

Discrete Time Averaging of Non-Ideal PWM DC-DC Converters Operating in DCM with Feedback

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Abstract

This paper presents a one-cycle-average (OCA) discrete-time model for PWM switched dc-dc converters operating in discontinuous conduction mode (DCM). The closed-loop system is considered in the presence of circuit parasitic. This model provides the exact OCA values of the output voltage and inductor current considering the conduction losses in the switching stage components. It also provides the exact discrete-time mathematical representation of the average values of other internal signals with little increase in simulation time. It is compared to some existing averaged models in terms of accuracy and speed through simulation results of boost converter.

Index terms— Simulation compared

1 Introduction

C-DC converters are widely used in various applications. These converters are usually operated in two modes of operation, namely continuous and discontinuous conduction modes [1]. The discontinuous conduction mode of operation of the converter is most frequently used for light load applications. It is also useful for extracting maximum power efficiently from the solar panel [2,3].

Due to various applications and needs of dc-dc converter operating in DCM, it is very essential to have a proper analytical model for this mode of operation for the analysis and design. The literature shows that more effort has been taken in this view for past three decades [2,4].

One of the most widely used techniques in the design procedure in power electronics is averaging technique. This technique provides the basic analytical foundation for the most power electronic design. In fact classical averaging procedure is not suitable when there are state discontinuities. At high switching operations the periodic solution has some amplitude ripple, and these ripples are not considered in the classical averaging theory. Due to this it has been found that the directly obtained averaged models are inaccurate for the converters operated in DCM [4]. This has inculcated to take efforts to obtain more accurate averaging method.

In the continuous-time averaging procedure, each circuit topology is modeled separately and then combined in on approximated model [4,5]. The duty cycle is a discrete-time variable, but treated as a continuous time variable in the existing continuous-time averaged models. Thus the orbital nature of the periodic solution is not obtained. Intern the periodic solution of the converters is averaged to equilibrium to form a nominal solution. In contrast, no such approximations are made in the sample date model. This provides the most accurate result, which replicates the actual behavior of PWM systems and is also suitable for digital control process. Sampled-data models allow us to focus on cycle -to-cycle behavior, ignoring intra cycle ripples. This makes them effective in general simulation, analysis and design. These models predict the values of signals at the beginning of each switching period, which most of the times represent peaks or valleys of the signals rather than average values. To better understand the average behavior of the system, a discrete-time model for the OCA signals was presented in [6].

In this paper, a sampled-data model for nonideal closed loop PWM converters operating in DCM is formulated. This gives the exact discrete-time mathematical representation of the values of the output and internal signals at

104 at each time instant, where the sawtooth function is shown in Figure 2 and mathematically represented by
 105 $\text{tri}(A(d_1, d_2) \cdot 0 \times p \ C(d_1, d_2) \cdot 0 \times p \ V_{\text{ref}} \ K_x(t) \ t^* \ T_s \ k \ \text{III}$.

106 4 Proposed Model

107 This section introduces the new averaged discrete-time model for closed loop PWM dc-dc converter operating in
 108 DCM considering conduction losses. Description of the original system and derivation of the proposed model are
 109 discussed here. The onecycle average (OCA) representation of the output signal [6] is given by (10)

110 The signal,
 111 (\hat{I}^*) is used to develop a new discrete-time model for PWM converters operating in DCM. This model provides
 112 the basis for discrete-time simulation of the averaged value of any state in the DCM PWM system, even during
 113 transient non-periodic operating conditions. a) Proposed OCA Discrete-Time Model It is desired to compute,
 114 without approximation, the evolution of all system variables at the sampling instants, assuming three different
 115 topologies for the system. Since the state and output equations (\hat{I}^*) and $x_{k+1} = A(d_1 k, d_2 k)x_k + B(d_1$
 116 $k, d_2 k)u_k \ y_{k+1} = C(d_1 k, d_2 k)x_k + D(d_1 k, d_2 k)u_k \ A(d_1, d_2) := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \ B(d_1, d_2) :=$
 117 $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \ C(d_1, d_2) := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \ D(d_1, d_2) := \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \ \hat{I}^* := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \ \hat{I}^* := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \ \hat{I}^* := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
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120 floor For reasonably high switching frequency, the value of $x(kT + t^*)$ can be approximated by neglecting the
 121 higher order terms in the Taylor expansion of the nonlinear functions \hat{I}^* and \hat{I}^* . That is And hence, a good
 122 approximation of (??6) becomes

123 5 Numerical Example

124 To compare existing models with DCM OCA model, consider a Boost converter with feedback considering
 125 conduction loss shown in Figure 1. The $\hat{I}^*(t^*) = I + A_1 t^* + A_2 t^2 + \dots \hat{I}^*(t^*) = (I t^* + A_1 t^* + A_2 t^2 + \dots) B_1 \hat{I}^*(t^*) \hat{I}^*(t^*) = (I t^* + A_1 t^* + A_2 t^2 + \dots) B_1 \hat{I}^*(t^*)$
 126 $\hat{I}^*(t^*) = (I t^* + A_1 t^* + A_2 t^2 + \dots) B_1 \hat{I}^*(t^*) \hat{I}^*(t^*) = (I t^* + A_1 t^* + A_2 t^2 + \dots) B_1 \hat{I}^*(t^*)$
 127 $\hat{I}^*(t^*) = (I t^* + A_1 t^* + A_2 t^2 + \dots) B_1 \hat{I}^*(t^*) \hat{I}^*(t^*) = (I t^* + A_1 t^* + A_2 t^2 + \dots) B_1 \hat{I}^*(t^*)$
 128 $\hat{I}^*(t^*) = (I t^* + A_1 t^* + A_2 t^2 + \dots) B_1 \hat{I}^*(t^*) \hat{I}^*(t^*) = (I t^* + A_1 t^* + A_2 t^2 + \dots) B_1 \hat{I}^*(t^*)$
 129 $\hat{I}^*(t^*) = (I t^* + A_1 t^* + A_2 t^2 + \dots) B_1 \hat{I}^*(t^*) \hat{I}^*(t^*) = (I t^* + A_1 t^* + A_2 t^2 + \dots) B_1 \hat{I}^*(t^*)$
 130 $\hat{I}^*(t^*) = (I t^* + A_1 t^* + A_2 t^2 + \dots) B_1 \hat{I}^*(t^*) \hat{I}^*(t^*) = (I t^* + A_1 t^* + A_2 t^2 + \dots) B_1 \hat{I}^*(t^*)$

131 $\hat{I}^*(t^*) = (I t^* + A_1 t^* + A_2 t^2 + \dots) B_1 \hat{I}^*(t^*) \hat{I}^*(t^*) = (I t^* + A_1 t^* + A_2 t^2 + \dots) B_1 \hat{I}^*(t^*)$

132 $T_s = 50 \mu\text{s}$.

133 $R_{DS} = 0.17 \Omega, R_D = 0.15 \Omega, V_{DS} = 0.17 \text{ V}, V_D = 0.4 \text{ V}$ are periodically $A_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \ B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \ B_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
 134 $0 \ 0 \ 1 \ (R+R_C)C \ ? \ ? \ ? \ ; \ B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \ ; \ B_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

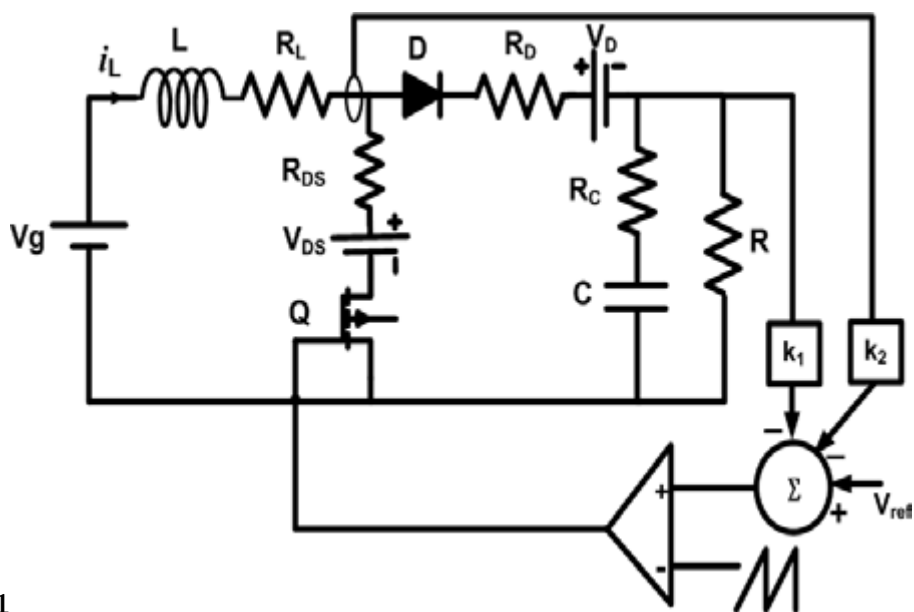
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Figure 1: D 2 F



1

Figure 2: Figure 1 :

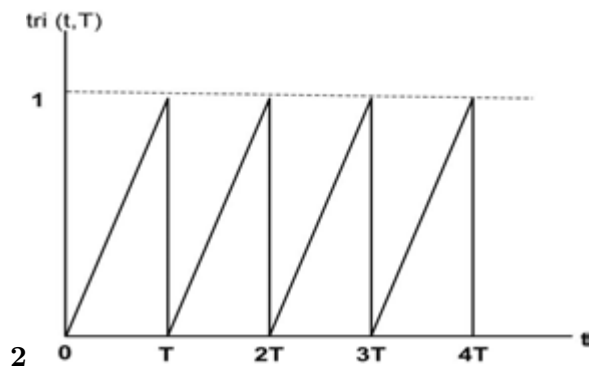


Figure 3: Figure 2 F

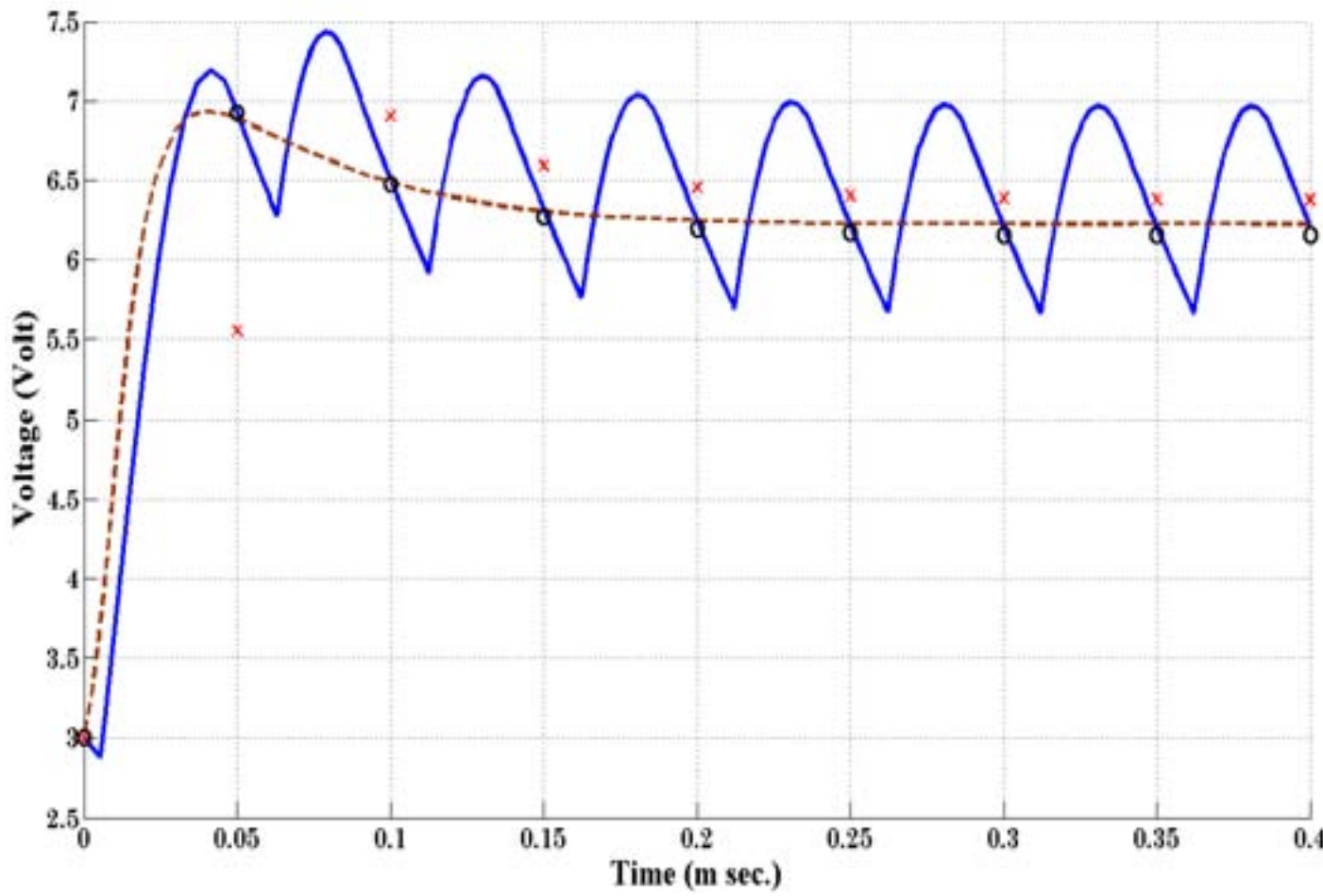
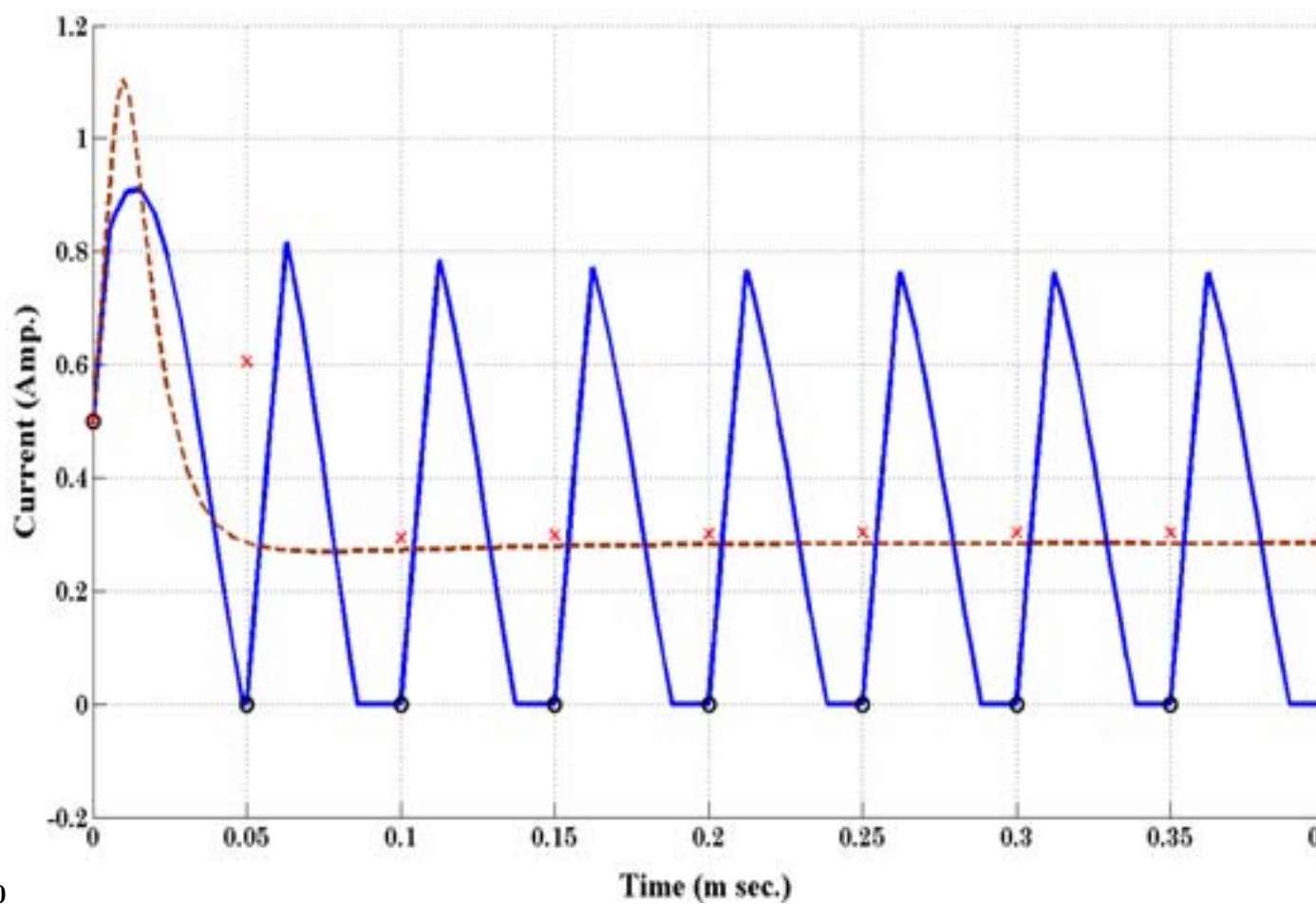
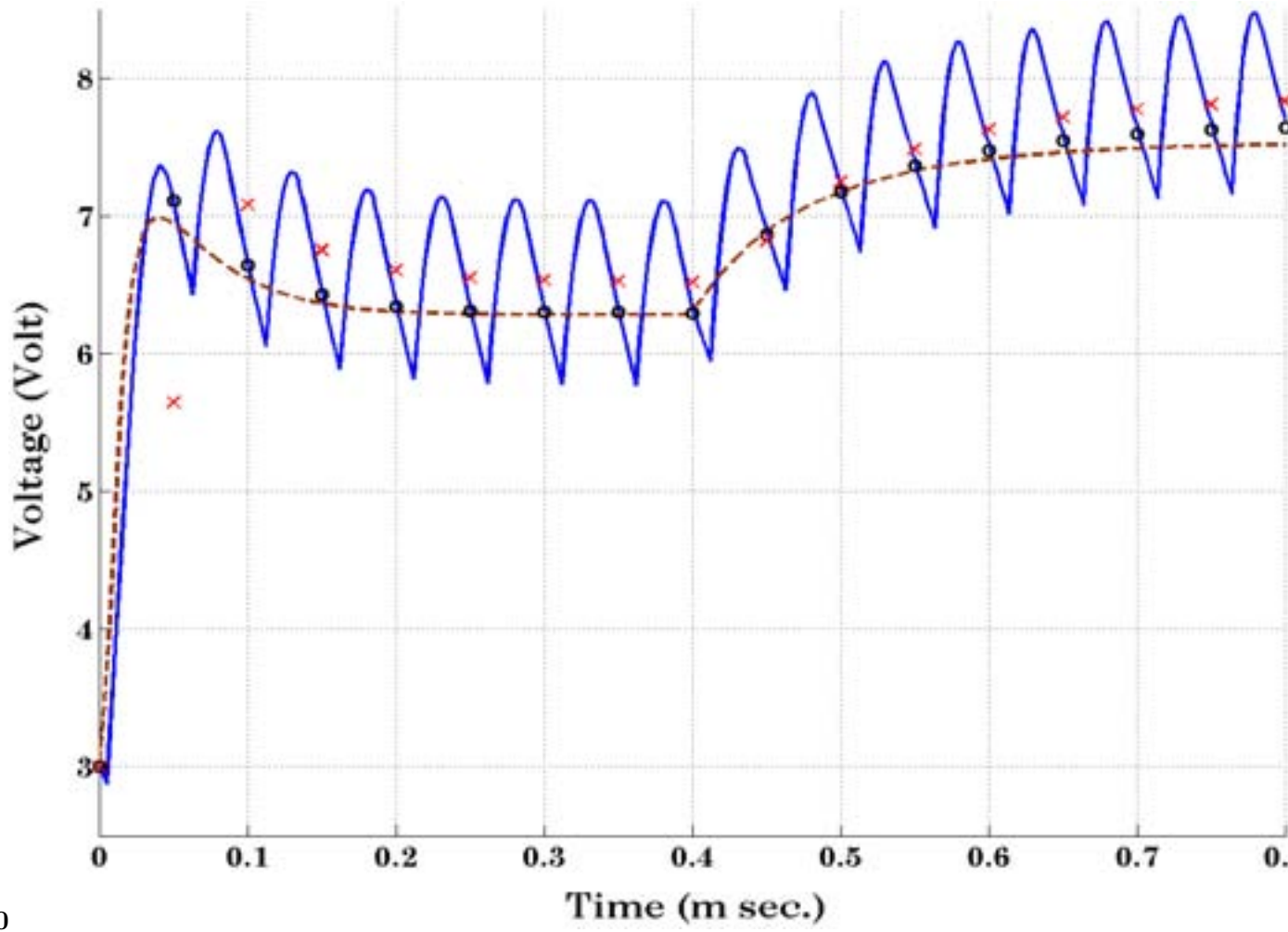


Figure 4:



0

Figure 5: 0 e



0

Figure 6: $t := t_0 e$

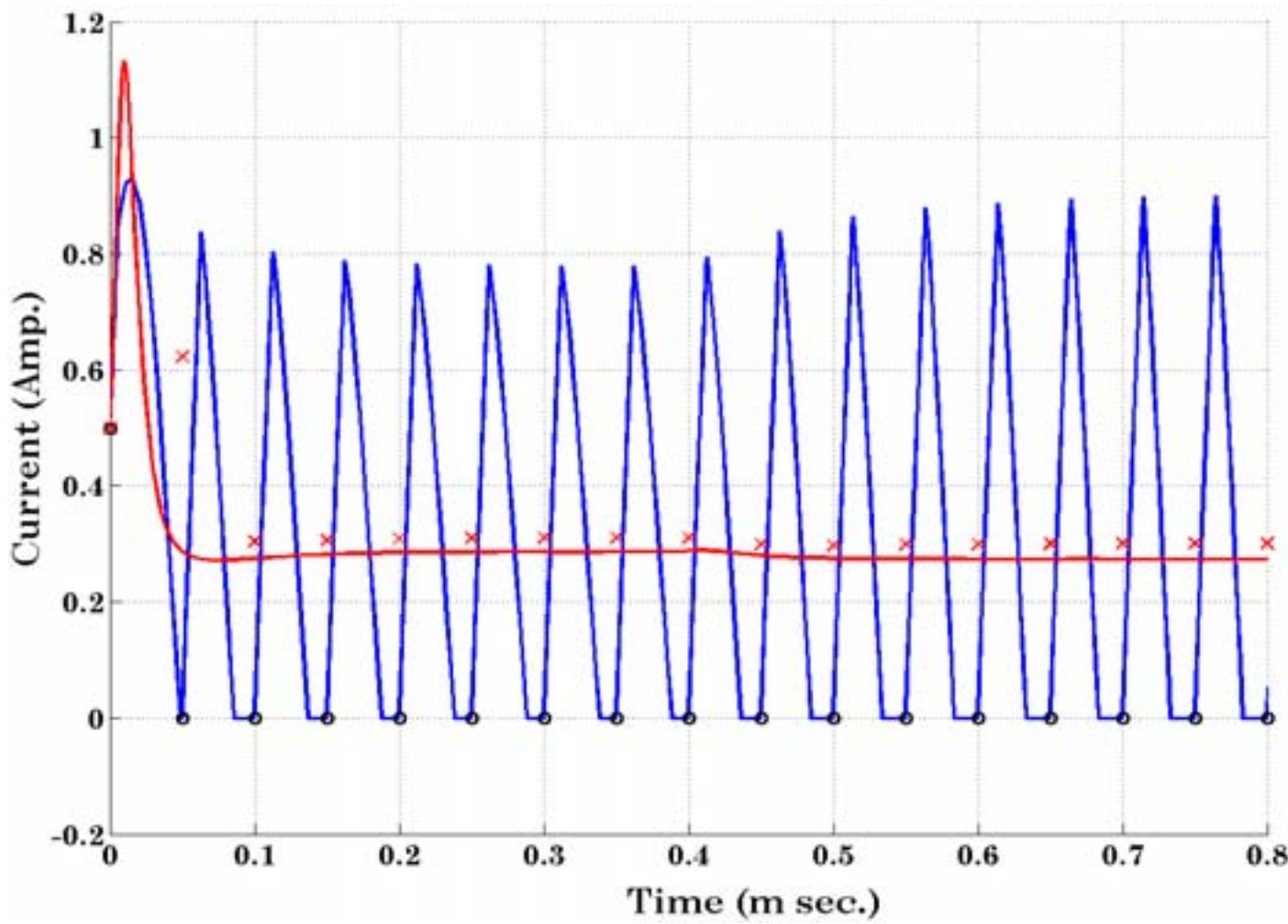


Figure 7:

statespace form. By defining the augmented state vector such that (12) is not in standard form

An equivalent (but standard form) representation of the OCA large-signal model is given by: (13)

(14)

(15)

Where

Figure 8:

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