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Acceleration Analysis of 3DOF Parallel Manipulators

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ACCELERATIONANALYSISOF3DOFPARALLELMANIPULATORS

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Acceleration Analysis of 3DOF Parallel Manipulators

Hassen Nigatu [α], Ajit Pal Singh [σ], Solomon Seid [ρ]

Abstract - This paper presents a new approach to the velocity and acceleration analyses 3DOF parallel manipulators. Building on the definition of the 'acceleration motor', the forward and inverse velocity and acceleration equations are formulated such that the relevant analysis can be integrated under a unified framework that is based on the generalized Jacobian. A new Hessian matrix of serial kinematic chains (or limbs) is developed in an explicit and compact form using Lie brackets. This idea is then extended to cover parallel manipulators by considering the loop closure constraints. A 3-PRS parallel manipulator with coupled translational and rotational motion capabilities is analyzed to illustrate the generality and effectiveness of this approach.

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I. Introduction

Lower mobility parallel manipulators having fewer than six degrees of freedom (DOF) continue to draw interest from both industry and academia because they regularly offer improved balance between speed, accuracy, rigidity and reconfigurability when compared with conventional machine tools and industrial robots having serial architectures.

Velocity, accuracy, stiffness and rigid body dynamic behaviours are important performance factors that's should be considered in the design of lower mobility parallel manipulators. Particularly in circumstances where high speed is the priority, rigid body dynamics become a major concern for the dynamic manipulability evaluation, motor sizing and controller design, all of which involve acceleration analysis as the prerequisite.

Although general, systematic approaches are available for velocity analysis of lower mobility parallel manipulators using either kinematic influence coefficient methods or screw theory based methods (Huang et al., 2000; Joshi & Tsai, 2002; Jhu et al., 2007) it is by no means an easy task to use these approaches for acceleration analysis owing to the nonlinearity arising from the second order partial derivatives.

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A number of approaches have been proposed for acceleration analysis of either serial or parallel manipulators. The most straightforward method is to take time derivatives of a set of velocity constraint equations. This, however, involves a tedious and laborious process as shown by many case-by-case studies (Tsai, 2000; Khalil & Guegan, 2004; Li et al., 2005; Callegari et al., 2006). Therefore, a recursive matrix method was proposed in order to reduce computational burdens (Staicu & Zhang, 2008; Staicu, 2009). Having a goal of achieving a general and compact form of the Hessian matrix, the kinematic influence coefficient method was proposed for dealing with the acceleration analysis of serial manipulators (Thomas & Twsar, 1982). This idea was then extended to cover full and lower mobility parallel manipulators (Huang, 1985a; Huang, 1985b; Zhu, 2005; Huang, 2006; Zhu et al., 2007). Along this track, kinematic analysis of a number of parallel manipulators with different architectures has been carried out (Fang & Huang, 1997; Lu, 2006; Lu, 2008). Despite the plausibility and merits of the kinematic influence coefficient method, only an implicit form of the Hessian matrix can be achieved because of the unavoidable partial derivative implementations. Recently, an approach for acceleration analysis was proposed that introduced an auxiliary Hessian matrix derived from the differentiation of the auxiliary Jacobian of a class of parallel manipulators containing a passive properly constrained limb (Lu & Hu, 2007a; Lu & Hu, 2007b; Lu & Hu, 2008). Its suitability for other types of parallel manipulators, however, remains an issue to be investigated. Screw theory based approaches (Hunt, 1978; Mohamed & Duffy, 1985; Kumar, 1992; Ling & Huang, 1995; Bonev et al., 2003; Fang & Tsai, 2003; Zoppi, 2006) could potentially be the most powerful method for acceleration analysis. In order to overcome the difficulty of expressing the twist derivatives in a screw form, a novel term named the "accelerator" (Sugimoto, 1990) or "acceleration motor" (Brand, 1947) was proposed and employed for the acceleration analysis of serial and parallel kinematic chains (Rico & Duffy, 1996; Rico & Duffy, 2000). However, the terms associated with the second derivatives in the acceleration equations can only be written in a lengthy form of Lie brackets rather than in a compact form of the Hessian matrix.

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Drawing primarily on the generalized Jacobian but also on the strengths of the kinematic influence coefficient method and the concept of accelerator, we propose a new approach for acceleration analysis of lower mobility parallel manipulators. Its goal is to achieve an explicit and compact form of the Hessian matrix. Having outlined in Section I the significance of acceleration analysis and its existing problems, the paper is organized as follows. Sections II and III systematically develop the formulations of forward/inverse velocity and acceleration models of serial and parallel kinematic chains, leading to new expressions of the Hessian matrices in a general and compact form. A practical illustration is presented in Section IV before conclusions are drawn in Section V.

II. Velocity Analysis

Based upon the authors' previous work (Huang et al., 2011), this section briefly addresses the velocity analysis of an f-DOF parallel manipulator using the "generalized Jacobian" and adds extensions necessary to its use in acceleration analysis. Without loss generality, assume that the manipulator is composed of l ($f \le l \le f+1$) limbs connecting the platform with the base, each essentially containing n_i $(i=1,2,\cdots,l)$ 1-DOF joints with at most one of them actuated. Thus, two families of parallel manipulators can be taken into account. The first family covers fully parallel manipulators having f constrained active limbs ($n_i < 6$ for all limbs). The second family contains those having f unconstrained active limbs (i.e. $n_i = 6$ for each of these f limbs) plus one properly constrained passive limb designated by $l = f+1$. Any other parallel manipulators not belonging to these two families can be dealt with in a manner similar to that used below.

It has been shown (Huang et al., 2011) that entire set of the variational twists of the platform spans a 6-dimensional vector space T, known as the twist space. As the dual space of T, the entire set of wrenches exerted on the platform spans a 6-dimensional vector space W, named the wrench space. For an f-DOF manipulator, T can be decomposed into an f dimensional subspace, $\mathsf{T}_a \subseteq \mathsf{T}$, and a 6-$f$ dimensional subspace, $\mathsf{T}_c \subseteq \mathsf{T}$, known respectively as the twist subspaces of permissions and restrictions. Correspondingly, W can also be decomposed into two subspaces, $\mathsf{W}_a \subseteq \mathsf{W}$ and $\mathsf{W}_c \subseteq \mathsf{W}$, known as the wrench subspaces of actuations and constraints. It has been proved that the following commutative relationships hold:

Direct sum: $\mathsf{T} = \mathsf{T}_a \oplus \mathsf{T}_c$, $\mathsf{W} = \mathsf{W}_a \oplus \mathsf{W}_c$ (1a)

Orthogonality: $\mathsf{W}_a = \mathsf{T}_c^{\perp}$, $\mathsf{W}_c = \mathsf{T}_a^{\perp}$ (1b)

Duality: $\mathsf{W}_a = \mathsf{T}_a^{*}$, $\mathsf{W}_c = \mathsf{T}_c^{*}$ (1c)

a) Velocity analysis of a limb

Let $\hat{\$}_{ta,j_a,i} \in \mathsf{T}_{a,i}$ $(j_a = 1,2,\cdots,n_i)$, $\hat{\$}_{wa,k_a,i} \in \mathsf{W}_{a,i}$ $(k_a = 1,2,\cdots,n_i)$, $\hat{\$}_{tc,j_c,i} \in \mathsf{T}_{c,i}$ $(j_c = 1,2,\cdots,6-n_i)$ and $\hat{\$}_{wc,k_c,i} \in \mathsf{W}_{c,i}$ $(k_c = 1,2,\cdots,6-n_i)$ be the basis elements of four vector subspaces associated with the ith limb. Note that the commutative relationships given in Eq. (1a, b, c) also hold for each limb since all limbs share the same platform. The variational twist of the platform can then be represented by a linear combination of the basis elements of $\mathsf{T}_{a,i}$ and $\mathsf{T}_{c,i}$

$$\begin{aligned}\boldsymbol{\$}_t &= \boldsymbol{\$}_{ta} + \boldsymbol{\$}_{tc} = \boldsymbol{\$}_{ta,i} + \boldsymbol{\$}_{tc,i} \\ &= \sum_{j_a=1}^{n_i} \delta\rho_{a,j_a,i}\hat{\$}_{ta,j_a,i} + \sum_{j_c=1}^{6-n_i} \delta\rho_{c,j_c,i}\hat{\$}_{tc,j_c,i}\ ,\quad i=1,2,\cdots,l \\ &= \boldsymbol{J}_i \delta\boldsymbol{\rho}_i\end{aligned} \tag{2}$$

where,

$$\boldsymbol{\$}_t = \begin{pmatrix}\delta\boldsymbol{r}^{\mathrm{T}} & \delta\boldsymbol{\alpha}^{\mathrm{T}}\end{pmatrix}^{\mathrm{T}}$$

$$\boldsymbol{J}_i = \begin{bmatrix}\boldsymbol{J}_{a,i} & \boldsymbol{J}_{c,i}\end{bmatrix}$$

$$\boldsymbol{J}_i = \begin{bmatrix}\boldsymbol{J}_{a,i} & \boldsymbol{J}_{c,i}\end{bmatrix}$$

$$\boldsymbol{J}_{a,i} = \begin{bmatrix}\hat{\$}_{ta,1,i} & \cdots & \hat{\$}_{ta,n_i,i}\end{bmatrix}$$

$$\boldsymbol{J}_{c,i} = \begin{bmatrix}\hat{\$}_{tc,1,i} & \cdots & \hat{\$}_{tc,6-n_i,i}\end{bmatrix}$$

$$\delta\boldsymbol{\rho}_i = \begin{pmatrix}\delta\boldsymbol{\rho}_{a,i}^{\mathrm{T}} & \delta\boldsymbol{\rho}_{c,i}^{\mathrm{T}}\end{pmatrix}^{\mathrm{T}}$$

$$\delta\boldsymbol{\rho}_{a,i} = \begin{pmatrix}\delta\rho_{a,1,i}\ \delta\rho_{a,2,i} \cdots \delta\rho_{a,n_i,i}\end{pmatrix}^{\mathrm{T}}$$

$$\delta\boldsymbol{\rho}_{c,i} = \begin{pmatrix}\delta\rho_{c,1,i}\ \delta\rho_{c,2,i} \cdots \delta\rho_{c,6-n_i,i}\end{pmatrix}^{\mathrm{T}}$$

$\delta\boldsymbol{r}$ and $\delta\boldsymbol{\alpha}$ denote, respectively, the linear variation of the reference point and the angular variation of the platform. $\hat{\$}_{ta,j_a,i}$ and $\delta\rho_{a,j_a,i}$ ($\hat{\$}_{tc,j_c,i}$ and $\delta\rho_{c,j_c,i}$) are the j_ath (j_cth) unit screw of permissions (restrictions) and its intensity. $\boldsymbol{J}_i$ is a 6×6 matrix known as the "generalized Jacobian" of a limb having connectivity of $n_i \le 6$.

For velocity analysis, which considers only ideal motions of the platform, relevant terms in Eq. (2) are replaced by:

$$\delta\boldsymbol{\rho}_i = \begin{pmatrix}\delta\boldsymbol{\rho}_{a,i}^{\mathrm{T}} & \delta\boldsymbol{\rho}_{c,i}^{\mathrm{T}}\end{pmatrix}^{\mathrm{T}} \rightarrow \dot{\boldsymbol{\theta}}_i = \begin{pmatrix}\dot{\boldsymbol{\theta}}_{a,i}^{\mathrm{T}} & \boldsymbol{0}^{\mathrm{T}}\end{pmatrix}^{\mathrm{T}}$$

$$\delta\boldsymbol{\rho}_{a,i} \rightarrow \dot{\boldsymbol{\theta}}_{a,i}$$

$$\delta\boldsymbol{\rho}_{c,i} \rightarrow \boldsymbol{0}$$

$$\dot{\boldsymbol{\theta}}_{a,i} = \begin{pmatrix}\dot{\theta}_{a,1,i} & \dot{\theta}_{a,1,i} & \cdots & \dot{\theta}_{a,n_i,i}\end{pmatrix}^{\mathrm{T}}$$

$$\boldsymbol{\$}_t = \begin{pmatrix}\delta\boldsymbol{r}^{\mathrm{T}} & \delta\boldsymbol{\alpha}^{\mathrm{T}}\end{pmatrix}^{\mathrm{T}} \rightarrow \boldsymbol{\$}_t = \begin{pmatrix}\boldsymbol{v}^{\mathrm{T}} & \boldsymbol{\omega}^{\mathrm{T}}\end{pmatrix}^{\mathrm{T}}$$

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where, $\dot{\theta}_i$ are individual joint velocities. Thus (Huang et al., 2011)

$$\$_t = \boldsymbol{J}_i \dot{\boldsymbol{\theta}}_i ,\ i = 1,2,\cdots,l \tag{3}$$

where, $\$_t = \left(\boldsymbol{v}^{\mathrm{T}} \quad \boldsymbol{\omega}^{\mathrm{T}}\right)^{\mathrm{T}}$ becomes the velocity twist: $\boldsymbol{v}$ and $\boldsymbol{\omega}$ are the linear velocity of the reference point and the angular velocity of the platform. $\dot{\theta}_{a,j_a,i}$ ($k_a = 1,2,\cdots,n_i$) represents the joint rate of the j_ath joint in limb i.

Using the commutative relationships given in Eq. (1) and taking inner products on both sides of Eq. (2) with $\hat{\$}_{wa,k_a,i}$ $(k_a = 1,2,\cdots,n_i)$ and $\hat{\$}_{wc,k_c,i}$ ($k_c = 1,2,\cdots,6-n_i$) leads, after the same replacements, to

$$\dot{\boldsymbol{\theta}}_i = \boldsymbol{J}_i^{-1} \$_t ,\ i = 1,2,\cdots,l \tag{4}$$

$$\boldsymbol{J}_i^{-1} = \begin{bmatrix} \boldsymbol{J}_{a,i}^L \\ \boldsymbol{J}_{c,i}^L \end{bmatrix}$$

$$\boldsymbol{J}_{a,i}^L = \begin{bmatrix} \hat{\$}_{wa,1,i}^{\mathrm{T}} \big/ \hat{\$}_{wa,1,i}^{\mathrm{T}} \hat{\$}_{ta,1,i} \\ \vdots \\ \hat{\$}_{wa,n_i,i}^{\mathrm{T}} \big/ \hat{\$}_{wa,n_i,i}^{\mathrm{T}} \hat{\$}_{ta,n_i,i} \end{bmatrix}$$

$$\boldsymbol{J}_{c,i}^L = \begin{bmatrix} \hat{\$}_{wc,1}^{\mathrm{T}} \big/ \hat{\$}_{wc,1}^{\mathrm{T}} \hat{\$}_{tc,1} \\ \vdots \\ \hat{\$}_{wc,6-n_i,i}^{\mathrm{T}} \big/ \hat{\$}_{wc,6-n_i,i}^{\mathrm{T}} \hat{\$}_{tc,6-n_i,i} \end{bmatrix}$$

Here the superscript L simply identifies that the matrix applies to a single limb. Thus, we have

$$\dot{\boldsymbol{\theta}}_{a,i} = \boldsymbol{J}_{a,i}^L \$_t ,\ i = 1,2,\cdots,l \tag{5}$$

b) Velocity analysis of a parallel manipulator

Building upon the work in Section 2.1, the velocity modeling of a parallel manipulator can be carried out with little extra effort. Let $\hat{\$}_{wa,g_k,k}$ be the unit wrench of actuations associated with the one actuated joint, numbered g_k, in the kth ($k = 1,2,\cdots,f$) limb and $\hat{\$}_{wc,k_c,i}$ be the k_cth ($k_c = 1,2,\ldots,6-n_k$) unit wrench of constraints in the ith ($i = 1,2,\cdots,l$) limb. Again, using the commutative relationships given in Eq. (1), taking inner products on both sides of Eq. (2) with $\hat{\$}_{wa,g_k,k}$ and $\hat{\$}_{wc,k_c,i}$, respectively, and making replacements as at Eq. (3) results in

$$\boldsymbol{J} \$_t = \dot{\boldsymbol{q}} \tag{6}$$

where,

$$\boldsymbol{J} = \begin{bmatrix} \boldsymbol{J}_a \\ \boldsymbol{J}_c \end{bmatrix}$$

$$\boldsymbol{J}_a = \begin{bmatrix} \hat{\$}_{wa,g_1,1}^{\mathrm{T}} \big/ \hat{\$}_{wa,g_1,1}^{\mathrm{T}} \hat{\$}_{ta,g_1,1} \\ \hat{\$}_{wa,g_2,2}^{\mathrm{T}} \big/ \hat{\$}_{wa,g_2,2}^{\mathrm{T}} \hat{\$}_{ta,g_2,2} \\ \vdots \\ \hat{\$}_{wa,g_f,f}^{\mathrm{T}} \big/ \hat{\$}_{wa,g_f,f}^{\mathrm{T}} \hat{\$}_{ta,g_f,f} \end{bmatrix}$$

$$\boldsymbol{J}_c = \begin{bmatrix} \boldsymbol{J}_{c,1} \\ \boldsymbol{J}_{c,2} \\ \vdots \\ \boldsymbol{J}_{c,l} \end{bmatrix}$$

$$\boldsymbol{J}_{c,i} = \begin{bmatrix} \hat{\$}_{wc,1,i}^{\mathrm{T}} \big/ \hat{\$}_{wc,1,i}^{\mathrm{T}} \hat{\$}_{tc,1,i} \\ \hat{\$}_{wc,2,i}^{\mathrm{T}} \big/ \hat{\$}_{wc,2,i}^{\mathrm{T}} \hat{\$}_{tc,2,i} \\ \vdots \\ \hat{\$}_{wc,6-n_i,i}^{\mathrm{T}} \big/ \hat{\$}_{wc,6-n_i,i}^{\mathrm{T}} \hat{\$}_{tc,6-n_i,i} \end{bmatrix}$$

$$\dot{\boldsymbol{q}} = \left(\dot{\boldsymbol{q}}_a^{\mathrm{T}} \quad \boldsymbol{0}^{\mathrm{T}}\right)^{\mathrm{T}}$$

$$\dot{\boldsymbol{q}}_a = \left(\dot{q}_{a,g_1,1} \quad \dot{q}_{a,g_2,2} \quad \cdots \quad \dot{q}_{a,g_f,f}\right)^{\mathrm{T}}$$

In order to distinguish the joint rates of actuated joints from those of the passive joints, we use $\dot{q}_{a,g_k,k}$ to represent the rate of actuated joint numbered g_k in the kth limb. For convenience, $\dot{q}_{a,g_k,k}$ will be simplified as $\dot{q}_k$ in what follows. $\boldsymbol{J}$ is an $\left(f + \sum_{i=1}^{l}(6-n_i)\right) \times 6$ matrix known as the generalized Jacobian of parallel manipulators with $f \le 6$ DOF. Since $\left(f + \sum_{i=1}^{l}(6-n_i)\right) \ge 6$, the left pseudo-inverse of $\boldsymbol{J}$ exists. Using superscript P to identify explicitly platform terms, this leads to

$$\$_t = \boldsymbol{J}^{+} \dot{\boldsymbol{q}} = \boldsymbol{J}_a^P \dot{\boldsymbol{q}}_a ,\ \boldsymbol{J}^{+} = \left(\boldsymbol{J}^{\mathrm{T}} \boldsymbol{J}\right)^{-1} \boldsymbol{J}^{\mathrm{T}} = \begin{bmatrix} \boldsymbol{J}_a^P & \boldsymbol{J}_c^P \end{bmatrix} \tag{7}$$

Substituting Eq. (7) into Eq. (4), $\dot{\boldsymbol{\theta}}_i$ can then be expressed in terms of $\dot{\boldsymbol{q}}$

$$\dot{\boldsymbol{\theta}}_i = \boldsymbol{J}_i^{-1} \boldsymbol{J}^{+} \dot{\boldsymbol{q}} = \boldsymbol{J}_i^{LP} \dot{\boldsymbol{q}} ,\ i = 1,2,\cdots,l \tag{8}$$

where, $\boldsymbol{J}_i^{LP} = \boldsymbol{J}_i^{-1} \boldsymbol{J}^{+}$ is a $6 \times \left(f + \sum_{i=1}^{l}(6-n_i)\right)$ matrix.

Furthermore, the linear map between $\dot{\boldsymbol{q}}_a$ and $\dot{\boldsymbol{\theta}}_{a,i}$ is

$$\dot{\boldsymbol{\theta}}_{a,i} = \boldsymbol{J}_{a,i}^L \boldsymbol{J}_a^P \dot{\boldsymbol{q}}_a = \boldsymbol{J}_{a,i}^{LP} \dot{\boldsymbol{q}}_a ,\ i = 1,2,\cdots,l \tag{9}$$

where,

$\boldsymbol{J}_{a,i}^{LP} = \boldsymbol{J}_{a,i}^L \boldsymbol{J}_a^P$ is a $n_i \times f$ matrix.

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III. Acceleration Analysis

Following the scheme in Section II, acceleration analysis will first be carried out on an n_i-DOF limb, with the results then being extended to cover an f-DOF parallel manipulator.

a) Acceleration analysis of a limb

Taking the variation of Eq. (2) and expressing the derivatives of screws in the form of Lie brackets as given in (Gallardo et al., 2003), yields

$$\boldsymbol{A}=\boldsymbol{J}_i\delta^2\boldsymbol{\rho}_i+\$_i,\ \ i=1,2,\cdots,l \tag{10}$$

where,

$$\boldsymbol{A}=\left(\left(\delta^2\boldsymbol{r}-\delta\boldsymbol{\alpha}\times\delta\boldsymbol{r}\right)^{\mathrm{T}}\quad\left(\delta^2\boldsymbol{\alpha}\right)^{\mathrm{T}}\right)^{\mathrm{T}}$$

$$\delta^2\boldsymbol{\rho}_i=\left(\left(\delta^2\boldsymbol{\rho}_{a,i}\right)^{\mathrm{T}}\quad\left(\delta^2\boldsymbol{\rho}_{c,i}\right)^{\mathrm{T}}\right)^{\mathrm{T}}$$

$$\delta^2\boldsymbol{\rho}_{a,i}=\left(\delta^2\rho_{a,1,i}\quad\delta^2\rho_{a,2,i}\quad\cdots\quad\delta^2\rho_{a,n_i,i}\right)^{\mathrm{T}}$$

$$\delta^2\boldsymbol{\rho}_{c,i}=\left(\delta^2\rho_{c,1,i}\quad\delta^2\rho_{c,2,i}\quad\cdots\quad\delta^2\rho_{c,6-n_i,i}\right)^{\mathrm{T}}$$

$$\begin{aligned}\$_i=&\left[\delta\rho_{a,1,i}\hat{\$}_{ta,1,i}\quad\delta\rho_{a,2,i}\hat{\$}_{ta,2,i}+\cdots+\delta\rho_{c,6-n_i,i}\hat{\$}_{tc,6-n_i,i}\right]\\&+\left[\delta\rho_{a,2,i}\hat{\$}_{ta,2,i}\quad\delta\rho_{a,3,i}\hat{\$}_{ta,3,i}+\cdots+\delta\rho_{c,6-n_i,i}\hat{\$}_{tc,6-n_i,i}\right]\\&+\cdots+\left[\delta\rho_{c,5-n_i,i}\hat{\$}_{tc,5-n_i,i}\quad\delta\rho_{c,6-n_i,i}\hat{\$}_{tc,6-n_i,i}\right]\end{aligned}$$

$\delta^2\boldsymbol{r}$, $\delta^2\boldsymbol{\alpha}$, $\delta^2\rho_{a,j_a,i}$, and $\delta^2\rho_{c,j_c,i}$ denote, respectively, the variation of $\delta\boldsymbol{r}$, $\delta\boldsymbol{\alpha}$, $\delta\rho_{a,j_a,i}$, and $\delta\rho_{c,j_c,i}$. The bracket $[*\quad *]$ in $\$_i$ denotes the Lie product (Gallardo, 2006).

From the properties of the Lie product, $\$_i$ can also be written as

$$\$_i=\$_{a,i}+\$_{ac,i}+\$_{c,i} \tag{11}$$

where,

$$\begin{aligned}\$_{a,i}=&\delta\rho_{a,1,i}\delta\rho_{a,2,i}\left[\hat{\$}_{ta,1,i}\ \hat{\$}_{ta,2,i}\right]+\cdots+\delta\rho_{a,1,i}\delta\rho_{a,n_i,i}\left[\hat{\$}_{ta,1,i}\ \hat{\$}_{ta,n_i,i}\right]\\&+\delta\rho_{a,2,i}\delta\rho_{a,3,i}\left[\hat{\$}_{ta,2,i}\ \hat{\$}_{ta,n_i,i}\right]+\cdots+\delta\rho_{a,2,i}\delta\rho_{a,n_i,i}\left[\hat{\$}_{ta,2,i}\ \hat{\$}_{ta,n_i,i}\right]\\&+\cdots+\delta\rho_{a,n_i-1,i}\delta\rho_{a,n_i,i}\left[\hat{\$}_{ta,n_i-1,i}\ \hat{\$}_{ta,n_i,i}\right]\end{aligned}$$

$$\begin{aligned}\$_{ac,i}=&\delta\rho_{a,1,i}\delta\rho_{c,1,i}\left[\hat{\$}_{ta,1,i}\ \hat{\$}_{tc,1,i}\right]+\cdots+\delta\rho_{a,1,i}\delta\rho_{c,6-n_i,i}\left[\hat{\$}_{ta,1,i}\ \hat{\$}_{tc,6-n_i,i}\right]\\&+\delta\rho_{a,2,i}\delta\rho_{c,1,i}\left[\hat{\$}_{ta,2,i}\ \hat{\$}_{tc,1,i}\right]+\cdots+\delta\rho_{a,2,i}\delta\rho_{c,6-n_i,i}\left[\hat{\$}_{ta,2,i}\ \hat{\$}_{tc,6-n_i,i}\right]\\&+\cdots\\&+\delta\rho_{a,n_i,i}\delta\rho_{c,1,i}\left[\hat{\$}_{ta,n_i,i}\ \hat{\$}_{tc,1,i}\right]+\cdots+\delta\rho_{a,n_i,i}\delta\rho_{c,6-n_i,i}\left[\hat{\$}_{ta,n_i,i}\ \hat{\$}_{tc,6-n_i,i}\right]\end{aligned}$$

$$\begin{aligned}\$_{c,i}=&\delta\rho_{c,1,i}\delta\rho_{c,2,i}\left[\hat{\$}_{tc,1,i}\ \hat{\$}_{tc,2,i}\right]+\cdots+\delta\rho_{c,1,i}\delta\rho_{c,6-n_i,i}\left[\hat{\$}_{tc,1,i}\ \hat{\$}_{tc,6-n_i,i}\right]\\&+\delta\rho_{c,2,i}\delta\rho_{c,3,i}\left[\hat{\$}_{tc,2,i}\ \hat{\$}_{tc,3,i}\right]+\cdots+\delta\rho_{c,2,i}\delta\rho_{c,6-n_i,i}\left[\hat{\$}_{tc,2,i}\ \hat{\$}_{tc,6-n_i,i}\right]\\&+\cdots+\delta\rho_{c,5-n_i,i}\delta\rho_{c,6-n_i,i}\left[\hat{\$}_{tc,5-n_i,i}\ \hat{\$}_{tc,6-n_i,i}\right]\end{aligned}$$

Then, Eq. (10) can be rewritten as

$$\boldsymbol{A}=\boldsymbol{J}_i\delta^2\boldsymbol{\rho}_i+\delta\boldsymbol{\rho}_i^{\mathrm{T}}\boldsymbol{H}_i\delta\boldsymbol{\rho}_i,\ \ i=1,2,\cdots,l \tag{12}$$

$$\boldsymbol{H}_i=\begin{bmatrix}\boldsymbol{H}_{a,i} & \boldsymbol{H}_{ac,i}\\ \boldsymbol{0} & \boldsymbol{H}_{c,i}\end{bmatrix}$$

$$\boldsymbol{H}_{a,i}=\begin{bmatrix}\boldsymbol{0} & \left[\hat{\$}_{ta,1,i}\ \hat{\$}_{ta,2,i}\right] & \left[\hat{\$}_{ta,1,i}\ \hat{\$}_{ta,3,i}\right] & \cdots & \left[\hat{\$}_{ta,1,i}\ \hat{\$}_{ta,n_i,i}\right]\\ \boldsymbol{0} & \boldsymbol{0} & \left[\hat{\$}_{ta,2,i}\ \hat{\$}_{ta,3,i}\right] & \cdots & \left[\hat{\$}_{ta,2,i}\ \hat{\$}_{ta,n_i,i}\right]\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \left[\hat{\$}_{ta,n_i-1,i}\ \hat{\$}_{ta,n_i,i}\right]\\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}$$

$$\boldsymbol{H}_{ac,i}=\begin{bmatrix}\left[\hat{\$}_{ta,1,i}\ \hat{\$}_{tc,1,i}\right] & \cdots & \left[\hat{\$}_{ta,1,i}\ \hat{\$}_{tc,6-n_i,i}\right]\\ \vdots & \vdots & \vdots\\ \left[\hat{\$}_{ta,n_i,i}\ \hat{\$}_{tc,1,i}\right] & \cdots & \left[\hat{\$}_{ta,n_i,i}\ \hat{\$}_{tc,6-n_i,i}\right]\end{bmatrix}$$

$$\boldsymbol{H}_{c,i}=\begin{bmatrix}\boldsymbol{0} & \left[\hat{\$}_{tc,1,i}\ \hat{\$}_{tc,2,i}\right] & \left[\hat{\$}_{tc,1,i}\ \hat{\$}_{tc,3,i}\right] & \cdots & \left[\hat{\$}_{tc,1,i}\ \hat{\$}_{tc,6-n_i,i}\right]\\ \boldsymbol{0} & \boldsymbol{0} & \left[\hat{\$}_{tc,2,i}\ \hat{\$}_{tc,3,i}\right] & \cdots & \left[\hat{\$}_{tc,2,i}\ \hat{\$}_{tc,6-n_i,i}\right]\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \left[\hat{\$}_{tc,5-n_i,i}\ \hat{\$}_{tc,6-n_i,i}\right]\\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}$$

$\boldsymbol{H}_i\in\mathbb{R}^{6\times6\times6}$ is known as the Hessian matrix of the ith limb. It is a three-dimensional matrix having six layers, each containing a 6×6 upper triangular matrix as shown in Fig. 1, where $[*\quad *]_{K_i}$ ($K_i=1,2,\cdots,6$) denotes the K_ith element of the Lie bracket $[*\quad *]$. The constituent parts of $\boldsymbol{H}_i$, $\boldsymbol{H}_{a,i}\in\mathbb{R}^{6\times n_i\times n_i}$, $\boldsymbol{H}_{ac,i}\in\mathbb{R}^{6\times n_i\times(6-n_i)}$ and $\boldsymbol{H}_{c,i}\in\mathbb{R}^{6\times(6-n_i)\times(6-n_i)}$, are also three-dimensional matrices having six layers.

In acceleration analysis where only ideal motions of the platform are considered, replacements can be made in Eq. (12) such that:

$$\delta\boldsymbol{\rho}_i=\left(\delta\boldsymbol{\rho}_{a,i}^{\mathrm{T}}\quad\delta\boldsymbol{\rho}_{c,i}^{\mathrm{T}}\right)^{\mathrm{T}}\rightarrow\dot{\boldsymbol{\theta}}_i=\left(\dot{\boldsymbol{\theta}}_{a,i}^{\mathrm{T}}\quad\boldsymbol{0}^{\mathrm{T}}\right)^{\mathrm{T}}$$

$$\delta^2\boldsymbol{\rho}_i=\left(\left(\delta^2\boldsymbol{\rho}_{a,i}\right)^{\mathrm{T}}\quad\left(\delta^2\boldsymbol{\rho}_{c,i}\right)^{\mathrm{T}}\right)^{\mathrm{T}}\rightarrow\ddot{\boldsymbol{\theta}}_i=\left(\ddot{\boldsymbol{\theta}}_{a,i}^{\mathrm{T}}\quad\boldsymbol{0}\right)^{\mathrm{T}}$$

$$\boldsymbol{A}=\left(\left(\delta^2\boldsymbol{r}-\delta\boldsymbol{\alpha}\times\delta\boldsymbol{r}\right)^{\mathrm{T}}\left(\delta^2\boldsymbol{\alpha}\right)^{\mathrm{T}}\right)^{\mathrm{T}}\rightarrow\boldsymbol{A}=\left(\left(\dot{\boldsymbol{v}}-\boldsymbol{\omega}\times\boldsymbol{v}\right)^{\mathrm{T}}\ \dot{\boldsymbol{\omega}}^{\mathrm{T}}\right)^{\mathrm{T}}$$

Thus,

$$\boldsymbol{A}=\boldsymbol{J}_i\ddot{\boldsymbol{\theta}}_i+\dot{\boldsymbol{\theta}}_i^{\mathrm{T}}\boldsymbol{H}_i\dot{\boldsymbol{\theta}}_i,\ \ i=1,2,\cdots,l \tag{13}$$

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or

$$\boldsymbol{A} = \boldsymbol{J}_{a,i}\ddot{\boldsymbol{\theta}}_{a,i} + \dot{\boldsymbol{\theta}}_{a,i}^{\mathrm{T}}\boldsymbol{H}_{a,i}\dot{\boldsymbol{\theta}}_{a,i}, \quad i = 1,2,\cdots,l \tag{14}$$

where, $\boldsymbol{A} = \left((\dot{\boldsymbol{v}} - \boldsymbol{\omega}\times\boldsymbol{v})^{\mathrm{T}} \ \dot{\boldsymbol{\omega}}^{\mathrm{T}} \right)^{\mathrm{T}}$ becomes the accelerator of an $n_i - \mathrm{DOF}$ limb with $\dot{\boldsymbol{v}}$ and $\dot{\boldsymbol{\omega}}$ being the linear acceleration of the reference point and the angular acceleration of the platform (Gallardo, 2003). The K_ith element in $\boldsymbol{A}$ has the expression

$$A_{K_i} = \boldsymbol{J}_{i,K_i}\ddot{\boldsymbol{\theta}}_i + \dot{\boldsymbol{\theta}}_i^{\mathrm{T}}\boldsymbol{H}_{i,K_i}\dot{\boldsymbol{\theta}}_i, \quad i = 1,2,\cdots,l \tag{15}$$

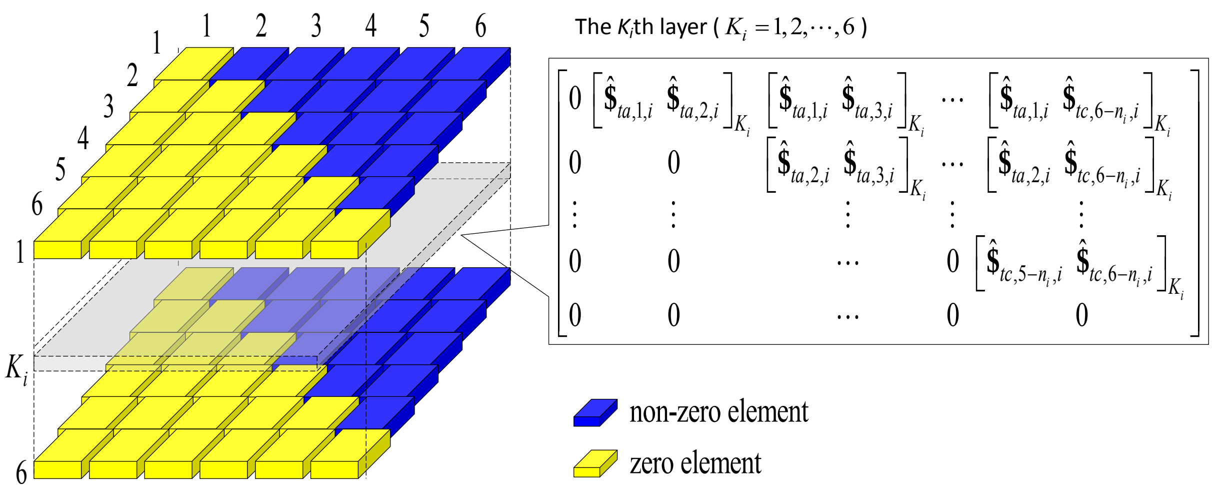

Fig. 1 : Hessian matrix, H_i, of the ith limb.

where, $\boldsymbol{J}_{i,K_i}$ is the K_ith row of $\boldsymbol{J}_i$, while $\boldsymbol{H}_{i,K_i}$ is the K_ith layer of $\boldsymbol{H}_i$.

In addition, the inverse acceleration equation of the ith limb can easily be obtained, by recalling Eq. (4), as

$$\ddot{\boldsymbol{\theta}}_i = \boldsymbol{J}_i^{-1}\left(\boldsymbol{A} - \$_t^{\mathrm{T}}\boldsymbol{J}_i^{-\mathrm{T}}\boldsymbol{H}_i\boldsymbol{J}_i^{-1}\$_t \right), \quad i = 1,2,\cdots,l \tag{16}$$

i. *Acceleration analysis of a parallel manipulator*

Following the same approach as at Eq. (6), the acceleration equation of a parallel manipulator is obtained by taking inner products on both sides of Eq. (13) with $\hat{\$}_{wa,g_k,k}$ and $\hat{\$}_{wc,k_c,i}$, respectively, and noting the relationship given in Eq. (8), to give

$$\boldsymbol{J}\boldsymbol{A} = \ddot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{H}\dot{\boldsymbol{q}} \tag{17}$$

where,

$$\ddot{\boldsymbol{q}} = \left(\ddot{\boldsymbol{q}}_a^{\mathrm{T}} \quad \boldsymbol{0} \right)^{\mathrm{T}}$$

$$\ddot{\boldsymbol{q}}_a = \left(\ddot{q}_{a,g_1,1} \quad \ddot{q}_{a,g_2,2} \quad \cdots \quad \ddot{q}_{a,g_f,f} \right)^{\mathrm{T}}$$

Here, $\ddot{q}_{a,g_k,k}$ is the acceleration of the actuated joint numbered g_k in the kth limb ($k = 1,2,\cdots,f$). For convenience, $\ddot{q}_{a,g_k,k}$ will be simplified as $\ddot{q}_k$ in what follows. $\boldsymbol{H} \in \mathbb{R}^{N\times N\times N}$ ($N = f + \sum_{i=1}^{l}(6-n_i)$) is known as the Hessian matrix of an f-DOF parallel manipulator. It is a three dimensional matrix composed of $\boldsymbol{H}_a \in \mathbb{R}^{f\times N\times N}$ and $\boldsymbol{H}_c \in \mathbb{R}^{\sum_{i=1}^{l}(6-n_i)\times N\times N}$. The expression for the K_ath (K_cth) layer of $\boldsymbol{H}_a$ ($\boldsymbol{H}_c$) is given in Fig. 2. Eq. (17) readily yields the inverse acceleration equation of a parallel manipulator,

$$\ddot{\boldsymbol{q}} = \boldsymbol{J}\boldsymbol{A} - \$_t^{\mathrm{T}}\boldsymbol{J}^{\mathrm{T}}\boldsymbol{H}\boldsymbol{J}\$_t \tag{18}$$

Moreover, multiplying both sides of Eq. (17) by the left pseudo-inverse of $\boldsymbol{J}$ gives the forward acceleration equation of a parallel manipulator:

$$\boldsymbol{A} = \boldsymbol{J}^{+}\left(\ddot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{H}\dot{\boldsymbol{q}} \right) \tag{19}$$

Furthermore, substituting Eqs. (8) and (19) into Eq. (16) expresses the joint acceleration in the ith limb in terms of the velocity and acceleration of the actuated joints:

$$\ddot{\boldsymbol{\theta}}_i = \boldsymbol{J}_i^{LP}\left(\ddot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{H}\dot{\boldsymbol{q}} \right) - \boldsymbol{J}_i^{-1}\dot{\boldsymbol{q}}^{\mathrm{T}}\left(\boldsymbol{J}_i^{LP} \right)^{\mathrm{T}}\boldsymbol{H}_i\boldsymbol{J}_i^{LP}\dot{\boldsymbol{q}}, \tag{20}$$

$i = 1,2,\cdots,l$

The above analyses formulate the forward/inverse velocity and acceleration equations of lower mobility parallel manipulators in a consistent manner under the umbrella of the generalized Jacobian. The velocity and acceleration of each $1-\mathrm{DOF}$ actuated joint of the manipulator can be evaluated using Eqs. (8) and (20). Note, also, that the velocity and acceleration analyses given for a limb are also valid for serial manipulators with $f \le 6$ DOF.

IV. An Example

Detailed execution of velocity and acceleration analyses for a 3-PRS parallel manipulator serves to

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illustrate the generality and effectiveness of the proposed approach.

Fig. 3 shows a schematic diagram of a 3-PRS parallel manipulator which is used as a 3-axis module named the Sprint Z3 (Wahl, 2002) as part of a 5-axis high-speed machining cell for extra large components. The manipulator consists of a base, a platform, and three identical limbs, each connecting the base with the platform in sequence by an actuated prismatic joint, a revolute joint, and a spherical joint.

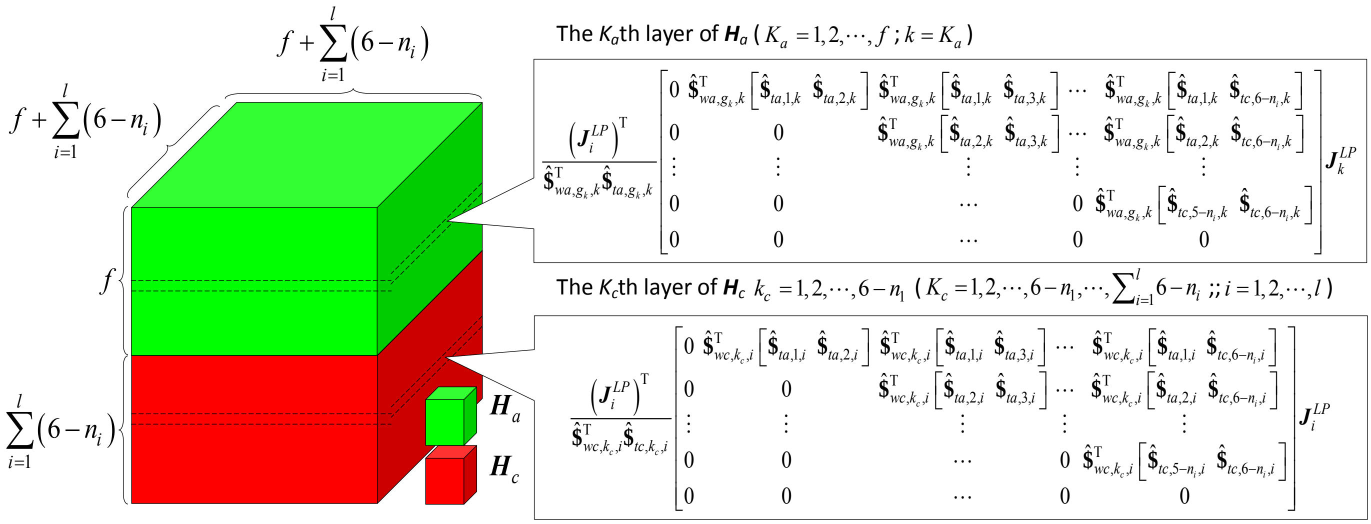

Fig. 2 : Hessian matrix, *H*, of a parallel manipulator.

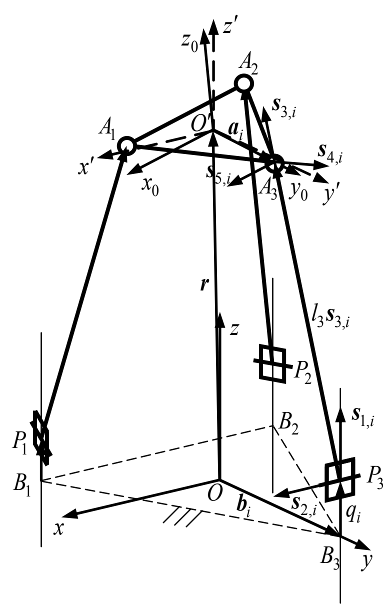

Fig. 3 : Schematic diagram of Sprint Z3 head.

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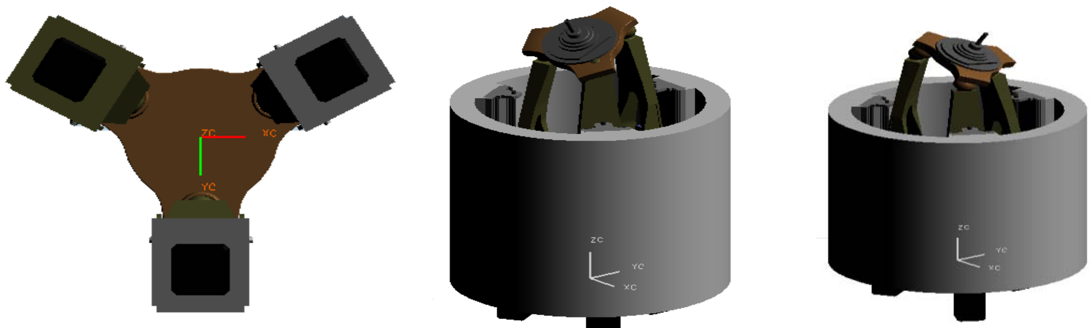

4-a 4-b 4-c

Fig. 4 : (a) Shows the motor configuration, (b) The platform tilted with $\theta=20^0$ about X_c, and (c) The platform tilted with 20^0 about Y_c.

Fig. 4 (a, b, and c) shows the CAD model of the selected example 3-PRS parallel kinematic machine, which helps to visualize and internalize the physical mechanism.

a) Inverse kinematics

A reference frame R is attached to the base and a body fixed frame R_0 to the platform, with O and O' located at the centers of the equilateral triangles $\Delta B_1B_2B_3$ and $\Delta A_1A_2A_3$, as shown. The z and z_0 axes are normal to the planes of those triangles, the x axis is parallel to $\overline{B_2B_1}$ and the x_0 axis is parallel to $\overline{A_2A_1}$. Also, an instantaneous reference frame R' is set with its origin at point O' and its three orthogonal axes remaining always parallel to those of R. Consequently, the orientation matrix of R_0 with respect to R can be obtained using three Euler angles, ψ, θ and ϕ in terms of precession, nutation, and body rotation according to the *z-x-z* convention

$$\boldsymbol{R}=\mathrm{Rot}(z,\psi)\mathrm{Rot}(x',\theta)\mathrm{Rot}(z'',\phi)$$

$$=\begin{bmatrix} \mathrm{C}\psi\mathrm{C}\phi-\mathrm{S}\psi\mathrm{C}\theta\mathrm{S}\phi & -\mathrm{C}\psi\mathrm{S}\phi-\mathrm{S}\psi\mathrm{C}\theta\mathrm{C}\phi & \mathrm{S}\psi\mathrm{S}\theta \\ \mathrm{S}\psi\mathrm{C}\phi+\mathrm{C}\psi\mathrm{C}\theta\mathrm{S}\phi & -\mathrm{S}\psi\mathrm{S}\phi+\mathrm{C}\psi\mathrm{C}\theta\mathrm{C}\phi & -\mathrm{C}\psi\mathrm{S}\theta \\ \mathrm{S}\theta\mathrm{S}\phi & \mathrm{S}\theta\mathrm{C}\phi & \mathrm{C}\theta \end{bmatrix} \quad (21)$$

where, 'S' and 'C' represent 'sin' and 'cos', respectively. Then, the position vector, $\boldsymbol{r}=\begin{pmatrix}x & y & z\end{pmatrix}^{\mathrm{T}}$, of O' can be expressed as

$$\boldsymbol{r}=\boldsymbol{b}_i+q_i\boldsymbol{s}_{1,i}+l_3\boldsymbol{s}_{3,i}-\boldsymbol{a}_i\,,\ i=1,2,3 \quad (22)$$

where,

$$q_i\boldsymbol{s}_{1,i}=\overline{B_iP_i}$$

$$\boldsymbol{s}_{1,i}=\begin{pmatrix}0 & 0 & 1\end{pmatrix}^{\mathrm{T}}$$

$$l_3\boldsymbol{s}_{3,i}=\overline{P_iA_i}$$

$$\boldsymbol{b}_i=\begin{pmatrix}b_{ix} & b_{iy} & b_{iz}\end{pmatrix}^{\mathrm{T}}=b\begin{pmatrix}\cos\beta_i & \sin\beta_i & 0\end{pmatrix}^{\mathrm{T}}$$

$$\boldsymbol{a}_i=\boldsymbol{R}\boldsymbol{a}_{i0}=\begin{pmatrix}a_{ix} & a_{iy} & a_{iz}\end{pmatrix}^{\mathrm{T}}$$

$$\boldsymbol{a}_{i0}=a\begin{pmatrix}\cos\beta_i & \sin\beta_i & 0\end{pmatrix}^{\mathrm{T}}$$

$$\beta_i=\frac{11\pi}{6}-(i-1)\frac{2\pi}{3}\,,\ i=1,2,3$$

$\boldsymbol{b}_i$ and $\boldsymbol{a}_i$ are the position vectors of A_i and B_i measured in R; $\boldsymbol{a}_{i0}$ is the position vector of A_i measured in R_0; *a* and *b* are the radii of the platform and base, respectively; $q_i=\theta_{a,1,i}$ is the joint variable of the actuated prismatic joint in the *i*th limb.

The constraint imposed by the revolute joint restricts the translational motion of A_i along the axis of the revolute joint in limb *i*. This leads to three additional constraint equations,

$$(\boldsymbol{r}+\boldsymbol{a}_i)^{\mathrm{T}}\boldsymbol{s}_{2,i}=0\,,\ i=1,2,3 \quad (23)$$

where, $\boldsymbol{s}_{2,i}=\begin{pmatrix}\sin\beta_i & -\cos\beta_i & 0\end{pmatrix}^{\mathrm{T}}$. Taking ψ, θ, and z as three generalized coordinates, Eq. (23) requires that

$$\phi=-\psi \quad (24)$$

$$x=-\frac{1}{2}a\mathrm{S}2\psi(1-\mathrm{C}\theta) \quad (25)$$

$$y=-\frac{1}{2}a\mathrm{C}2\psi(1-\mathrm{C}\theta) \quad (26)$$

Thus the three desired motions, ψ, θ, and z, can be considered as three independent Cartesian variables, leaving the translations along the x and y axes, and rotation about the z' axis (angle ϕ) as the constrained variables. Given a set of ψ, θ, and z, the inverse position problem is determined by

$$q_i=(\boldsymbol{r}+\boldsymbol{a}_i-\boldsymbol{b}_i-l_3\boldsymbol{s}_{3,i})^{\mathrm{T}}\boldsymbol{s}_{1,i}\,,\ i=1,2,3 \quad (27)$$

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$$\boldsymbol{s}_{3,i}=\begin{pmatrix}s_{3x,i} & s_{3y,i} & s_{3z,i}\end{pmatrix}^{\mathrm{T}},\ s_{3x,i}=\frac{1}{l_3}\left(x+a_{ix}-b_{ix}\right)$$

$$s_{3y,i}=\frac{1}{l_3}\left(y+a_{iy}-b_{iy}\right),\ s_{3z,i}=\sqrt{1-s_{3x,i}^2-s_{3y,i}^2}$$

b) Velocity analysis

Given the bases for the four vector subspaces of its *i*th limb (Huang et al., 2011), the generalized Jacobians of the *i*th limb and the generalized Jacobian of the manipulator can be formulated as follows.

For the *i*th limb ($i=1,2,3$):

$$\boldsymbol{J}_i=\begin{bmatrix}\boldsymbol{J}_{a,i} & \boldsymbol{J}_{ci}\end{bmatrix} \tag{28}$$

$$\boldsymbol{J}_{a,i}=\begin{bmatrix}\hat{\$}_{ta,1,i} & \hat{\$}_{ta,2,i} & \hat{\$}_{ta,3,i} & \hat{\$}_{ta,4,i} & \hat{\$}_{ta,5,i}\end{bmatrix}$$

$$=\begin{bmatrix}\boldsymbol{s}_{1,i} & \left(\boldsymbol{a}_i-l_3\boldsymbol{s}_{3,i}\right)\times\boldsymbol{s}_{2,i} & \boldsymbol{a}_i\times\boldsymbol{s}_{3,i} & \boldsymbol{a}_i\times\boldsymbol{s}_{4,i} & \boldsymbol{a}_i\times\boldsymbol{s}_{5,i}\\ \boldsymbol{0} & \boldsymbol{s}_{2,i} & \boldsymbol{s}_{3,i} & \boldsymbol{s}_{4,i} & \boldsymbol{s}_{5,i}\end{bmatrix}$$

$$\boldsymbol{J}_{c,i}=\hat{\$}_{tc,1,i}=\begin{pmatrix}\left(\boldsymbol{a}_i-l_3\boldsymbol{s}_{3,i}\right)\times\boldsymbol{n}_{1,i}\\ \boldsymbol{n}_{1,i}\end{pmatrix}$$

For the parallel manipulator:

$$\boldsymbol{J}=\begin{bmatrix}\boldsymbol{J}_a\\ \boldsymbol{J}_c\end{bmatrix} \tag{29}$$

$$\boldsymbol{J}_a=\begin{bmatrix}\boldsymbol{s}_{3,1}^{\mathrm{T}}/\boldsymbol{s}_{1,1}^{\mathrm{T}}\boldsymbol{s}_{3,1} & \left(\boldsymbol{a}_1\times\boldsymbol{s}_{3,1}\right)^{\mathrm{T}}/\boldsymbol{s}_{1,1}^{\mathrm{T}}\boldsymbol{s}_{3,1}\\ \boldsymbol{s}_{3,2}^{\mathrm{T}}/\boldsymbol{s}_{1,2}^{\mathrm{T}}\boldsymbol{s}_{3,2} & \left(\boldsymbol{a}_2\times\boldsymbol{s}_{3,2}\right)^{\mathrm{T}}/\boldsymbol{s}_{1,2}^{\mathrm{T}}\boldsymbol{s}_{3,2}\\ \boldsymbol{s}_{3,3}^{\mathrm{T}}/\boldsymbol{s}_{1,3}^{\mathrm{T}}\boldsymbol{s}_{3,3} & \left(\boldsymbol{a}_3\times\boldsymbol{s}_{3,3}\right)^{\mathrm{T}}/\boldsymbol{s}_{1,3}^{\mathrm{T}}\boldsymbol{s}_{3,3}\end{bmatrix}$$

$$\boldsymbol{J}_c=\frac{1}{l_3}\begin{bmatrix}\boldsymbol{s}_{2,1}^{\mathrm{T}}/\boldsymbol{s}_{1,1}^{\mathrm{T}}\boldsymbol{s}_{3,1} & \left(\boldsymbol{a}_1\times\boldsymbol{s}_{2,1}\right)^{\mathrm{T}}/\boldsymbol{s}_{1,1}^{\mathrm{T}}\boldsymbol{s}_{3,1}\\ \boldsymbol{s}_{2,2}^{\mathrm{T}}/\boldsymbol{s}_{1,2}^{\mathrm{T}}\boldsymbol{s}_{3,2} & \left(\boldsymbol{a}_2\times\boldsymbol{s}_{2,2}\right)^{\mathrm{T}}/\boldsymbol{s}_{1,2}^{\mathrm{T}}\boldsymbol{s}_{3,2}\\ \boldsymbol{s}_{2,3}^{\mathrm{T}}/\boldsymbol{s}_{1,3}^{\mathrm{T}}\boldsymbol{s}_{3,3} & \left(\boldsymbol{a}_3\times\boldsymbol{s}_{2,3}\right)^{\mathrm{T}}/\boldsymbol{s}_{1,3}^{\mathrm{T}}\boldsymbol{s}_{3,3}\end{bmatrix}$$

where, $\boldsymbol{s}_{j_a,i}$ is a unit vector along the j_ath 1-DOF joint of the *i*th limb; $\boldsymbol{n}_{1,i}=\boldsymbol{s}_{1,i}\times\boldsymbol{s}_{2,i}$. The joint axes are arranged such that $\boldsymbol{s}_{1,i}\perp\boldsymbol{s}_{2,i}$ and $\boldsymbol{s}_{2,i}\perp\boldsymbol{s}_{3,i}$; $\boldsymbol{s}_{3,i}$, $\boldsymbol{s}_{4,i}$ and $\boldsymbol{s}_{5,i}$ are coincident with three rotational axes of the spherical joint, with $\boldsymbol{s}_{3,i}$ aligned along the rod. Substituting Eq. (29) into Eqs. (6) and Eq. (7) generates the inverse and forward velocity equations of the manipulator

$$\dot{\boldsymbol{q}}=\boldsymbol{J}\$_t \tag{30}$$

$$\$_t=\boldsymbol{J}^{-1}\dot{\boldsymbol{q}} \tag{31}$$

where, $\dot{\boldsymbol{q}}=\begin{pmatrix}\dot{\boldsymbol{q}}_a^{\mathrm{T}} & \boldsymbol{0}^{\mathrm{T}}\end{pmatrix}^{\mathrm{T}}$ and $\dot{\boldsymbol{q}}_a=\begin{pmatrix}\dot{q}_1 & \dot{q}_2 & \dot{q}_3\end{pmatrix}^{\mathrm{T}}$.

c) Acceleration analysis

The Hessian matrix, $\boldsymbol{H}$, of the manipulator can be found by substituting the expressions just found for $\hat{\$}_{ta,j_a,i}$ ($j_a=1,2,\ldots,5$), $\hat{\$}_{tc,1,i}$ and Jacobians $\boldsymbol{J}_i$ and $\boldsymbol{J}$ into the forms shown in Figure 2 and Eq. (17) to give

$$\boldsymbol{H}_{a,K_a}=\frac{\left(\boldsymbol{J}_i^{LP}\right)^{\mathrm{T}}\boldsymbol{M}_{a,i}\boldsymbol{J}_i^{LP}}{\boldsymbol{s}_{1,i}^{\mathrm{T}}\boldsymbol{s}_{3,i}},\ K_a=i=1,2,3 \tag{32}$$

$$\boldsymbol{H}_{c,K_c}=\frac{\left(\boldsymbol{J}_i^{LP}\right)^{\mathrm{T}}\boldsymbol{M}_{c,i}\boldsymbol{J}_i^{LP}}{l_3\boldsymbol{s}_{1,i}^{\mathrm{T}}\boldsymbol{s}_{3,i}},\ K_c=i=1,2,3 \tag{33}$$

where,

$$\boldsymbol{J}_i^{LP}=\boldsymbol{J}_i^{-1}\boldsymbol{J}^{-1}$$

$$\boldsymbol{M}_{a,i}=\begin{bmatrix}\boldsymbol{M}_{a1,i} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{M}_{a2,i}\end{bmatrix}$$

$$\boldsymbol{M}_{a1,i}=\begin{bmatrix}0 & \boldsymbol{s}_{3,i}^{\mathrm{T}}\boldsymbol{n}_{1,i} & 0 & \boldsymbol{s}_{3,i}^{\mathrm{T}}\left(\boldsymbol{s}_{1,i}\times\boldsymbol{s}_{4,i}\right) & \boldsymbol{s}_{3,i}^{\mathrm{T}}\left(\boldsymbol{s}_{1,i}\times\boldsymbol{s}_{5,i}\right)\\ 0 & 0 & 0 & l_3\boldsymbol{s}_{4,i}^{\mathrm{T}}\boldsymbol{s}_{2,i} & l_3\boldsymbol{s}_{5,i}^{\mathrm{T}}\boldsymbol{s}_{2,i}\end{bmatrix}$$

$$\boldsymbol{M}_{a2,i}=\begin{bmatrix}0\\ -l_3\boldsymbol{s}_{4,i}^{\mathrm{T}}\boldsymbol{n}_{1,i}\\ l_3\left(\boldsymbol{s}_{5,i}\times\boldsymbol{s}_{3,i}\right)^{\mathrm{T}}\left(\boldsymbol{s}_{3,i}\times\boldsymbol{n}_{1,i}\right)\\ 0\end{bmatrix}$$

$$\boldsymbol{M}_{c,i}=\begin{bmatrix}\boldsymbol{0} & \boldsymbol{M}_{c1,i}\\ \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}$$

$$\boldsymbol{M}_{c1,i}=\begin{bmatrix}-\boldsymbol{s}_{3,i}^{\mathrm{T}}\boldsymbol{n}_{1,i} & -\boldsymbol{s}_{4,i}^{\mathrm{T}}\boldsymbol{n}_{1,i} & -\boldsymbol{s}_{5,i}^{\mathrm{T}}\boldsymbol{n}_{1,i} & -1\\ -l_3 & 0 & -l_3\boldsymbol{s}_{5,i}^{\mathrm{T}}\boldsymbol{s}_{3,i} & -l_3\boldsymbol{s}_{3,i}^{\mathrm{T}}\boldsymbol{n}_{1,i}\end{bmatrix}$$

Here, $\boldsymbol{H}_{a,K_a}$ ($\boldsymbol{H}_{c,K_c}$) represents the K_ath (K_cth) layer of $\boldsymbol{H}_a$ ($\boldsymbol{H}_c$). Then, substituting Eqs. (32) and (33) into Eqs. (18) and (19), the inverse and forward acceleration equations of the manipulator are

$$\ddot{\boldsymbol{q}}=\boldsymbol{J}\boldsymbol{A}-\$_t^{\mathrm{T}}\boldsymbol{J}^{\mathrm{T}}\boldsymbol{H}\boldsymbol{J}\$_t \tag{34}$$

$$\boldsymbol{A}=\boldsymbol{J}^{-1}\left(\ddot{\boldsymbol{q}}+\dot{\boldsymbol{q}}^{\mathrm{T}}\boldsymbol{H}\dot{\boldsymbol{q}}\right) \tag{35}$$

where, $\ddot{\boldsymbol{q}}=\begin{pmatrix}\ddot{\boldsymbol{q}}_a^{\mathrm{T}} & \boldsymbol{0}\end{pmatrix}^{\mathrm{T}}$ and $\ddot{\boldsymbol{q}}_a=\begin{pmatrix}\ddot{q}_1 & \ddot{q}_2 & \ddot{q}_3\end{pmatrix}^{\mathrm{T}}$.

d) Coordinate transformation for numerical simulation

Numerical simulations for the inverse velocity and acceleration, require the explicit relationships of the velocity twist and accelerator to the first and second derivatives of three independent coordinates, ψ, θ, and z because they are used for path planning.

Taking the time derivative of Eqs. (25) and (26) gives

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$$\boldsymbol{v} = \dot{\boldsymbol{r}} = \boldsymbol{J}_v \dot{\boldsymbol{g}}_c \tag{36}$$

$$\boldsymbol{J}_v = \begin{bmatrix} -a\mathrm{C}2\psi(1-\mathrm{C}\theta) & -0.5a\mathrm{S}2\psi\mathrm{S}\theta & 0 \\ a\mathrm{S}2\psi(1-\mathrm{C}\theta) & -0.5a\mathrm{C}2\psi\mathrm{S}\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \dot{\boldsymbol{g}}_c = \begin{pmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{z} \end{pmatrix}$$

Then, taking the time derivative of Eq. (36) results in

$$\dot{\boldsymbol{v}} = \boldsymbol{J}_v \ddot{\boldsymbol{g}}_c + \dot{\boldsymbol{g}}_c^{\mathrm{T}} \boldsymbol{H}_v \dot{\boldsymbol{g}}_c \tag{37}$$

where, $\boldsymbol{H}_v \in \mathbb{R}^{3\times3\times3}$ is a three dimensional matrix with $\boldsymbol{H}_{v,i}$ ($i = 1,2,3$) being its ith layer;

$$\ddot{\boldsymbol{g}}_c = \begin{pmatrix} \ddot{\psi} \\ \ddot{\theta} \\ \ddot{z} \end{pmatrix}, \quad \boldsymbol{H}_{v,1} = \begin{bmatrix} 2a\mathrm{S}2\psi(1-\mathrm{C}\theta) & -a\mathrm{C}2\psi\mathrm{S}\theta & 0 \\ -a\mathrm{C}2\psi\mathrm{S}\theta & -0.5a\mathrm{S}2\psi\mathrm{C}\theta & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{H}_{v,2} = \begin{bmatrix} 2a\mathrm{C}2\psi(1-\mathrm{C}\theta) & a\mathrm{S}2\psi\mathrm{S}\theta & 0 \\ a\mathrm{S}2\psi\mathrm{S}\theta & -0.5a\mathrm{C}2\psi\mathrm{C}\theta & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{H}_{v,3} = \boldsymbol{0}_{3\times3}$$

The angular velocity vector of the platform, $\boldsymbol{\omega} = (\omega_x \quad \omega_y \quad \omega_z)^{\mathrm{T}}$, can be derived by recalling, e.g. (Angeles, 2003), the standard matrix expression for the $\square\times$ operator

$$\begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} = \dot{\boldsymbol{R}}\boldsymbol{R}^{\mathrm{T}} \tag{38}$$

and directly comparing elements, to give

$$\boldsymbol{\omega} = \boldsymbol{J}_\omega \dot{\boldsymbol{g}}_c \tag{39}$$

$$\boldsymbol{J}_\omega = \begin{bmatrix} -\mathrm{S}\psi\mathrm{S}\theta & \mathrm{C}\psi & 0 \\ \mathrm{C}\psi\mathrm{S}\theta & \mathrm{S}\psi & 0 \\ 1-\mathrm{C}\theta & 0 & 0 \end{bmatrix}$$

Taking the time derivative of Eq. (39) gives

$$\dot{\boldsymbol{\omega}} = \boldsymbol{J}_\omega \ddot{\boldsymbol{g}}_c + \dot{\boldsymbol{g}}_c^{\mathrm{T}} \boldsymbol{H}_\omega \dot{\boldsymbol{g}}_c \tag{40}$$

where, $\boldsymbol{H}_\omega \in \mathbb{R}^{3\times3\times3}$ is also a three dimensional matrix with $\boldsymbol{H}_{\omega,i}$ ($i = 1,2,3$) being its ith layer;

$$\boldsymbol{H}_{\omega,1} = \begin{bmatrix} -\mathrm{C}\psi\mathrm{S}\theta & -\mathrm{S}\psi & 0 \\ -\mathrm{S}\psi\mathrm{C}\theta & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{H}_{\omega,2} = \begin{bmatrix} -\mathrm{S}\psi\mathrm{S}\theta & \mathrm{C}\psi & 0 \\ \mathrm{C}\psi\mathrm{C}\theta & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{H}_{\omega,3} = \begin{bmatrix} 0 & 0 & 0 \\ \mathrm{S}\theta & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Then, given $\dot{\boldsymbol{g}}_c$ and $\ddot{\boldsymbol{g}}_c$, $\dot{\boldsymbol{q}}_a = (\dot{q}_1 \quad \dot{q}_2 \quad \dot{q}_3)^{\mathrm{T}}$ and $\ddot{\boldsymbol{q}}_a = (\ddot{q}_1 \quad \ddot{q}_2 \quad \ddot{q}_3)^{\mathrm{T}}$ can be evaluated using Eqs. (36)-(40), (6), and (18).

Consider now a specific system having the geometry: $a = 250\ \mathrm{mm}$, $b = 312.5\ \mathrm{mm}$, and $l_3 = 540\ \mathrm{mm}$

Also, assume as an example that the path and motion rules of the platform are:

$$\ddot{\psi}(t) = \begin{cases} \ddot{\psi}_{\max}\sin\left(\dfrac{2\pi}{T}t\right) & 0 \le t \le t_1 \\ 0 & t_1 < t \le t_2 \\ -\ddot{\psi}_{\max}\sin\left(\dfrac{2\pi}{T}t\right) & t_2 < t \le t_3 \end{cases} \tag{41}$$

$$\dot{\psi}(t) = \begin{cases} -\ddot{\psi}_{\max}\left(\dfrac{T}{2\pi}\right)\cos\left(\dfrac{2\pi}{T}t\right) + \dfrac{\dot{\psi}_{\max}}{2} & 0 \le t \le t_1 \\ \dot{\psi}_{\max} & t_1 < t \le t_2 \\ \ddot{\psi}_{\max}\left(\dfrac{T}{2\pi}\right)\cos\left(\dfrac{2\pi(t-t_2)}{T}\right) + \dfrac{\dot{\psi}_{\max}}{2} & t_2 < t \le t_3 \end{cases} \tag{42}$$

$$\psi(t) = \begin{cases} -\ddot{\psi}_{\max}\left(\dfrac{T}{2\pi}\right)^2 \sin\left(\dfrac{2\pi}{T}t\right) + \dfrac{\dot{\psi}_{\max}}{2}t & 0 \le t \le t_1 \\ \dot{\psi}_{\max}t - \dfrac{\dot{\psi}_{\max}}{4}T & t_1 < t \le t_2 \\ \left(\begin{array}{l} \ddot{\psi}_{\max}\left(\dfrac{T}{2\pi}\right)^2 \sin\left(\dfrac{2\pi}{T}(t-t_2)\right) \\ +\dfrac{\dot{\psi}_{\max}}{2}(t+t_2) - \dfrac{\dot{\psi}_{\max}}{4}T \end{array}\right) & t_2 < t \le t_3 \end{cases} \tag{43}$$

$\theta = 40°$, $\dot{\theta} = 0\ \mathrm{rad/s}$, $\ddot{\theta} = 0\ \mathrm{rad/s^2}$, $z = 645\ \mathrm{mm}$, $\dot{z} = 0\ \mathrm{m/s}$, $\ddot{z} = 0\ \mathrm{m/s^2}$

where, T is the cycle time; $0 \square t_1$, $t_1 \square t_2$ and $t_2 \square t_3$ are the times used for acceleration, uniform motion, and deceleration.

$$T = \frac{\dot{\psi}_{\max}\pi}{\ddot{\psi}_{\max}}, \quad t_1 = t_3 - t_2 = \frac{T}{2}, \quad \psi(t_3) = 2\pi \tag{44}$$

Substituting into Eq. (44) the givens $\dot{\psi}_{\max} = 1.47\ \mathrm{rad/s}$ and $\ddot{\psi}_{\max} = 11.96\ \mathrm{rad/s^2}$ results in $T = 0.3852\ \mathrm{s}$, $t_1 = 0.1926\ \mathrm{s}$, $t_2 = 4.2857\ \mathrm{s}$, $t_3 = 4.4783\ \mathrm{s}$

When the platform of the manipulator moves according to the preceding rules, the velocity/acceleration of the actuated joints, the linear velocity/acceleration of the reference point O', and the

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angular velocity/acceleration of the platform versus time can be evaluated using the proposed approach. These results, shown in Fig. 4, have been verified by a CAD model of the manipulator. There was no discernable difference between the results obtained using this approach and the CAD software.

V. Conclusion

This paper presents a general and systematic approach for the forward and inverse velocity and acceleration analysis of lower mobility parallel manipulators using screw theory. With this approach, the process of acceleration modeling of serial and parallel kinematic chains can be integrated into the unified framework of the generalized Jacobian. It results in a new Hessian matrix being developed in a general and compact form. This allows rigid body dynamic modeling of lower mobility manipulators to be integrated into a single mathematical framework.

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