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## MRI Denoising Using Waveatom Shrinkage

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# MRI Denoising Using Waveatom Shrinkage

Geetika Dua<sup>a</sup> & Varun Raj<sup>a</sup>

**Abstract** - It is well known that noise in Magnetic Resonance Image has a Rician distribution. Unlike additive Gaussian noise, Rician noise is signal dependent, and separating signal from noise is a difficult task. In this paper, a denoising technique is used in order to remove Rician noise from MRI using Waveatom shrinkage. De-noising by any shrinkage technique is highly sensitive to the threshold selection. Here to estimate the noise variance, histogram based technique is used and to calculate the shrinkage threshold a new technique is proposed. This method is applied to both simulated images and real images. Wave atom transform has been applied for different noise levels. This has been done in order to find more accurate results. A comparative analysis of wave atom and wavelet is also performed.

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## I. INTRODUCTION

Magnetic resonance imaging (MRI) is a medical imaging technique that measures the response of atomic nuclei of body tissues to high frequency radio waves when placed in a strong magnetic field and that produces images of the internal organs. Magnetic Resonance Imaging has proven to be particularly valuable for examination of the soft tissues in the body and is a commonly used form of medical imaging. Because of the resolution of MRI and the technology being essentially harmless it has emerged as the most accurate and desirable imaging technology. It was shown that pure noise in MR magnitude images could be modeled as a Rayleigh distribution. Afterwards, the Rician model was proposed as a more general model of noise in MR images. Sources of MR noise include thermal noise, inductive losses, sample resolution, and field-of-view. Despite significant improvements in recent years, magnetic resonance images often suffer from low SNR especially in cardiac and brain imaging. Therefore, noise reduction techniques are of great interest in MR imaging.

## II. RELATED WORK

The image processing literature presents a variety of de-noising methods. Many of the popular de-noising algorithms suggested are based on wavelet thresholding [1]–[4]. These approaches attempt to separate significant features from noise in the frequency domain and simultaneously preserve them while removing noise. If the wavelet transform is

applied on MR magnitude data directly, both the wavelet and the scaling coefficients of a noisy MRI image become biased estimates of their noise-free counterparts. Therefore, it was suggested [2] that the application of the wavelet transform on squared MR magnitude image data would result in the wavelet coefficients no longer being biased estimates of their noise-free counterparts. Although the bias still remains in the scaling coefficients, it is not signal-dependent and can therefore be easily removed. The difficulty with wavelet or anisotropic diffusion algorithms is again the risk of over smoothing fine details particularly in low SNR images [5]. From these points, it is understood that all the algorithms have the drawback of over-smoothing fine details. In [6], stated that oscillatory functions or oriented textures have a significantly sparser expansion in wave atoms than in other fixed standard representations like Gabor filters, wavelets and curvelets. In [7], denoising using Wave Atom is done by estimating the noise variance by trial and error method. In [8], denoising using Wave Atom is done by estimating the noise variance by histogram technique.

## III. RICIAN NOISE

Magnetic resonance magnitude image data are usually modelled by the Rician distribution[9]. The magnetic resonance signals are acquired in quadrature channels. Each signal produces an image that is degraded by a zero-mean Gaussian noise of standard deviation as 0. The two images are then combined into a magnitude image and the Gaussian noise PDF is transformed into a Rician noise PDF. The joint probability density of the noise from two quadrature channels can be expressed as [10]:

$$p(n_r, n_i) = \frac{1}{2\pi\sigma_0^2} \exp\left(-\frac{n_r^2 + n_i^2}{2\pi\sigma_0^2}\right) \quad (1)$$

The expectation values for the mean magnitude and the variance are[2]:

$$\begin{aligned} I &= \sigma_0 \sqrt{\frac{\pi}{2}} \exp\left(-\frac{X^2}{4\sigma_0^2}\right) \times \\ &\left[ \left(1 + \frac{X^2}{2\sigma_0^2}\right) I_0\left(\frac{X^2}{4\sigma_0^2}\right) + \frac{X^2}{2\sigma_0^2} I_1\left(\frac{X^2}{4\sigma_0^2}\right) \right] \quad (2) \\ \sigma_I^2 &= X^2 + 2\sigma_0^2 - \frac{\pi\sigma_0^2}{2} \exp\left(\frac{X^2}{2\sigma_0^2}\right) \times \end{aligned}$$

$$\left[ \left( 1 + \frac{X^2}{2\sigma_0^2} \right) I_0 \left( \frac{X^2}{4\sigma_0^2} \right) + \frac{X^2}{2\sigma_0^2} I_1 \left( \frac{X^2}{4\sigma_0^2} \right) \right]^2 \quad (3)$$

where  $I_0$  and  $I_1$  are modified Bessel functions of the first kind and  $X$  denotes the MR magnitude image.

#### IV. WAVEATOM TRANSFORM

Wavelet transform is a well known multiresolution analysis tool capable of conveying accurate temporal and spatial information. Wavelets better represent objects with point singularities in 1D and 2D space but fail to deal with singularities along curves in 2D. Therefore wavelet representation does not offer sufficient sparseness for image analysis. Following the introduction of wavelet transform, research community has witnessed intense efforts for development of wave atoms, ridgelets[11], contourlets[12] and curvelets[13]. These tools have better directional and decomposition capabilities than wavelets. Wave atoms have a sharp frequency localization that cannot be achieved using a filter bank based on wavelet packets and offer a significantly sparser expansion for oscillatory functions[14]. Wave atoms capture coherence of pattern across and along oscillations whereas curvelets capture coherence only along oscillations. To make our discussion concrete, we need to classify various wave-packet transforms as phase-space tilings. Since a complete collection must span all positions and frequencies, we see that wave packets are actually tiles in phase-space. We say a tiling is universal if it treats democratically all positions and orientations as shown in Figure 1.

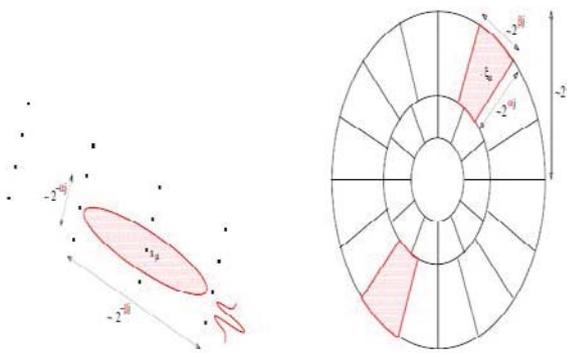


Figure 1 : Essential support of a wave packet with parameters  $(\alpha, \beta)$ , in space (left), and in frequency (right).

Two parameters should suffice to index a lot of known wave packet architectures:  $\alpha$  to index whether the decomposition is multiscale ( $\alpha = 1$ ) or not ( $\alpha = 0$ ); and  $\beta$  to indicate whether basis elements are localized and poorly directional ( $\beta = 1$ ) or, on the contrary, extended and fully directional ( $\beta = 0$ ). Wave Atoms corresponds to  $\alpha = \beta = 1/2$ , having an aspect ratio  $\sim 2^{j/2} \times 2^{j/2}$  in space, with oscillations of wavelength  $\sim 2^j$  in the codirection  $\xi_\mu$ .

Wave atoms are a variant of 2D wavelet packets which obey the parabolic scaling law: wavelength  $\sim$  (diameter)<sup>2</sup>.

#### V. EXPERIMENTS AND RESULTS

This section gives a detailed analysis of the proposed MRI de-noising algorithm. It compares and validates the performance of the proposed method using simulated and Real MR images and also compares the performance of the proposed method with Wavelet shrinkage.

Determination of threshold is very critical in this work. Input elements with absolute value greater than the set threshold value, are set to 1. In this work a new threshold is proposed which is better as compared to old threshold[15].

Old Threshold is given as:

$$\sqrt{\ln((\max val) - (\min val))} \sigma \quad (4)$$

New threshold is given as :

$$\sqrt{2 * \ln((\max val) - (\min val))} \sigma \quad (5)$$

Where  $\sigma$  the noise variance, maxval is the highest pixel value in the image and the minval is the lowest pixel value in the image. Noise variance is estimated by the method Automatic estimation of the noise variance from the histogram of an MR image[16]. Output of thresholding is given by

$$x = (\text{abs}(y) > \text{thld}) * y. \quad (6)$$

After applying threshold criterion inverse Wave Atom transform and inverse Wavelet transform is applied separately and performance of both is compared using four comparison parameters.

Four comparison parameters mean square error (MSE), peak signal to noise ratio (PSNR), signal to mean square error (S/MSE) and signal to noise ration(SNR) are used which are defined as:

Mean square error (MSE) is given as

$$MSE = \frac{1}{m * n} \sum_{i=1}^m \sum_{j=1}^n (N(i, j) - DN(i, j))^2 \quad (7)$$

Where  $m$  is number of rows in the image,  $N(i, j)$  is the noisy image and  $DN(i, j)$  is the de-noised image.

Peak Signal to Noise ratio (PSNR) is given as

$$PSNR = 10 \log_{10} \left( \frac{R^2}{MSE} \right) \quad (8)$$

Here  $R$  is the maximum fluctuation in the input image data type.

Signal to Noise ratio (SNR) is obtained by

$$SNR = 10 \log_{10} \left( \frac{\text{var}(x)}{\text{var}(\hat{x} - x)} \right) \quad (10)$$

(Where  $x$  is noise free simulated images and  $\hat{x}$  is the noisy image or de-noised images).

**FOR STIMULATED IMAGES:** For experiments with simulated images, images were loaded from Matlab software.

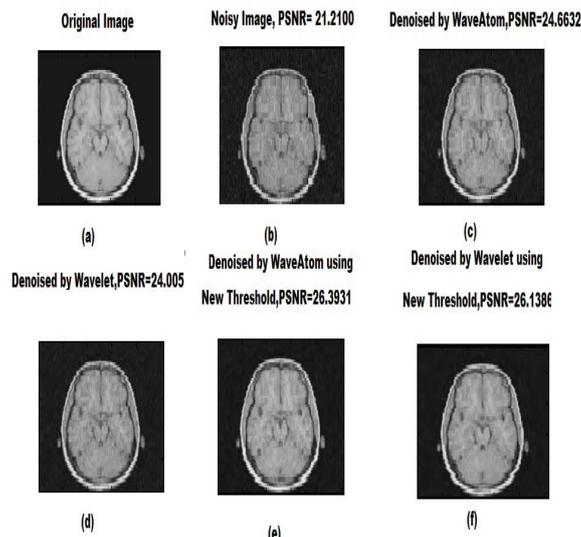


Figure.2 : (a) Stimulated Original Image (b) High Noise image (c) De-noised by Wave Atom with old threshold (d) De-noised by Wavelet with old threshold (e) De-noised by Wave Atom with new threshold (f) De-noised by Wavelet with new threshold.

Table 1 : Parameters for High Noise Image

Parameters	Denoised using Wave Atom with old threshold	Denoised using Wavelet with old threshold	Denoised using Wave Atom with new threshold	Denoised using Wavelet with new threshold
MSE(mean square error)	0.0034172	0.0036304	0.0022945	0.002433
PSNR( peak signal to noise ratio)	24.6632 dB	24.4005 dB	26.3931 dB	26.1386 dB
S/MSE(signal to mean square error)	17.0783 dB	16.8156 dB	18.8082 dB	18.5537 dB
SNR(signal to noise ratio)	12.4240 dB	12.0257 dB	15.2194 dB	14.6561 dB

Table 2 : Parameters for Low Noise Images

Parameters	Denoised using WaveAtom with old threshold	Denoised using Wavelet with old threshold	Denoised using WaveAtom with new threshold	Denoised using Wavelet with new threshold
MSE(mean square error)	3.5587e-005	3.8718e-005	3.3665e-005	3.5996e-005
PSNR( peak signal to noise ratio)	44.4871 dB	44.1208 dB	44.7282 dB	44.4375 dB
S/MSE(signal to mean square error)	36.9022 dB	36.5359 dB	37.1433 dB	36.8526 dB
SNR(signal to noise ratio)	30.6408 dB	30.1310 dB	30.7152 dB	30.1635 dB

**FOR REAL IMAGES:** The real images were down loaded from the Open Access Series of imaging Studies (OASIS) database[17].

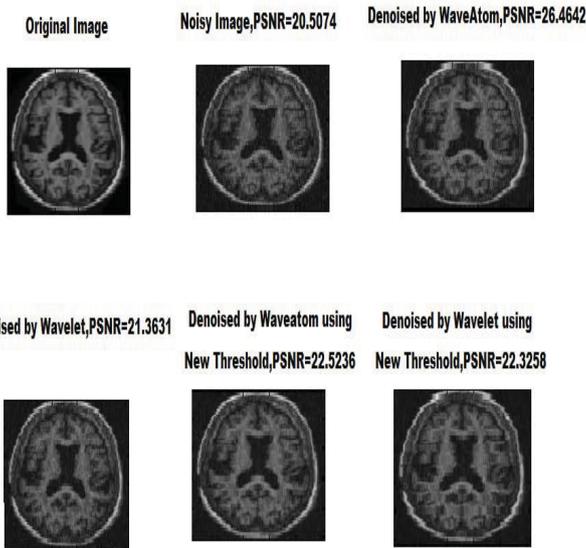


Figure.3 : (a) Real Original Image (b) High Noise image (c) De-noised by Wave Atom with old threshold (d) De-noised by Wavelet with old threshold (e) De-noised by Wave Atom with new threshold (f) De-noised by Wavelet with new threshold.

Table 3 : Parameters for High SNR Images

Parameters	Denoised using Wave Atom with old threshold	Denoised using Wavelet with old threshold	Denoised using Wave Atom with new threshold	Denoised using Wavelet with new threshold
MSE(mean square error)	0.0027496	0.0028503	0.0019821	0.0021131
PSNR( peak signal to noise ratio)	25.6072 dB	25.451 dB	27.0287 dB	26.7508 dB
S/MSE(signal to mean square error)	14.3304 dB	14.1742 dB	15.7519 dB	15.4739 dB
SNR(signal to noise ratio)	10.2556 dB	10.0420 dB	12.2912 dB	11.8558 dB

Table 4 : Parameters for Low Noise Image

Parameters	Denoised using WaveAtom with old threshold	Denoised using Wavelet with old threshold	Denoised using WaveAtom with new threshold	Denoised using Wavelet with new threshold
MSE(mean square error)	0.0071381	0.0073061	0.0055929	0.0058536
PSNR( peak signal to noise ratio)	21.4642 dB	21.3631 dB	22.5236 dB	22.3258 dB
S/MSE(signal to mean square error)	10.1873 dB	10.0863 dB	11.2468 dB	11.0489 dB
SNR(signal to noise ratio)	6.3210 dB	6.1699 dB	7.8936 dB	7.5525 dB

It is clear from table 1 and 2,3and 4 that de-noised image using wave atom with new threshold has lowest mean square error (MSE), highest peak signal to noise ratio (PSNR), highest signal to mean square error (S/MSE) and highest signal to noise ratio (SNR). It is also clear from the table that quality parameters of image de-noised by wave atom is better as comparison to the quality parameters of image de-noised by wavelet.

## VI. CONCLUSION

It have been concluded that noise removal on MRI by the proposed new threshold gives better results as compared to the old threshold. Also it is clear that wave atom transform gives better results as comparison to wavelet transform. Results are verified by taking Stimulated MRI images and Real images downloaded from Open Access Series of imaging Studies database. Four comparison parameters are taken which are: MSE, PSNR, S/MSE, and SNR. Comparison is shown in the form of tables in which wave atom transform provides lower mean square error (MSE), higher peak signal to noise ratio (PSNR), higher signal to mean square error (S/MSE) and higher signal to noise ratio (SNR) as comparison to wavelet transform.

## VII. FUTURE SCOPE

The field of image processing has been growing at a very fast pace. The day to day emerging technology requires more and more revolution and evolution in the image processing field. The work proposed in this paper also portrays a small contribution in this regard. This work can be further enhanced to de-noise the other type of images, like CT, Ultrasound, X ray images. It will provide a good add on to the already existing denoising techniques. Moreover, for future work we can train our algorithm using various techniques like fuzzy logic or neural network, in order to attain the best output without performing calculations for each and every combination.

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