Boundary Element Analysis of Pile Groups in Sand

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Abstract

This paper is devoted to make use of the boundary element method as a practical problem solving tool to analyze the soilpile interaction problems. A computer program was adopted for the analysis process. It is a computer package called PGroupN which is concerned mainly with the analysis of the pile group problems. A non-linear soil model has been adopted with this program to assess the pile-soil interaction within the group. Some parametric studies were carried out with this problem including the pile diameter, pile length, spacing to diameter ratio, soil type (sand) and the thickness of stratum.It was found that for 9 pile group analyzed, the corner pile carries about 4

Index terms— boundary element, pile group, sand, pile load

1 Boundary Element Technique

hile the finite element calculations constitute the major part of the computer studies and considered the most popular technique, the boundary element method (BEM) has been well examined as complementary or alternative system of analysis, with a view of reduce program running costs and enabling larger problems to be tackled.

The BEM is a boundary integral equation technique. The problem boundaries are discretized so that a singular solution of the governing differential equation can be integrated around them, yielding an appropriate source distribution which will generate the specified boundary conditions. Alternatively, the method may be regarded as a 'super' finite element technique, where each element models a homogenous zone (Hobbs et al., 1978).

Early attempts at employing numerical solution procedures for the solution of boundary integral equations have developed into two distinct and parallel ways. One of these is an intuitive, physical approach and the other is a more mathematical treatment based on concepts of classical potential theory (Banerjee and Butterfield, 1981) (Crouch and Starfield, 1983) (Venturini, 1983), (Beer, 1986) and (Beer and Watson, 1992). a) Boundary Element Method in pile-soil Interaction Problems Poulos (1971a) proposed integral equations for an elastic solution of laterally loaded pile. This problem is based on Mindlin's solution (Mindlin, 1936) for a point load in a homogeneous, isotropic elastic half space. The pile response shown in Figure 1 was calculated by integrating Mindlin's equation over the corresponding area of the soil. In Figure 1a, the pile is discretized into a number of elements. The transition of load through the pile is shown in Figure 1b, while the pile deformation (w) is shown in Figure 1c. The method could be extended to study the behavior of pile groups (Poulos, 1971b).

The integral equation mentioned above, was extended by Butterfield and Banerjee (1971b) using a BEM to treat a soil continuum and pile as two separate domains whose boundaries were discretized into a finite number of elements. A set of fictitious tractions were assumed at the pile -soil interface. It had been shown by Butterfield and Banerjee (1971b) that these fictitious tractions were identical to the real ones for pile slenderness ratio greater than five. Each boundary element was associated with known tractions and displacements. Some of these were known over parts of the boundary, the rest of them were computed using Kelvin' solutions (Lancellotta, 2009). Once these boundary values were obtained, the displacements and tractions at any point inside the domain could be computed.

Butterfield and Banerjee (1971a) presented an elastic analysis for the general compressible pile group problem including a rigid smooth ground contacting cap. The problems were formulated as an integral equation developed

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46 from Mindlin's analysis (Mindlin, 1936) for a point load embedded within a semi-infinite ideal elastic half space.
47 By distributing such point loads over the pile cap -supporting medium interface and the pile shaft and pile base
48 -medium interface, an integral representation was obtained given the vertical displacement at all points in the
49 medium in terms of fictitious stress intensities. Chin (2004) derived elastic design charts for axial pile settlement
50 response from the elastic response simplified BEM for piles imbedded in a two-layer soil continuum. These charts
51 cover the practical ranges of pile and soil parameters as shown in Figure **??**: b) Application of the Boundary
52 Element Method to Soil-

2   Pile Interaction Problems

This paper presents the results of a numerical analysis for the solution of soil-pile interaction problems using the
BEM.

A problem was chosen to investigate the behavior during the variation of different parameters. A summary
of the analysis procedure is presented here followed by a comprehensive discussion to all the obtained results in
order to evaluate the work, as much as possible, according to the adopted researches.

Because of the particulate nature of the mineral skeleton of the soil, the stress-strain behavior of the soil is
exceedingly complex. One way out from this complexity is to use concepts and formulas from the theory of
elasticity. This means that the actual non-linear behavior of soil is linearized which may lead to a conservative
design. Thus, the non-linearity represents the realistic behavior of the soil and it is adequate to simulate its
general problems.

Many researchers emphasized the importance of considering soil non-linearity in routine design. For pile
group problems, this issue has not been satisfactorily addressed yet, and current design practice is still generally
based on linear approaches (Basile, 2003). The main drawback to the application of linear models to pile group
problems is that they ignore the non-linear load-deformation characteristics of soil and hence misrepresent the
force in piles, specifically by giving higher stresses in group corners. The cost of this in practice is high and there
is an urgent need in industry for efficient non-linear analysis method (Basile, 2003).

A reasonable compromise between excessive complexity and unacceptable simplicity is provided by the BEM,
in which the pile-soil interface is discretized and the characteristics of soil response are represented in a lumped
form by ascribing the behavioral features of the soil to the interface elements (Poulos, 1989).

3   II. Method of Analysis

The analysis with the program PGroupN is based on complete non-linear BEM formulation. The analysis involves
discretization of only the pile-soil interface into a number of cylindrical elements, while the base is represented by
a circular (disc) element. The method employs a subtracting technique in which the piles and the surrounding
soil are considered separately and then compatibility and equilibrium conditions are imposed at the interface.

The external load is applied incrementally and, at each increment, a check is made that the stress state at
the pile-soil interfaces does not violate the yield criteria. This is achieved by specifying the limiting stresses
of the soil for the axial pile shaft capacity, and end-bearing resistance. The elements of the pile-soil interface
which have yielded can take no additional load and any increase in load is therefore redistributed between the
remaining elements until all elements have failed. Thus, by successive application of loading increments, the
entire load-displacement relationship for the pile group is determined.

4   a) BEM Program PGroupN

Repute calculation engine is the leading-edge program PGroupN which provides a complete 3D nonlinear
boundary element solution of the soil continuum. This overcomes limitations of traditional interactionfactor
methods and gives more realistic predictions of deformations and the load distribution between piles (Basile,
2003).

This program is based on a complete boundary element formulation extending an idea first proposed by
Butterfield and **??**anerjee (1971, a). The method employs a substructuring technique in which the piles and the
surrounding soil are considered separately and then compatibility and equilibrium conditions are imposed at the
interface. Given unit boundary conditions, i.e. pile group loads and moments; these equations are solved, thereby
leading to the distribution of stresses, loads and moments in the piles for any loading condition (Geocentrix Ltd,
2002).

5   b) Choice of Soil Parameters

The choice of soil parameters for PGroupN is simple and direct: for a linear analysis, it is only necessary to
define two soil parameters whose physical interpretation is clear, i.e. the soil modulus (E) and Poisson's ratio (
). If the effects of soil non-linearity are considered in elastic-plastic analysis, the strength properties of soil need
also to be specified, i.e. the undrained shear strength (c_u) for cohesive soils and the angle of friction ( ) for
cohesionless soils. Thus the applied method, by taking into account the continuous nature of pile-soil interaction,
removes the uncertainty of t-z and p-y approaches and provides a simple design tool based on conventional soil
parameters (Basile, 2003).

6 c) Nonlinear Soil Model

Soil is not a linear material. The relations between stress and strain are much more complicated than the simple, linear elastic material. Therefore, in order to represent geotechnical problems realistically, some form of nonlinear relation must be used (Christian and Desai, 1977).

The widely used function for simulation of stress-strain curves in the finite element analysis was formulated by Chang and Duncan (1970), and Duncan and Chang (1970) using Kondner's (1963) finding that the plot of stress versus strain in a triaxial compression In these equations, the subscripts have been removed from the stresses and strains for clarity, so that and represent vertical stress and strain, respectively. The latter form of the equation plots as a straight line (Figure **??**) and, conversely, a plot with axis / and can be used to check whether the data from a test do fit a hyperbola or to find the parameters of the hyperbola from the test data.

To find the tangent modulus, it is first helpful to observe that at very small strains:b (3)

So that $1/b$ is the initial Young's modulus E. At large strains, the relation becomes:a 1 (4)

So that $1/a$ is the compressive strength; actually, $1/a$ is the asymptote. Since the compressive strength will be reached before the curve becomes asymptotic as shown in Figure **??**, it is customary to require the compressive strength s to be R f /a, where R f is the failure ratio. Thus s R a f (5) Equation (1) can also be solved for :a b 1 (6)

The tangent modulus at any level of stress or strain is:2 2 2) 1 ( ) 1 (1) (s R E a b a b b E f i t (7)

For a Mohr-Coulomb material at failure:sin 1 cos 2 sin 2 3 3 1 c f (8)

The term /s is the ratio between the existing 1 -3 and s that would be available for the existing 3 . The ratio is: The tangent modulus now becomes: (10) The initial modulus has been found to vary with the confining pressure:2 3 3 1 cos 2 sin 2 sin 1 1 c R E E f i tn a a i p Kp E 3 (11)

where p a is the atmospheric pressure, K and n are constants to be determined.

The complete relation then becomes:2 3 3 1 3 sin 2 cos 2 sin 1 1 c R p Kp E f n a a t (12)

The PGroupN analysis adopts a non-linear model, which follows the well-established hyperbolic relationship between stress and strain proposed by Duncan and Chang (1970) and also applied to pile problems by Poulos (1989). This simple relationship assumes that the soil Young's modulus (E t) varies with the stress level at the pile-soil interface, i.e. it is a function of the initial tangent soil modulus (E i), the hyperbolic curve-fitting constant (R f), the current pile-soil stress (t) and the limiting value of pile-soil stress (lim), as shown in Figure **??**. The hyperbolic curve fitting constant R f defines the degree of non-linearity of the stress-strain response and can range between zero (an elasticperfectly plastic response) and one (an asymptotic hyperbolic response in which the limiting pile-soil stress is never reached. The best way to determine the value of R f is by fitting the PGroupN load-deformation curve with the data from the full-scale pile load test. In the absence of any test data, the value of R f can be initially estimated based on experience.

7 d) Soil Domain

The BEM involves the integration of an appropriate elementary singular solution for the soil medium over the surface of the problem domain, i.e. the pile-soil interface. With reference to the present problem, the well-established solution of Mindlin (1936) for a point load within a homogeneous, isotropic elastic half space has been adopted, yielding: (D D D D) {u s }= [G s]{t s }**(13)**

Where {u s } are the soil displacements, [G s] is the flexibility matrix obtained form Mindlin's solution and {t s } are the soil tractions. The singular part of the [G s] matrix is calculated via analytical integration of the Mindlin functions.

8 e) Pile Domain

If the piles are assumed to act as simple beamcolumns which are fixed at their heads to the pile cap, the displacements and tractions over each element can be related to each other via the elementary beam theory, yielding:{u p }= [G p]{t p }+{B}**(14)**

Where {u p } are the pile displacements, {t p } are the pile tractions, {B} are the pile displacements due to unit boundary displacements and rotations of the pile cap, and [G p] is the matrix of coefficients.

9 f) Solution of the System

The above soil and pile equations are coupled via compatibility and equilibrium constraints at the pilesoil interface. Thus, by specifying unit boundary conditions, i.e., unit values of vertical displacement, horizontal displacement and rotation of pile cap, these equations are solved, thereby leading to the distribution of stresses, loads and moments in the piles for any loading condition.

10 g) Extension to Non-linear Soil Behavior

Non-linear soil behavior is incorporated, in an approximated manner, by assuming that the soil Young's modulus varies with the stress level at the pilesoil interface (Basile, 2003). A simple and popular assumption is to adopt a hyperbolic stress-strain relationship, in which case the tangent Young modulus of the soil E t may be written as (Duncan and Chang, 1970;Poulos, 1989):2 t lim 1 f i R t E E t (15)

161   Where E i is the initial tangent soil modulus, R f is the hyperbolic curve-fitting constant, t is the pile-soil
162 shear stress, and t lim is the limiting value of the pile-soil stress.
163   Thus, the boundary element equations described above for the linear response are solved incrementally using
164 the modified values of soil Young's modulus of Equation (15) and enforcing the conditions of yield, equilibrium
165 and compatibility at the pile-soil interface.
166   Different values of R f should be used for the axial response of the shaft and base, and for the lateral response
167 of the shaft. For the axial response of the shaft, there is relatively small amount of nonlinearity, and nonlinear,
168 and the value of R f in the range 0.9-0.99 is recommended (Poulos, 1989).

11   h) General Description of the Problem

170 Group behavior is very complex. The response of each pile is modified by the stress condition imposed on the soil
171 by other members of the group. Therefore, the behavior is generally dependent on the pile spacing and length,
172 relative stiffness of the piles, number of piles in the group, in addition to the soil conditions.
173   In order to carry out parametric study and investigate the influence of these parameters on the behavior of
174 the piles, it is essential to start with basic problems. The problem which is chosen to be studied is a system of
175 pile group under axial load.
176   Two sets of pile groups consisting of 4 and 9 piles with circular cross sections are embedded in the soil. These
177 pile groups were assigned different internal and external variables in order to study the behavior of the piles. The
178 internal variables refer to pile diameter, pile length and spacing between piles, while the external variables refer
179 to the applied load, soil types and their thickness.
180   It is assumed that the pile cap is fully rigid and not in contact with soil. The free-standing length, which
181 represents the distance from the ground surface to the bottom of the cap, is assigned to be 0.5 m. This means
182 that embedded length is reduced by 0.5 m and the interface elements are not considered within this gap. Unlike
183 the 4 pile group, the behavior of a single pile in a group of 9 piles is associated with its location within the group.
184 The key of identification of the piles in that group is shown in Figure **??**.
185   The soil is assumed to be homogeneous and its parameters are based on subsoil idealization. The soil layer
186 rests on a rigid base. The level of water table is 1.0 m below the ground surface. The piles are not based on the
187 rigid layer which classified them as floating piles. A simple idealization of the pile-soil system is shown in Figure
188 **??**.

12   i) Parametric Study

190 Any designer is normally interested in the following aspects of the behavior of the pile groups:
191   Evaluation of the collapse load; Calculation of the settlement which leads to select a suitable factor of safety
192 in the design; and
193   The distribution of stress along the piles so that it can provide adequate reinforcement in the piles.
194   The above targets with their simple statements represent a summary for the long analysis journey through
195 many parameters which affect the pile group behavior. Some of these parameters adopted in this study are those
196 incorporated with the nonlinear analysis values of R f in the range 0-0.5 are appropriate, (Polous, 1989). The
197 axial response of the base is highly based on the BEM. Table 1 shows an outline of the analysis program for the
198 pile group problem.

13   j) Pile Length

200 The length of the pile (L) plays an important role in increasing the bearing capacity of the group. The analysis
201 begins with a pile length of 10 m and reaches a length of 25 m with increments of 5 m.

14   k) Pile Diameter

203 It is well known that the pile cross sectional area affects the capability of pile to sustain the loads. This parameter
204 is taken into account during the analysis of the pile with its circular cross section. Six diameters are chosen to
205 be the cases under study represented by (0.5 m, 0.6 m, 0.7 m, 0.8 m, 0.9 m and 1.0 m), respectively.

15   l) Spacing between Piles

207 Due to pile-soil-pile interaction, the group of piles tends to settle more than a proportionally loaded single pile.
208 This is because neighboring piles are within each others, so each pile interacts with the surrounding piles which
209 transfer the stresses to the other piles (Basile, 2003). Thus, the spacing between piles (S) is chosen as another
210 parameter in this study. The spacing is usually correlated with the pile diameter, so the values of spacing are
211 S=2D, S=3D, S=4D and S=5D.

16   m) Applied Axial Load

213 The failure of pile groups under axial loading has been extensively examined (Basile, 2003). The applied load is
214 increased gradually until the stresses along the pile reach the limiting state.

17 n) Soil Type

The soil type is sand taken from Kerbala city in Iraq with the properties determined by Ghalib, (1975). The sand was washed between the No.30 (0.59 mm) and No.50 (0.297 mm) sieves to obtain a more uniform material which would not segregate during sample preparation. A special drained triaxial test on a cylindrical sand samples (76 mm height and 76 mm diameter) was used. The axial force was increased up to an axial strain of 20%. The maximum void ratio was obtained by slowly pouring a sample of the dry sand into a Proctor mould from very low height. Minimum void ratio was determined by fabricating the sand in the mould under (0.141 kg/cm 2) surcharge as specified by the ASTM standards. The triaxial chamber was first filled with de-aired water, and then the soil specimen was first consolidated to equilibrium under all around confining pressure. The confining pressures were 100, 150 and 250 kN/m 2 . Volumetric changes were measured during the application of the cell pressure. Table 2 illustrates the properties of the soil to be considered in this study. The stress -strain curves from triaxial test are shown in Figure **??**.

respectively, in order to study the effect of soil layer thickness. The program shows incapability to deal with cases where the pile is standing directly upon the bottom rigid layer (i.e. H=L).

18 III. Results and Discussion

The results were normalized by taking the factor K p (stiffness factor) represented by the equation: This factor gives an indica tion to the load-settlement relationship during the loading process. Since the cap is assumed to be fully rigid, thus all the piles in the group settle by the same amount. The piles are arranged at different spacing for each load increment to find out their effects on the group behavior.

19 GWD P Kp

It is well known that increasing the pile length means that more shaft resistance is generated at the pile-soil interface along its length; moreover, the pile is penetrating the soil to settle down on a stiffer stratum. This increase in the pile capa city, if the load is still the same, leads to a reduction in the group settlement which is clearly shown in these figures. At an applied load of (8000 kN), Figure **??** checks that an increase in the pile length form (10 m) to (25 m) results in 53% decrease in settlement, whereas this percentage becomes 63% as the load increases up to 12000 kN. One can also notice the loss of the first points (i.e. at L=10 m) in some figures. This means that as the applied load is continuously increased, the shorter piles will be failed first.

These figures also show the role of the pile diameter during the loading process. The enlargement of the pile diameter produces a reduction in the intensity of loads carried by each pile in the group. The result is a reduction in loads to be transferred to the soil and less settlement is to be produced.

20 o) Soil Thickness

The groups of piles are embedded in a soil layer with a thickness (H) of 30 m, 45 m, 60 m and 75 m single pile in the group is associated with the others by means of strong bond, which is the cap. Thus, as the distance between the neighboring piles decreases, the interaction between these adjacent piles increases and vise versa. The results indicate that settlement of the short pile of a length (10 m) is decreased by 11% due to doubling the spacing from (S=2D) to (S=4D) while for pile of a length (25 m), the doubling process did not make a considerable effect.

Figure 12 shows the effects of the presence of a pile within a group as compared with a single pile having the same properties and dimensions. It is clearly shown that the interaction between piles in a group tends to increase the settlement more than the single pile even if they carry the same amount of loads.

Figure 13 shows the variation of the skin friction along a single pile in the group. Here, the pile length is chosen to be (25 m) and the friction at the pile interface is taken as the accumulative forces along the pile length. In close examination of this figure, one can notice the high convergence among the curves at each pile diameter. This gives an impression that the frictional resistance at the shaft is developed first until it reaches the limiting stress, then the pile base will take the rest of the applied load.

It is evident that the shaft resistance increased with increasing pile diameter due to the increase in the surface area which is in contact with the surrounding soil. This will contribute in a more frictional resistance to be added to the pile capacity with each diameter increment. The direct relation with depth reflects the influence of the effective overburden stress which increases with depth.

Figure 14 shows the corresponding plots of the percentage of load carried by the pile base. For a group of 4 piles and connected with a square cap, the load on each pile is equal since the total load is applied axially at the center of group. The miscellaneous trends of the curves from the linear to nonlinear and, return back finally to the linear path indicate the sharing process between the shaft and the base of the pile. The middle portion of the curves illustrates the beginning of the shaft resistance to take its share of the total load until it is fully mobilized.

This feature is compatible with the loadsettlement curves shown in Figure 15. The nonlinear path is approximately diminished as the pile reaches its ultimate bearing capacity.

Figures 16 to 18 show the plot of the nondimensional stiffness (K p) as a function of pile depth. These relations are given for 9 pile group embedded in sand and subjected to sequence of axial loads applied at the center of the cap. Generally, the curves follow similar trends as in the case of 4 pile group during the variation

274 of different parameters. Although the two mentioned series of the pile groups have similar trends but they are
275 different in their values. The results are in good agreement with those of Butterfield and Banerjee (1971 b) and
276 Paiva and Trondi (1999).

277 As a supplement to these figures, Figure 19 shows the relation of the pile head settlement against the total
278 applied load for 9 pile group embedded in sand. A comparison with the load-settlement curves of 4 pile group
279 shown in Figure 15 clarifies the influence of increasing the number of piles in the group in causing a larger amount
280 of settlement. This result makes sense because as the number of piles in the group increases with constant spacing,
281 the interaction between the adjacent piles certainly increases and the effects are more pronounced.

282 A rigid cap commonly offers uniform displacements for the group, but, on the other hand, a nonuniform
283 distribution of loads appears. This fact is clearly shown in Figure **??**0 which indicates the load distribution on
284 group of 9 piles embedded in sand. The load on each pile (P) is normalized by the average load of all piles in the
285 group (P av). It is evident that the greatest loads are carried out by the corner piles, followed by the border
286 piles (mid-side piles) and the center pile which is compatible with Basile (2003) and Matos et al. (2005). The
287 reason is that the center pile, due to pile-soil-pile interaction, will need a smaller amount of load to settle of the
288 same amount as the corner pile. It can be seen that the corner pile carries about 4% more than the average load,
289 while the border and the center piles carry 2% and 10% less than the average load, respectively. As the ratio of
290 pile spacing to diameter (S/D) increases, the difference between loads on each pile decreases, especially between
291 the corner and the center piles.

292 One can find a noticeable effect of the pile spacing to diameter ratio (S/D) on the load distributed for all
293 the piles in the group but for the border pile. As the spacing ratio increases form (S/D=2) to (S/D=5), the
294 difference between the corner and the center piles is reduced from 13% to 6% with approximately a horizontal
295 path for the border piles. At a spacing ratio (S/D=3), the effect of length seems to be diminished through the
296 high convergence between the different lengths behind this ratio. This refers to that moving the pile apart from
297 the adjacent piles in a group tends to reduce the interaction among them which allow the pile to go through more
298 independence.

299 Ideally, for an axially loaded pile group, all piles will carry the same amount of load as the total applied load
300 approaches the ultimate load capacity of the group (Basile, 2003). This feature becomes clearer in Figure 17 as
301 the total load increases towards its ultimate value.

302 Figure 21 illustrates the share of load carried by the base of each pile in the group. It can be seen that the
303 pile's base takes different portions of loads according to their locations but follow the same trend during the
304 gradual increase in the applied load. This may lead to think, as it was mentioned before in the problem of 4 pile
305 group, that the shear stress along the pile's interface is developed equally regardless of the number and location
306 of piles.

307 IV.

21 Conclusions

309 1. A rigid cap offers uniform displacements for all the piles in the group, but on the other hand, a non uniform
310 distribution of loads appears. For the group of 9 piles, the maximum loads are carried by the corner piles, followed
311 by the border piles and the center piles. This conclusion obtained by the boundary element analysis coincides
312 with the same findings obtained by other approaches. 2. For 9 pile group analyzed, the corner pile carries about
313 4% more than the average load, while the border and the center piles carry 2% and 10% less than the average
314 load, respectively. As the ratio of pile spacing to diameter (S/D) increases, the difference between loads on each
315 pile decreases, especially between the corner and the center piles. 3. As the applied load reaches the ultimate
316 capacity of the group, all the piles will share the same amount of load. 4. For a pile group embedded in sand, the
317 shear stress along the pile interface increases gradually during the loading process until it become fully mobilized,
318 then the pile base takes the rest of the applied load. This shear stress is found to be equally developed regardless
319 of the number and the location of the piles in the group. [1] [2] [3]

Figure 1:

Figure 2:

Hyperbolic relationship:

$$E_{tan} = E_i \left(1 - \frac{R_f t}{t_{lim}} \right)^2$$

- Rf = 1
- Rf = 0.75
- Rf = 0.5
- Rf = 0.25
- Rf = 0 (Elastic-plastic model)

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Figure 3: Figures 9 to 11
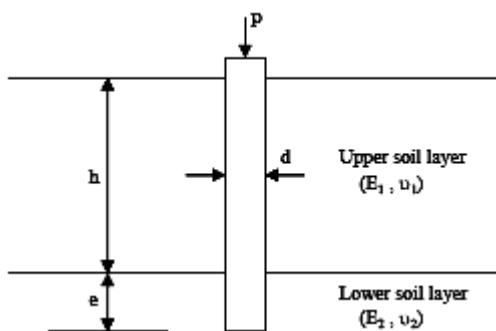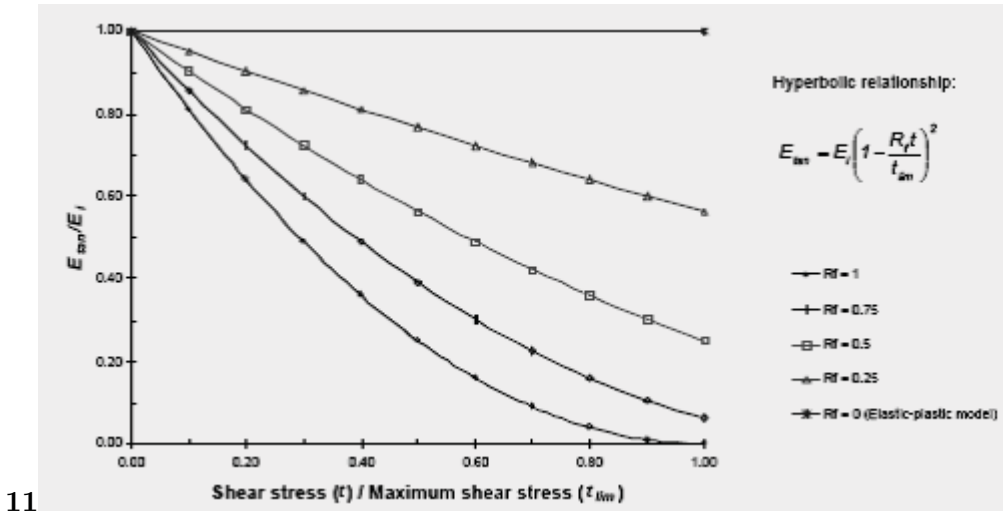
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Figure 4: Table 1 :

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Global Journal of Researches in Engineering (	Soil properties i E (kN/m 2) bulk , (kN/m 3) Poisson's ratio 3 (kN/m 2) Ø (degree)	Max. void ratio, e max Relative density, Dr % b. Strength properties 80.835 Line

Figure 5: Table 2 :

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