Optimal Pricing, Shipments and Ordering Policies for Single-Supplier Single-Buyer Inventory System with Price Sensitive Stock-Dependent Demand and Order-Linked Trade Credit

By Nita H. Shah, Dushyantkumar G. Patel & Digeshkumar B. Shah

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I. Introduction

The conventional economic order quantity (EOQ) model is formulated under the assumption that the demand of the commodity is deterministic and constant; and the buyer must follow cash-on-delivery as a mode of payment. Both these assumptions are unrealistic in business world. The supplier aims at lowering investment tie-up for warehousing inventory, to estimate demand by offering sales promotional incentives and attracting more buyer. Goyal (1985) modeled the concept of permissible delay period to compute economic order quantity. He assumed demand to be constant and calculated interest earned by the buyer on the purchase cost of the unit. Thereafter, several researchers studied different variants of inventory models with allowable trade credits to discuss advantages of this promotional tool. One can refer review article on trade credit by Shah et al. (2010).

In the cited articles, it is observed that offer of credit from the supplier to the buyer is independent of the size of the order. Once again this is also somewhat impractical.

Khouja and Mehraj (1996) analyzed the vendor's credit polices to determine the optimal order quantity when credit terms are linked to order quantity. Shinn and Hwang (2003), Chang (2004), Chang et al. (2005) Chang and Liao (2004, 2006), Chang et al. (2009), Shah and Shukla (2010, 2011), Shah (2010), Shah et al.(2012) considered order-dependent trade credit and quadratic demand structure in their analysis. These models were derived either from vendor's or from buyer's point of view. Here, only one player takes decision and that is to be followed by the other willingly or un-willingly.


Levin et al. (1992) quoted that “large piles of goods attract more customers”. This demand is treated as stock-dependent demand. Urban (2005) analyzed effect of displayed goods on order quantity. Roy and Chaudhari (2006) assumed finite planning horizon by allowing shortages to study the effect of stock-dependent parameter on decision variable and objective function.

In this study, the goal is to analyze an integrated inventory system comprising of single-vendor single-buyer when demand rate is price sensitive stock-dependent demand.
dependent. The production rate is proportion to demand rate. The buyer qualifies for delay payments only if the order is greater than that of pre-specified as announced by the vendor. The joint total profit per unit time of the supply chain is maximized with respect to retail price of the unit, order quantity for the buyer and number of shipments from the vendor to the buyer. An algorithm is outlined to decide the best optimal solution. The numerical example is given to elaborate the mathematical development of the proposed problem. Sensitivity analysis is carried out to discuss managerial insights.

II. Notation and Assumptions

a) Notation

\[ R(I(t), s) \]  Price-sensitive Stock-dependent demand rate; \((\alpha + \beta I(t))s^{-\eta} \) where \(\alpha > 0\) denotes constant scale demand, \(0 < \beta < 1\) denotes stock-dependent parameter, \(\eta > 1\) denotes price-elasticity and \(s\) denotes retail price of the product per unit by the buyer. \((s\) is a decision variable\)

- \(A_v\) Vendor’s set-up cost per set-up
- \(A_b\) Buyer’s ordering cost per order
- \(C_p\) Production cost per unit
- \(C_b\) Buyer’s purchase cost per unit

Note: \(s > C_b > C_p\)

- \(I_v\) Vendor’s inventory holding cost rate per unit per year, excluding interest charges
- \(I_b\) Buyer’s inventory holding cost rate per unit per year, excluding interest charges
- \(I_{vp}\) Vendor’s opportunity cost /$/ unit time
- \(I_{bp}\) Buyer’s opportunity cost /$/ unit time
- \(I_{be}\) Buyer’s interest earned /$/ unit time
- \(\rho\) Capacity utilization which is ratio of demand rate to the production rate; \(\rho < 1\) is known constant
- \(M\) Permissible credit period for the buyer given by the vendor
- \(Q\) Buyer’s order quantity per order (a decision variable)
- \(Q_d\) Pre-specified order quantity to qualify for delayed payment
- \(T\) Cycle time (a decision variable)
- \(T_d\) The duration of time when \(Q_d\) - units are sold off

b) Assumptions

The following assumptions are made in deriving the proposed model.

1. Single - vendor and single - buyer supply chain is studied for a single-item.
2. Lead - time is zero. Shortages are not allowed.
3. The demand rate is price - sensitive stock-dependent.
4. The buyer qualifies for settlement of account at a later date if the order is equal or larger than the pre-specified quantity \(Q_d\) by the vendor. Otherwise, the buyer must pay for the purchases immediately.
5. During the credit period, the buyer earns interest at the rate \(I_{be}\) per unit on the generated revenue. At the end of the credit period, the buyer incurs an opportunity cost at a rate of \(I_{bp}\) on the unsold items in the warehouse.

III. Mathematical Model

In this section, we develop an integrated inventory model when demand is price-sensitive stock-dependent demand and trade credit is offered only if buyer’s order quantity is equal or greater than a pre-specified quantity.

a) Vendor’s total profit per unit time

The total profit per unit time for the vendor comprises of sales revenue, set-up cost, holding cost and opportunity cost as follows:

1. Sales revenue: The total sales revenue per unit time is \( (s - C_p)Q \). (See Appendix A for computation of \(Q\))

2. Set-up cost: \(nQ\) -units are manufactured in one production run by the vendor. Therefore, the set-up cost per unit time is \(\frac{A_v}{nT}\).

3. Holding cost: Using Joglekar (1988), the vendor’s average inventory per unit time is
(4) Opportunity cost: If \( Q_d \) or more units are ordered by the buyer, the credit period of \( M - \) units is permissible to settle the account. In this scenario, vendor endures a capital and payment received. Equivalently, when \( T \geq T_d \), the delay in payments is permissible and corresponding opportunity cost per unit time is \( \frac{C_b I_{vp} Q M}{T} \). On the other hand, when \( T < T_d \) the vendor receives payments on deliver and so no opportunity cost will occur.

Hence, the total profit per unit time for the vendor is

\[
TVP(n) = \begin{cases} 
TVP_1(n), & T < T_d \\
TVP_2(n), & T \geq T_d 
\end{cases}
\]

Where

\[
TVP_1(n) = \left( \frac{(C_b - C_p)}{T} - \frac{A_v}{nT} \right) - \frac{C_p (I_v + I_{vp}) [(n-1)(1-\rho) + \rho]}{T \beta^2} \left( e^{\beta s^{-\eta} T} s^{-\eta} - s^{-\eta} - \beta T \right)
\]

\[
TVP_2(n) = \left( \frac{(C_b - C_p)}{T} \right) - \frac{A_v}{nT} - \frac{C_p (I_v + I_{vp}) [(n-1)(1-\rho) + \rho]}{T \beta^2} \left( e^{\beta s^{-\eta} T} s^{-\eta} - s^{-\eta} - \beta T \right)
\]

b) Buyer’s total profit per unit time

The total profit per unit time for the buyer comprises of sales revenue, ordering cost, holding cost, opportunity cost and interest earned. These costs are computed as follows:

(1) Sales revenue: The total sales revenue per unit time is \( (s - C_b) \frac{Q}{T} \). (See Appendix A for computation of \( Q \))

(2) Ordering cost: The ordering cost per unit time is \( \frac{A_b}{T} \).

(3) Holding cost: The buyer’s holding cost (excluding interest charges) per unit time is

\[
\frac{C_b I_{bp} O}{T}
\]

Opportunity cost per unit time

\[
= \begin{cases} 
\frac{C_b I_{bp} O}{T}, & 0 < T < T_d \\
0, & T_d \leq T \leq M \\
\frac{C_b I_{bp} \left( e^{\eta \rho - M \beta s^{-\eta} T} + \beta s^{-\eta} T + \rho M - s^{-\eta} - \beta T \right)\{ T_d \leq M \leq T \lor M \leq T_d \leq T. 
\end{cases}
\]
Interest earned: As discussed in opportunity cost interest earned per unit time in all the four cases is as follows.

Interest earned per unit time

\[
\begin{cases}
0, & 0 < T < T_d \text{ (because payment is to be made on delivery)} \\
\frac{sI_{be}}{T} \left[ \int_{0}^{T} R(I(t),s) \, dt + Q(M-T) \right], & T_d \leq T \leq M \text{ (figure 2)} \\
\frac{sI_{be}}{T} \left[ \int_{0}^{M} R(I(t),s) \, dt \right], & T_d \leq M \leq T \text{ or } M \leq T_d \leq T \text{ (figure 3)}
\end{cases}
\]

Figure 1: Opportunity cost for \( T_d \leq M \leq T \) or \( M \leq T_d \leq T \)

Figure 2: Interest earned by buyer when \( T_d \leq T \leq M \)
Figure 3: Interest earned by buyer when $T_d \leq M \leq T$

Hence, the buyer’s total profit per unit time is

$$TBP(T) = \begin{cases} 
TBR_1(T), & 0 < T < T_d \\
TBP_2(T), & T_d \leq T \leq M \\
TBP_3(T), & T_d \leq M \leq T \\
TBP_4(T), & M \leq T_d \leq T 
\end{cases}$$

(4)

Where

$$TBP_1(T) = \frac{(s-C_b)Q}{T} - \frac{A_b}{T} - \frac{C_b I_b}{T} \alpha \left( e^{\beta s^{-\eta}T} s^\eta - s^\eta - \beta T \right) - \frac{C_b I_{hp}Q}{T}$$

(5)

$$TBP_2(T) = \frac{(s-C_b)Q}{T} - \frac{A_b}{T} - \frac{C_p I_b}{T \beta^2} \alpha \left( e^{\beta s^{-\eta}T} s^\eta - s^\eta - \beta T \right)$$

$$+ \frac{s I_{bc}}{T} \int_0^T R(I(t),s) t dt + Q(M - T)$$

(6)

$$TBP_3(T) = TBP_4(T) = \frac{(s-C_b)Q}{T} - \frac{A_b}{T} - \frac{C_b I_b}{T \beta^2} \alpha \left( e^{\beta s^{-\eta}T} s^\eta - s^\eta - \beta T \right)$$

$$- \frac{C_b I_{hp}}{T \beta^2} \alpha \left( s^{-\eta} e^{-M \beta s^{-\eta}} + s^{-\eta} e^{-M \beta s^{-\eta}T} + \beta M - s^\eta - \beta T \right)$$

$$- \frac{s I_{bc}}{T \beta^2} \alpha \left( -e^{\beta s^{-\eta}T} s^\eta + s^{-\eta} e^{-M \beta s^{-\eta}} + e^{-M \beta s^{-\eta}} + \beta s^{-\eta} T M \beta \right)$$

(7)

c) Joint total profit per unit time

In integrated system, the vendor and buyer decide to take joint decision which maximizes the profit of the supply chain. The joint total profit per unit time for the integrated system is
\[ \pi(n, T) = \begin{cases} 
\pi_1(n, T) = TVP_1(n) + TBR_1(T), & 0 < T < T_d \\
\pi_2(n, T) = TVP_2(n) + TBP_2(T), & T_d \leq T \leq M \\
\pi_3(n, T) = TVP_2(n) + TBP_3(T), & T_d \leq M \leq T \\
\pi_4(n, T) = TVP_2(n) + TBP_3(T), & M \leq T_d \leq T.
\end{cases} \]

Where

\[ \pi_1(n, T) = \left(s - C_p - C_b I_{bp}\right) \frac{Q}{T} - \frac{A}{T} \left(\phi + \psi\right) \alpha \left(e^{\beta s^{-\eta}T} s^{-\eta} - s^{-\eta} - \beta T\right) \]

\[ \pi_2(n, T) = \left(s - C_p - (C_b I_{vp} - s I_{be}) M\right) \frac{Q}{T} - s I_{be} Q - \frac{A}{T} \left(\phi + \psi\right) \alpha \left(e^{\beta s^{-\eta}T} s^{-\eta} - s^{-\eta} - \beta T\right) + \frac{s I_{be}}{T} \int_0^T R(I(t), s) t dt \]

\[ \pi_3(n, T) = \left(s - C_p - C_b I_{vp} M\right) \frac{Q}{T} - \frac{A}{T} \left(\phi + \psi\right) \alpha \left(e^{\beta s^{-\eta}T} s^{-\eta} - s^{-\eta} - \beta T\right) \]

\[ - \frac{C_b I_{bp} \alpha}{\beta^2 T} \left(e^{\beta s^{-\eta}(T-M)} s^{-\eta} - s^{-\eta} - \beta (T-M)\right) \]

\[ + \frac{s I_{be} \alpha}{\beta^2 T} \left(e^{\beta s^{-\eta}T} s^{-\eta} - (s^{-\eta} + \beta M) e^{\beta s^{-\eta}(T-M)}\right) \]

\[ \bar{A} = A_b + \frac{A_v}{n} \]

\[ \phi = \frac{C_p (I_v + I_{vp}) [(n-1)(1 - \rho) + \rho]}{\beta^2} \]

\[ \psi = \frac{C_b I_{bp}}{\beta^2} \]

IV. Computational Procedure

For fixed \( T \), we note that \( \pi(n, T) \) is a concave function of \( n \) because
\( \frac{\partial^2 \pi(n, T)}{\partial n^2} = -2 \frac{A_v}{n^3 T} < 0 \).

Therefore to find optimum number of shipments \( n^* \) we will have a local optimal solution. The optimum value of cycle time and retail price can be obtained by setting
\( \frac{\partial \pi}{\partial T} = 0 \) and \( \frac{\partial \pi}{\partial s} = 0 \) simultaneously for fixed \( n \).

Algorithm:
Step 1: Set parametric values.
Step 2: Compute \( T_d \) using \( \frac{1}{\beta s^{-\eta}} \ln\left(1 + \frac{\beta Q_d}{\alpha}\right) \) for given value of \( Q_d \).
Step 3: Set \( n = 1 \).
Step 4: Knowing \( T_d \) and \( M \), compute \( T \) and \( s \) by solving \( \frac{\partial \pi_j}{\partial T} = 0 \) and \( \frac{\partial \pi_j}{\partial s} = 0 \) simultaneously for \( j = 1, 2, 3 \).
Step 5: Find corresponding profit \( \pi_j \) for \( j = 1, 2, 3 \).
Step 6: Increment \( n \) by 1.
Step 7: Repeat step 4 and 6 until
\( \pi(n-1, T(n-1), s(n-1)) \leq \pi(n, T(n), s(n)) \geq \pi(n+1, T(n+1), s(n+1)) \).
Once the optimal solution \( (n^*, T^*, s^*) \) is obtained, the optimal order quantity can be obtained.

V. **Numerical Examples and Interpretations**

Example 1: Consider, \( \alpha = 10000 \) units, \( \beta = 10\% = 0.1 \), \( \eta = 1.25 \), \( \rho = 0.7 \), \( C_b = $10/\text{unit} \), \( C_p = $5/\text{unit} \), \( C_{v} = $400/\text{setup} \), \( C_{b} = $50/\text{order} \), \( I_v = 10\%/\text{unit/annum} \), \( I_b = 10\%/\text{unit/annum} \), \( I_{bp} = 8\%/\text{unit/annum} \), \( I_{be} = 5\%/\$/annum \), \( I_{vp} = 2\%/\text{unit/annum} \), \( s = $25/\text{unit} \) and \( M = 30 \) days.

The optimal shipments and ordering units with buyer, vendor and joint profit for different values of \( Q_d \) are exhibited in Table 1.

<table>
<thead>
<tr>
<th>( Q_d )</th>
<th>( Q^* )</th>
<th>( n^* )</th>
<th>( T^* ) (days)</th>
<th>Profit($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>74</td>
<td>11</td>
<td>184</td>
<td>Buyer $2661, Vendor $576, Joint $3237</td>
</tr>
<tr>
<td>200</td>
<td>74</td>
<td>11</td>
<td>184</td>
<td>Buyer $2661, Vendor $576, Joint $3237</td>
</tr>
<tr>
<td>300</td>
<td>300</td>
<td>9</td>
<td>219</td>
<td>Buyer $2690, Vendor $566, Joint $3256</td>
</tr>
<tr>
<td>400</td>
<td>400</td>
<td>9</td>
<td>219</td>
<td>Buyer $2690, Vendor $566, Joint $3256</td>
</tr>
<tr>
<td>500</td>
<td>74</td>
<td>11</td>
<td>184</td>
<td>Buyer $2661, Vendor $576, Joint $3237</td>
</tr>
<tr>
<td>600</td>
<td>74</td>
<td>11</td>
<td>184</td>
<td>Buyer $2661, Vendor $576, Joint $3237</td>
</tr>
</tbody>
</table>

From Table 1, it is seen that the vendor’s total profit and joint total profit of the system increase with increase in \( Q_d \) and then further increase in prespecified units lower their profits whereas for the buyer, it is opposite trend. It is seen that the buyer’s optimal order quantity \( Q^* \) is equal to \( Q_d \) when and less than \( Q_d \) when \( Q_d \geq 500 \). Thus, vendor is advised to set threshold which is effective. If the threshold set by the vendor is too high, the buyer will be reluctant to order a quantity greater than the threshold to take advantage of delayed payments. The concavity of joint total profit for \( (n, s) \) for obtained \( T^* \) is exhibited in fig. 4, for \( (n, T) \) for \( s = 29.63 \) in fig. 5 and for \( (s, T) \) for 9 - shipments in fig. 6.

![Figure 4: Concavity of joint total profit for (n,s) for obtained T^*](image-url)
Example 2 Consider the data given in Example 1. We study the effect of delayed payments for $Q_d = 300$ units.

**Figure 5:** Concavity of joint total profit for $(n, T)$ for $s$.

**Figure 6:** Concavity of joint total profit for $(s, T)$ for $n$. 
Table 2: Optimal solutions for different $M(Q_d = 300)$

<table>
<thead>
<tr>
<th>$M$ (days)</th>
<th>$Q^*$</th>
<th>$n^*$</th>
<th>$T^*$ (days)</th>
<th>Profit($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>300</td>
<td>9</td>
<td>219</td>
<td>2684</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>567</td>
</tr>
<tr>
<td>30</td>
<td>300</td>
<td>9</td>
<td>219</td>
<td>2687</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>567</td>
</tr>
<tr>
<td>40</td>
<td>300</td>
<td>9</td>
<td>219</td>
<td>2690</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>566</td>
</tr>
<tr>
<td>50</td>
<td>300</td>
<td>9</td>
<td>218</td>
<td>2693</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>566</td>
</tr>
<tr>
<td>60</td>
<td>300</td>
<td>9</td>
<td>218</td>
<td>2696</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>566</td>
</tr>
</tbody>
</table>

From Table 2, it is observed that longer credit period increases buyer's total profit and joint profit of the supply chain. The longer credit period reduces vendor's total profit because payment will be received late for the purchases made. This suggests that late payment increases risk of cash shortage for the vendor.

Example 3. In this example, we carry out sensitivity analysis to find the critical inventory parameters. The changes in the optimal cycle time, purchase quantity and joint profit are studied by varying inventory parameters as −20%, −10%, 10% and 20%. The results are exhibited in Figure 7.

![Figure 7: Variation in joint total profit](image)

It is observed from fig. 7 that joint profit increases positively with increase in scale demand. It is evident that both the player should take advantage of demand increase and setting agreeable selling price. Production cost of supplier reduced joint total profit. It is advised to the supplier to use advanced technology which reduces this production cost. Other inventory parameters have very small perturbations in profit of the supply chain.

Example 4. In table 3, we compare independent vs. joint decision, for pre-specified quantity $Q_d = 300$ units at which buyer qualifies for getting delay period facility.
Table 3: Optimal Solution of independent and integrated scenario

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Total Shipments</th>
<th>Buyer's Total</th>
<th>Vendor's Total</th>
<th>Joint Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independent</td>
<td>9</td>
<td>2901</td>
<td>236</td>
<td>3137</td>
</tr>
<tr>
<td></td>
<td>Ordering Quantity</td>
<td>62</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cycle Time</td>
<td>329</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Integrated</td>
<td>9</td>
<td>2690</td>
<td>567</td>
<td>3256</td>
</tr>
<tr>
<td></td>
<td>Ordering Quantity</td>
<td>87</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cycle Time</td>
<td>219</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total Annual Profit</td>
<td></td>
<td>2901</td>
<td>236</td>
</tr>
<tr>
<td>Readjusted</td>
<td>Total Annual Profit</td>
<td></td>
<td>3011</td>
<td>245</td>
</tr>
</tbody>
</table>

Where

\[ \pi(n, T) = \frac{TBP(P, T)}{TBP(P, T) + TVP(n)} \]

Buyer's profit = \[ \pi(n, T) \times \frac{2901}{2901 + 236} = 3011 \]

Supplier's profit = \[ \pi(n, T) \times \frac{236}{2901 + 236} = 245 \]

Table 3 shows that the total annual profit under joint decision $3256 (=$2690+$567) which is greater than the total profit under independent decision $3137 (=$2901+$236). It establishes that joint decision is advantageous to both the players. The last row of table 3 is about readjustment of the profits (Goyal (1976)) to encourage players for joint decision.

VI. Conclusion

A single-vendor single-buyer inventory policy is analyzed. The demand is considered to be price-sensitive stock-dependent. The vendor offers order-dependent credit time to settle the account for the purchases made. The computational algorithm is outlined to maximize the joint total profit per unit time with respect to number of transfers from the vendor to the buyer, retail price of the product to be set by the buyer and cycle time. Based on the results, it is established that longer credit period helps buyer while out-flow risk is for vendor. To entice the buyer for the co-ordinate decision, vendor should set proper threshold for pre-specified order units. It is observed that the order-dependent trade credit attracts the buyer for larger order and thereby saving in transportation cost which is part of the ordering cost.

References Références Referencias

6. Chung, K. J., Goyal, S. K. and Huang, Y. F., 2005. The optimal inventory policies under permissible delay in payments depending on the ordering
Appendix A

The rate of change of inventory at any instant of time can be discussed by differential equation

\[ \frac{dI(t)}{dt} = -\left(\alpha + \beta I(t)\right)s^{-\eta}, 0 \leq t \leq T \]

with \( I(0) = Q \) and \( I(T) = 0 \). Using \( I(T) = 0 \), the solution of the differential equation is

\[ I(t) = \frac{\alpha}{\beta} \left( e^{\beta s^{-\eta} (T-t)} - 1 \right), \quad 0 \leq t \leq T \]

The units to be purchased \( Q = I(0) = \frac{\alpha}{\beta} \left( e^{\beta s^{-\eta} T} - 1 \right) \).