$	$10 \times 10^{-6} / ^\circ\text{C}$

The study of the behaviour of an FGM plate under mechanical loads is done for a square plate whose constituent materials are taken to be Aluminum and zirconia. The top surface of the plate is ceramic (zirconia) rich and the bottom surface is metal (Aluminium) rich. Variation of effective young's modulus, Thermal conductivity and Thermal expansion with respect to parameter  $z/h$  for various material index as shown in figure 2,3 and 4 respectively.

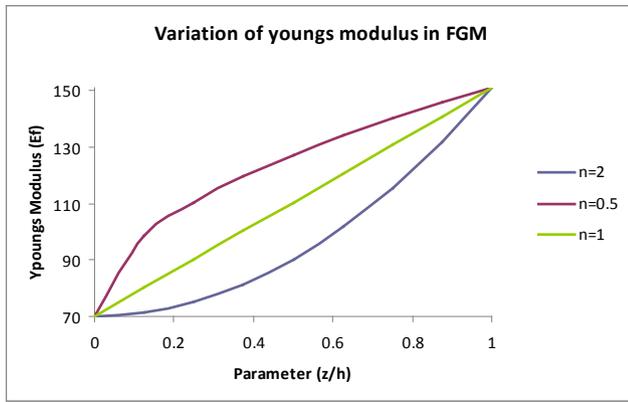


Figure 2 : Variation of effective young's modulus with respect to parameter z/h for various material index

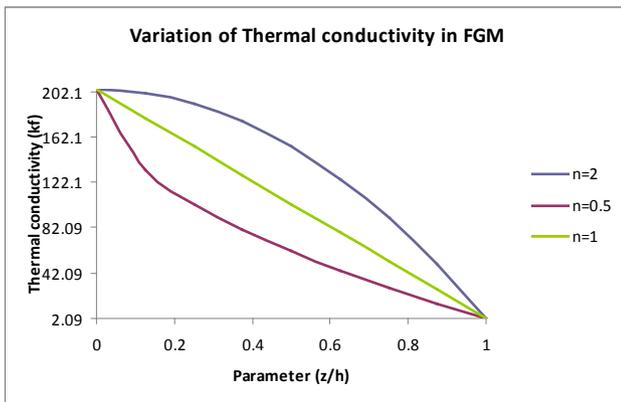


Figure 3 : Variation of effective Thermal conductivity with respect to parameter z/h for various material index

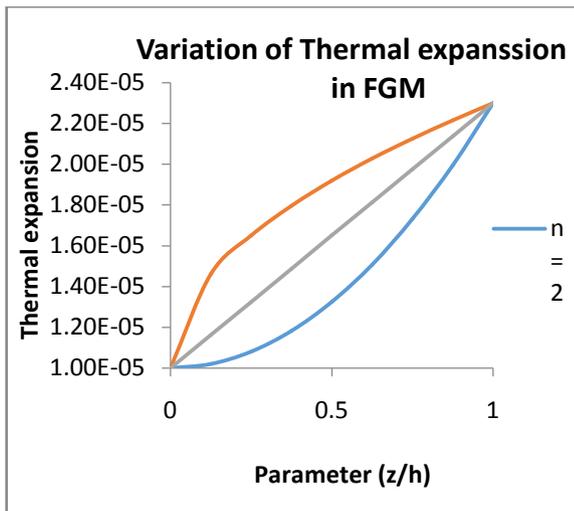


Figure 4 : Variation of effective Thermal expansion with respect to parameter z/h for various material index

c) Analysis

The static analysis was performed on a square plate of side length  $a=b = 0.2\text{m}$  and thickness  $h = 0.01\text{m}$ . The plate is assumed to be simply supported on all its edges. A regular 8 by 8 mesh of linear elements in a full size plate was chosen after convergence studies.

The value of the uniformly distributed loading chosen was equal to  $q_0 = 0.01 \times 10^6 \text{ N/m}^2$ . The results were plotted. The analysis is performed for fix values of the volume fraction exponent i.e  $n=2$ . The results are presented in terms of non-dimensional stress and deflection. The various non dimensional parameters used are non dimensional center deflection  $w = w_0 E_t h^3 / (q_0 a^4)$  and non dimensional shear stress  $\sigma_{xz} = \sigma_{xz} h^2 / (q_0 a^2)$ .

In the present analysis, in addition to the uniform loading, the plate is subjected to a temperature field where the uniform temperature up to  $300^\circ\text{C}$  is given and the reference surface temperature is held at  $20^\circ\text{C}$ . The materials are assumed to be perfectly elastic throughout the deformation. A simply supported FG plate subjected to a uniformly distributed mechanical load and thermal loading as shown in figure 5.

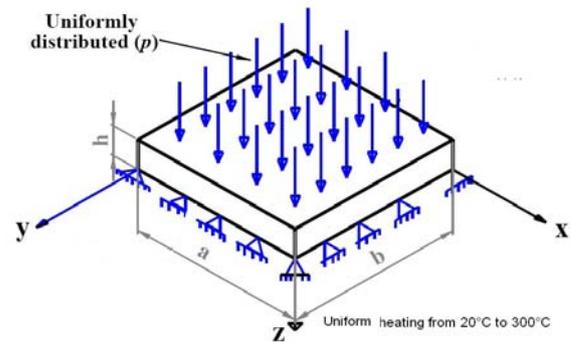


Figure 5 : A simply supported FG plate subjected to a uniformly distributed mechanical load and thermal loading

d) Boundary Conditions and Meshing

The square plate modeled is meshed using the mesh tool. The mesh tool provides a convenient path to many of the most common mesh controls, as well as to the most frequently performed meshing operations. The plate modeled throughout this project is subjected to simply supported Boundary condition i.e. along the X direction,  $U_y = U_z = 0$  and along the Y direction  $U_x = U_z = 0$ . It is illustrated in Figure 6.

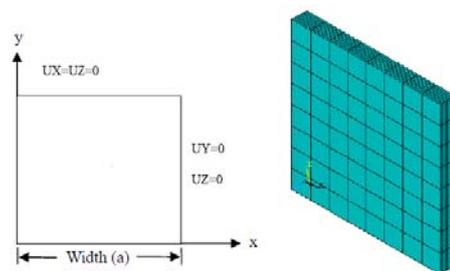


Figure 6 : Square plate with 8 layers, 8x8 mesh and Simply supported boundary condition

Using the APDL tool, the layer of the model along the thickness are divided into the number of layers desired; the other layers are then selected and divided

depending on the mesh size required. The following figure 6 shows an FGM plate modeled with 8 layers and a mesh count of size  $8 \times 8$  along the x-y plane. Once the model is meshed; the model is modified in order to create layers with different material properties. This is done with the help of shell section. The material properties are then assigned to the respective layers defined along the thickness. It is to be noted that each layer is isotropic in nature.

#### IV. RESULT

In this section we present several numerical simulations, in order to assess the behavior of functionally graded plates subjected to mechanical, thermal and thermo-mechanical loads. A simple supported plate is considered for the investigation. The plate is made up of a ceramic material at the top, a metallic at the bottom. The simple power law with different values of  $n = 2$  is used for the through-the-thickness variation. Following trends obtained as shown in various graphs.

##### a) Non dimensional deflection

Here the non dimensional deflection parameter is plotted against non dimensional parameter ( $z/h$ ) for mechanical loading, thermal loading and thermo mechanical loading for metal plate, ceramic plate and functionally graded material plate as shown in figure 7, 8 and 9 respectively.

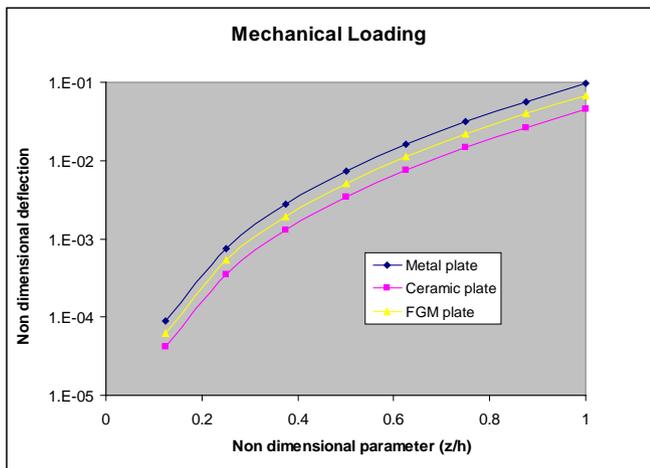


Figure 7: Variation of Non dimensional deflection for mechanical loading with  $z/h$  for  $n=2$

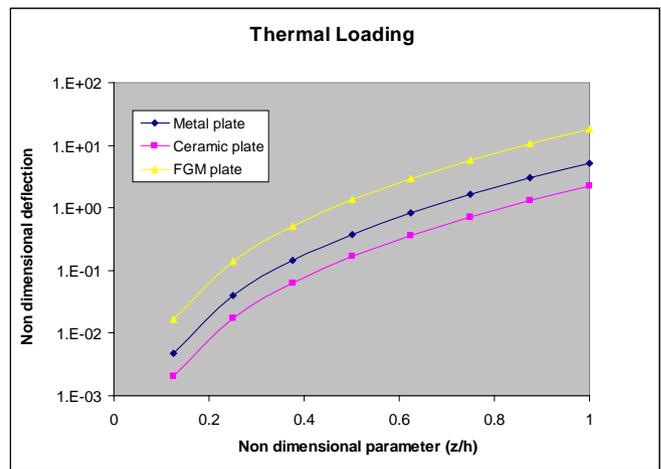


Figure 8: Variation of Non dimensional deflection for thermal loading with  $z/h$  for

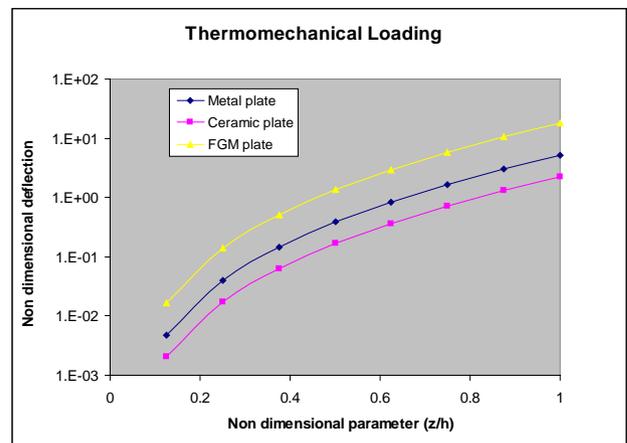


Figure 9: Variation of Non dimensional deflection for thermo mechanical loading with  $z/h$  for  $n=2$

##### b) Ansys Diagram

In this section we present several numerical simulations diagram of deflection in figure 10, 11 and 12 for mechanical, thermal and thermomechanical loading respectively to assess the behavior of functionally graded plates.

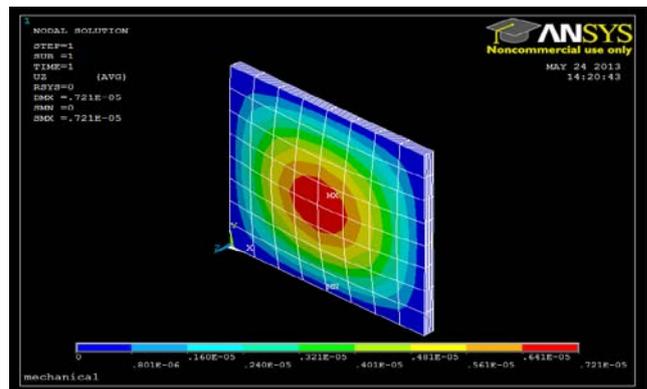


Figure 10: Variation of Non dimensional deflection for mechanical loading in FGM

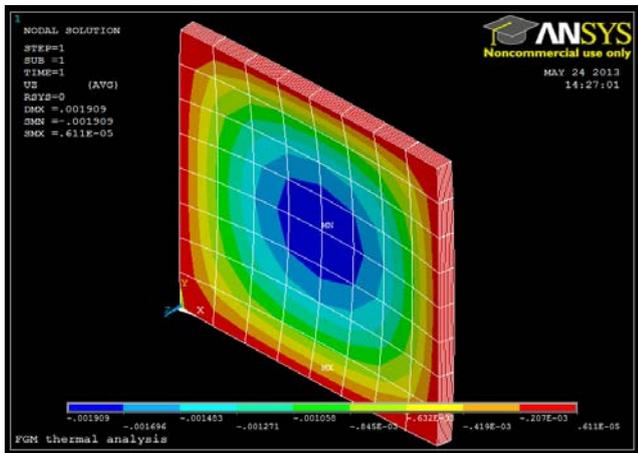


Figure 11 : Variation of Non dimensional deflection for thermal loading in FGM plate

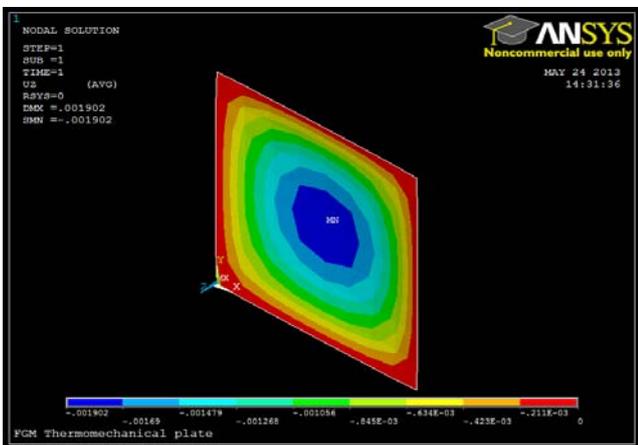


Figure 12 : Variation of Non dimensional deflection for thermo mechanical loading in FGM plate

## V. CONCLUSION

In this report analysis is carried out on a functionally gradient material square plate made of Aluminium/Zirconia. The plate considered is thick plate with  $a/h=20$  and  $a/b=1$ . The structural response of this plate is studied with respect to mechanical, thermal and thermomechanical loads. The structural response of functionally gradient material plate is also compared with pure metal and pure ceramic plate under mechanical, thermal and thermomechanical loading. The properties of functionally gradient material are calculated for each layer according to power law. The material index, number of layers and mesh size is kept constant. The following points are summarized:

1. The modeling of functionally gradient material plate in step wise variation in properties is successfully developed.
2. It is observed that the response of plates depends upon the intermediate properties of the metal and the ceramic.

3. In case of pure mechanical loading the non dimensional deflection of functionally gradient material plate is in between pure metal and ceramic plate.
4. In case of pure thermal and thermomechanical loading the non dimensional deflection is same nature. Ceramic plate having minimum deflection in mechanical, thermal and thermomechanical loading.
5. As for as review of literature it is also concluded that fine the number of mesh better the results. Also ANSYS gives faster approximate results and degree of accuracy depends on the mesh size, layers and solver.
6. In this report first order shear deformation theory has been used for formulation of the problem, it is concluded from the review that higher order theory can give approximate better results.

### a) Future Scope

The plate modeled here was a step wise graded structure, with each layer being isotropic with specific material properties. The material properties for each layer have been calculated by any other methods like Mori-Tanka etc. which may give better estimation of properties. Also we can go for the coding of the material to get the continuous variation of the properties. The material index, no of layers and meshing size can also be changed to get the better results. The position of natural axis and its eccentricity can also be considered for perfect analysis.

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