

On the Calculation of Crack Width in RC Linear Elements under Eccentric Load

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Abstract

Proof of controlling crack width is a basic condition for securing suitable performance in serviceability limit state. Most codes struggle with offering procedure for crack width calculation. So did the former Euro Code [ENV 1992-1-1:dec. 1991] and the present [BS EN 1992-1-1:2004]. Both contain a procedure, rendering almost identical calculation results, however aiming mainly to pure bending while eccentric load is practically out of the scope. A simplified procedure is offered here aiming to fill this gap via a very simple transformation leaving the principles of the Euro Code unchanged. Numerical examples demonstrate the application of the suggested procedure. Comparison with parallel analytical tools support the validity of the results thus obtained. The procedure is simple, user friendly and ready to be involved in code drafting.

Index terms— concrete structures, structural design, crack control, crack width calculation, steel reinforcement, constitutive laws, serviceability limit state.

1 I.

2 Notation

A_s -the area of reinforcement close to the tension face of the section A_s' -the area of reinforcement close to the compression face of the section d -effective height of the section d_s -distance from the center of the tensile reinforcement A_s to the extreme fiber in tension d_s' -distance from the center of the compression reinforcement A_s' to the extreme fiber in compression e -eccentricity of the normal force relative to section center E_c -concrete modulus of elasticity E_s -reinforcing bars modulus of elasticity f_{ctm} -the mean tensile strength of the concrete f_{yk} -yield strength of reinforcing bars K_1 - K_2 -coefficients for calibration of S_{rm} (bond and stress distribution) [ENV 1992-1-1:dec. 1991] $M_{d,ser}$ -service moment acting on the section resulting from static analysis $M_{d,ser}$ -moment acting on the section after normal force being transferred to A_s $N_{d,ser}$ -service normal force acting on the section resulting from static analysis $s_{rm,avg}$ -final crack spacing [ENV 1992-1-1:dec. 1991] w_k -the design crack width y -distance from the extreme fiber in tension to the section center y' -distance from the extreme fiber in compression to the section center η -coefficient relating the average crack width to the design crack width [ENV 1992-1-1:dec. 1991] ϵ_{sm} -mean strain in the reinforcement at the crack allowing for tension stiffening σ_s -bar's diameter (or the average scaled bars diameters) ρ -reinforcement ratio relative to the effective concrete section in tension $A_{c,eff}$ σ_{sr} -stress in the tensile reinforcement under the cracking moment M_{cr} [ENV 1992-1-1:dec. 1991] σ_s -stress in the tensile reinforcement under the service moment including the axial force transferred to the tensile reinforcement II.

3 Introduction

Limiting crack width is one of the two basic conditions (but not only) for securing suitable performance in serviceability limit state: deformation and cracking limitation. The later is no less important since crack width requirements are more relaxed than in the past, but the need of verification is essential. Some codes, like the ACI [ACI 318M-05], have given up calculating crack width, assuming that control may be attained indirectly.

8 SOLUTION:

The EN 2 in its for-mer [ENV 1992-1-1:dec. 1991] and present [BS EN 1992-1-1:2004] versions, has pursued in providing procedures for calculating the crack width, however, focusing on pure bending mainly. Considering ec-centrally loaded sections is important in both RC and PC elements. A simple procedure is offered here that allows a straightforward crack width calculation in linear concrete elements, eccentrically loaded. The results are compared with the limitations im-posed by EN2 [1992, 2004] and with the stress state of sections eccentrically loaded obtained by nonli-near material analysis [Farhat, R., 1995] and found to be in very good agreement.

4 III.

Calculation of Crack Width According to En 2 [ENV 1992] EN 2 [ENV 1992] offered the following procedure for calculating crack width:

(1)

The average final crack spacing defined as:

(2) Note: $k_2 = 0.5$ for bending and 1.0 for pure tension with possible interpolation for intermediate cases according to: $k_2 = (\epsilon_1 + \epsilon_2)/2$ with ϵ_1 & ϵ_2 being the greater and the lesser tensile strains at the boundaries of the sec-tion considered (quote).

Though this definition leaves the impression that ec-centric load is dealt with, it appears not to be the case, as eccentric compression is not included in this consideration and in a cracked section under eccen-tric tension there hardly is any possibility to calcu-late ϵ_1 , while undoubtedly ϵ_2 will be in compression.

The mean strain in the reinforcement defined as: Essentially there is difference in the cracks spacing and the strains, however the final calculation results according both renders almost identical results.

5 IV. Proposed Method for Calculation of Crack Width under Eccentric Load

The proposed herein method, follows the procedures as given in [ENV 1992-1-1:dec. 1991] (detailed above) or [BS EN 1992-1-1:2004], except for a transformation suggested that allows for easy and simple consideration of the eccentricity in loading. Only the procedure given in EN2 [ENV 1992-1-1:dec. 1991] is discussed in the following, however in the numerical examples that follow crack width is calculated according both EN2 versions.

A symmetrical with reference to vertical axis section is given in Figures 1a&2a (see notation). On the section acts a normal force in service $N_{d,ser}$ at eccentricity e_d vs. the section center, as obtained from elastic In order to maintain equili-brium, after transfer, the moment will be:

(5)

for eccentric compression -see Fig. 2b (6)

for eccentric tensionsee Fig. 1b From here on the section analysis for cracking will be conducted under the action of $M_{sd,ser}$ and $N_{d,ser}$. 2. The stress in the tensile face is to be checked assuming uncracked section. If it exceeds f_{ctm} (the mean tensile strength of the concrete) the section is cracked. 3. The stress in the tensile reinforcement will be: $\sigma_{sm} = \frac{M_{sd,ser}}{I_{cr}} - \frac{N_{d,ser}}{A_s} \leq \sigma_{s,lim}$ (1) [EN 2] $\sigma_{s,lim} = \frac{f_{yk}}{1.1}$ for tension and $\sigma_{s,lim} = \frac{f_{yk}}{1.1} - \frac{f_{ctm}}{1.1}$ for compression. 4. The average strain in the tensile reinforcement is calculated as given in (3) above. ϵ_1 and ϵ_2 remain as recommended there. 5. The average distance between cracks s_{rm} is calculated according to (2) above, with $k_1 = 0.8$ for high bond bars and $k_2 = 0.5$ for pure bending. ϵ_1 and ϵ_2 as defined in EN2 [ENV 1992]. 6. Finally the maximum crack width, according to EN2 [ENV 1992] is:

(10)

V.

6 Numerical Examples

The examples given in the following aim to cover a variety of problems that may rise applying the of-fered procedure. In all examples the concrete type is $f_{ck,cl} = 25\text{MPa}$ with mean concrete tensile strength $f_{ctm} = 2.6\text{MPa}$ and $E_{cm} = 31000\text{MPa}$. The reinforcement consists of ribbed single bars (?) with $f_{yk} = 400\text{MPa}$ and/or welded mats of high strength welded bars () with $f_{yk} = 500\text{MPa}$.

7 Example 1

A section of a wall, 300 mm thick, contains 2000 mm²/m tensile reinforcement in the form of ?16@100mm at a distance $d_s = 50\text{mm}$ from the in-ner face of the wall, (See Figure 3). The section effective depth d is 250mm.

8 Solution:

The section is under tensile load with eccentricity $e_d = 75.3/115.9 = 0.65\text{m}$

Transferring the load to the center of the tensile rein-forcement results in a moment:

The stress at service in the tensile reinforcement will be: With a cracking moment $M_{cr} = 39.0\text{kNm/m}$

The mean strain in the reinforcement is: An estimate of $A_{c,eff}$ gives 80000 mm², therefore

The average distance between cracks will be: Therefore the calculated maximum crack width is:

The maximum crack width assessed according to EN2 [BS EN 1992-1-1:2004] is 0.196 mm.

The result was reviewed with the aid of: a. Nonlinear section analysis developed by Farhat [Farhat, 1995] wherefrom the stresses and strains in the cracked section are as follows:

The difference between the suggested here analysis and the nonlinear analysis for ϵ_s is 3.3% -within very reasonable level of accuracy. b. According to Table 7.2N [BS EN 1992-1-1:2004] for a maximum bar diameter of 16 mm and at a stress level of 200 MPa the crack width to be expected will be approximately 0.2 mm. Also, according to Table 7.3N [BS EN 1992-1-1:2004] for a 400 mm thick section of a floor is given (Figure 4) where the tensile reinforcement at a distance 50 mm from the upper Any bottom floor reinforcement is ignored for the purpose of this analysis. The reinforcement placed in the form of a matt is not fully embedded in the support therefore one half of the amount is considered active, however scaling the amount in terms of strength to an equivalent of ribbed bars the total amount of reinforcement is 1760 mm² (1200+560).

Due to the most extreme load combination the following was obtained: Solution:

The section is under tensile load with eccentricity Transferring the load to the center of the tensile reinforcement results in a moment:

The stress at service in the tensile reinforcement will be :

The cracking moment is 69.33 kNm/m Therefore the mean strain in the reinforcement is: An estimate of ϵ_{cr} gives 113300 $\mu\epsilon$, therefore

The average distance between cracks will be (with an average bars diameter -13mm):

Therefore the calculated maximum crack width is: w_{max} calculated according to EN2 [BS EN 1992-1-1:2004] is 0.160 mm.

Discussion of the results: a. Stresses and strains resulting from nonlinear section analysis [Farhat, 1995] produce:

Again the difference between ϵ_s from the analysis offered and the nonlinear analysis [Farhat, 1995] is 4.1% -a very fair level of accuracy. b. According to table 7.2N [BS EN 1992-1-1:2004] for a stress level of 180 MPa in the reinforcement a maximum bar size of over 16 mm is allowed for limiting crack width to 0.2 mm. According to Table 7.3N [BS EN 1992-1-1:2004] for the stress level of 180 MPa a maximum bar spacing exceeds 150 mm, but in the current example the distance is 125 mm, therefore it may be concluded that the max. crack width is lower than 0.2 mm (here -0.158 mm).

9 Example 3

A portion of the ceiling of a buried underground structure is given, 400 mm thick, having 2320 mm² /m at distance 50 mm from the bottom face and 1111 mm²/m at distance 50 mm from the upper face.

The effective depth d is 350 mm. See Figure 5. Addressing loading combination 1:

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The section is under compressive load with eccentricity $e/d = 120.3/123.7 = 0.973m$

Transferring the load to the center of the tensile reinforcement results in a moment:

The stress at service in the tensile reinforcement will be The cracking moment is 69.33 kNm/m Therefore the mean strain in the reinforcement is: With:

The average distance between cracks will be: Therefore the calculated maximum crack width is: w_{max} according to EN2 [BS EN 1992-1-1:2004] is 0.109 mm Addressing load combination 2:

Checking stresses assuming uncracked state under eccentric tension proves that in the upper face the stress is 1.50 MPa and at the bottom face the stress is -0.82 MPa. Discussion of the results for load combination 1: a. The stresses and strains obtained in nonlinear analysis [Farhat, 1995] are:

The stress in the reinforcement for the proposed analysis is 143.

11 Conclusions

A simple procedure is presented; modifying slightly the proposed procedure in EN2 in it's both versions, allowing calculating directly crack width in linear concrete members, with sections under eccentric tensile or compressive normal force. Several examples offered demonstrate the simplicity and practicality in application of the procedure. Nonlinear section analysis proves a very good correspondence with the results obtained with the simplified method offered. A good correspondence is obtained also with forecasts from EN2 [ENV 1992-1-1:dec. 1991] and EN2 [BS EN 1992-1-1:2004] based themselves on similar calculations.

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Figure 1: (3)

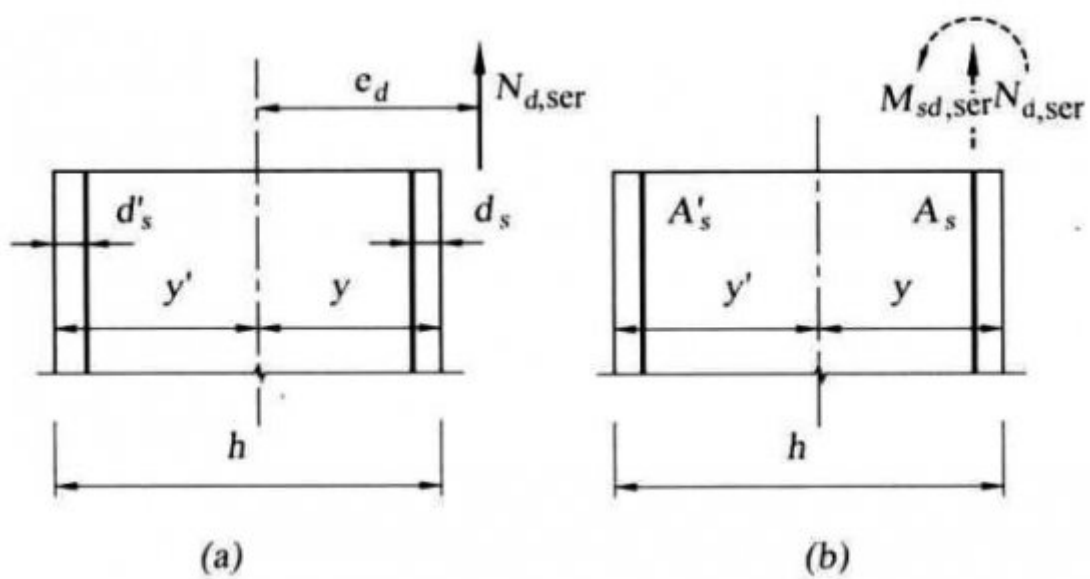
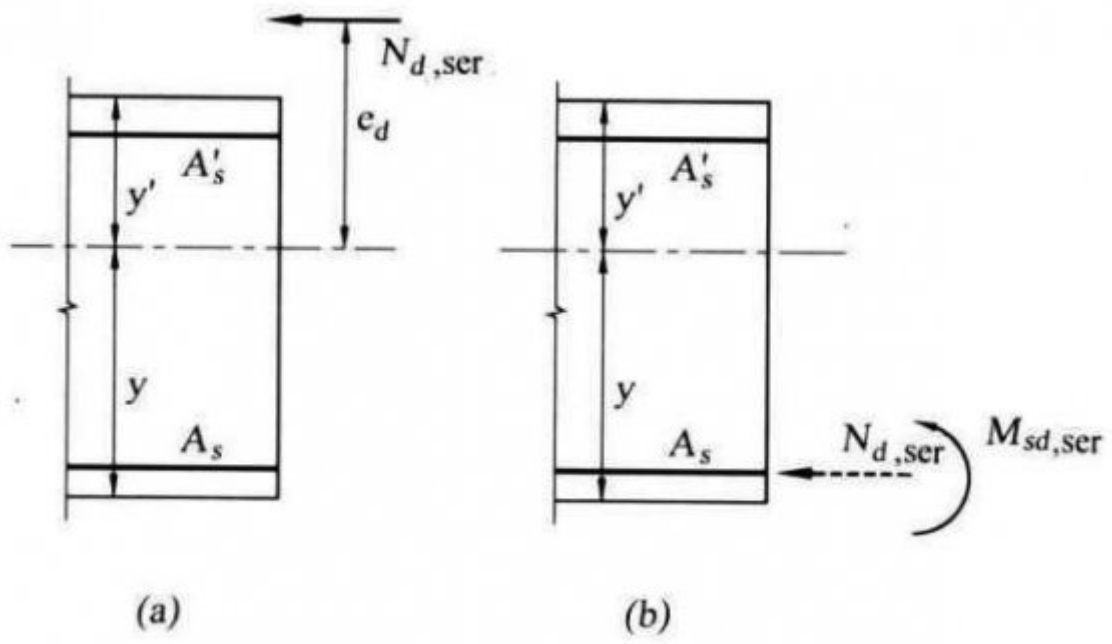


Figure 2: Figure 1 :



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Figure 3: Figure 2 :

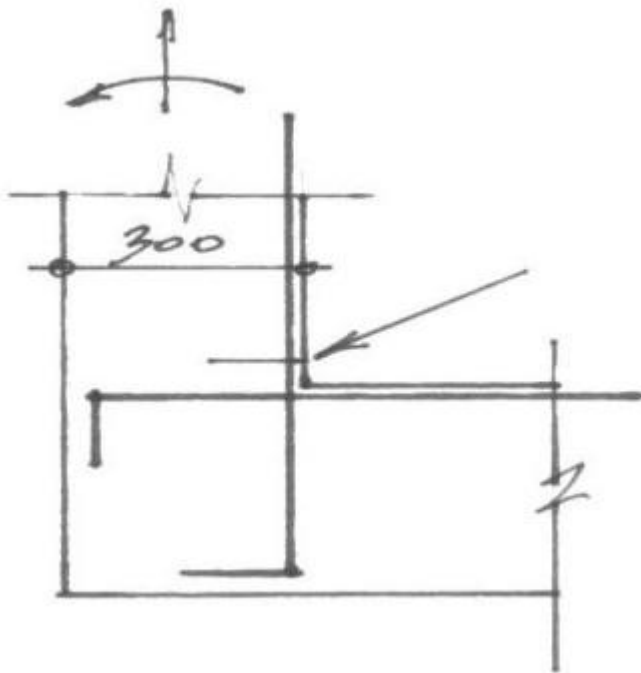
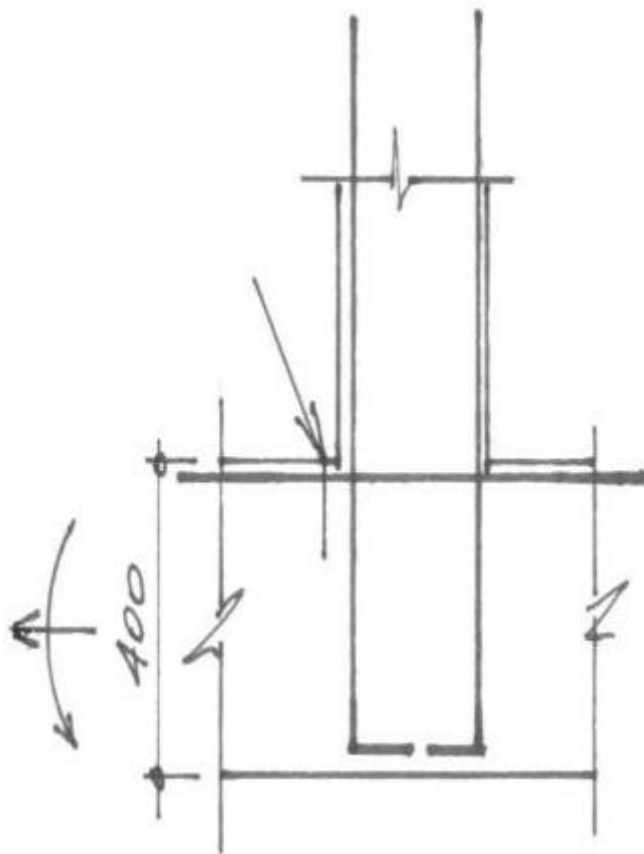


Figure 4:



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Figure 5: Figure 3 :

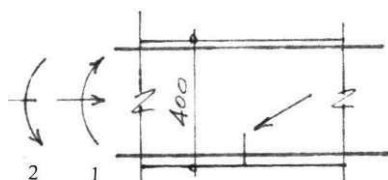


Figure 6:

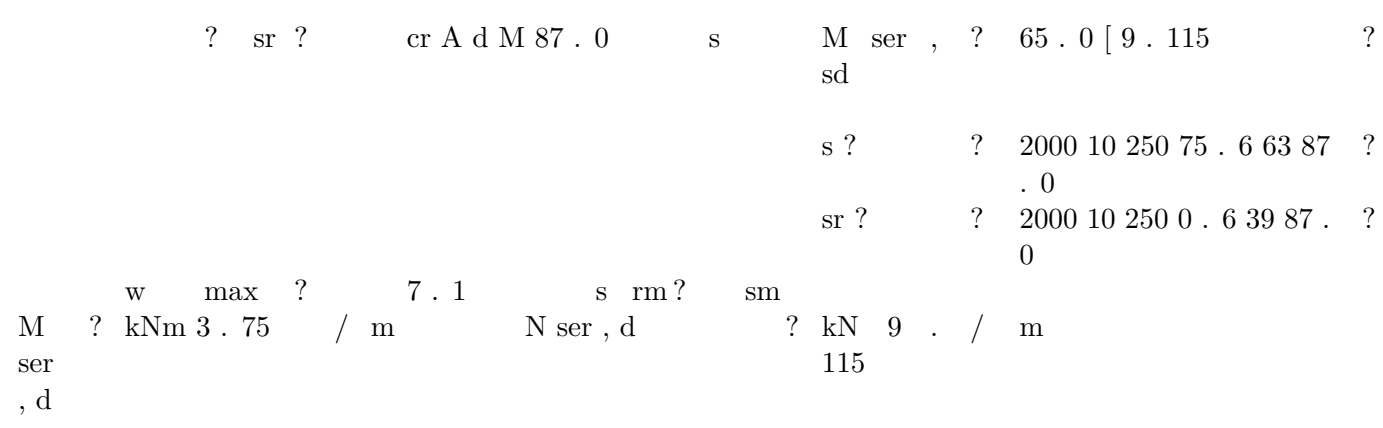


Figure 7:

VI.

Figure 8:

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