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## Effect of Structure Parameters on the Signal Strength in a Solid Beam Driven Plasma-Loaded Backward Wave Oscillator

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# Effect of Structure Parameters on the Signal Strength in a Solid Beam Driven Plasma-Loaded Backward Wave Oscillator

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**Abstract** - The effect of structure parameters on the group velocity and signal strength of a solid beam driven plasmaloded backward wave oscillator is investigated analytically for a particular mode. The theory of approximate cubic dispersion equation as derived earlier for a solid beam driven plasmaloded backward wave oscillator (BWO) is used for this investigation. The effect of variation of structure parameters on the temporal and spatial growth rates and group velocity result a change in the signal strength of BWO.

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## I. INTRODUCTION

In this paper, the effects of waveguide parameters on the group velocity and signal strength of a plasmaloded BWO with sinusoidally corrugated slow wave structure having very smaller corrugation depth driven by a solid intense relativistic electron beam have been investigated analytically. This study is based on the approximate linear theory of absolute instability derived for a solid beam driven plasma-loaded BWO[1]. In the previous works, most of the researchers conducted investigations for efficiency and resonance enhancement, power enhancement and frequency shifting of microwave emission in a plasma filled BWO[2-5]. Most of them carried out their analysis keeping the structure parameters constant. Some of the researchers have devoted their interests on the investigations comprising slow-wave instability by numerical analysis [6 - 10]. Some researchers work on absolute instability for annular electron- beam driven plasma-loaded BWO[11,12]. Here, the authors are

interested to study the effect of variation of structure parameters on the group velocity and signal strength of a solid beam driven plasma loaded BWO by absolute instability analysis for  $TM_{01}$  mode. Formulation of the analytical dispersion relation is presented in Section II. Section III describes the analytical results of the analysis. Discussion and conclusions are stated in section IV.

## II. FORMULATION

In deriving the expression of group velocity and arbitrary signal strength, an waveguide model consisting of a sinusoidally corrugated-wall structure having very smaller corrugation depth is considered [1]. The waveguide inner space is filled completely with plasma of uniform density  $N_p$ . A relativistic electron beam of density  $N_b$ , that is assumed covers the entire inner space of the waveguide, is moving along the waveguide axis with a velocity  $v_b$  relative to the background plasma. The entire system is assumed to be immersed in a strong infinite axial guiding magnetic field  $B_0$ . The numerical dispersion relation of this beam-plasma waveguide system for  $TM_{01}$  mode is  $D(k, \omega) = 0$ [13]. Where,  $D$  is the value of the determinant of a square matrix with elements  $D_{mn}$ . and  $k$  and  $\omega$  are respectively the wavenumber and frequency. The approximate dispersion relation of this system for the resonance interaction of the zeroth beam harmonic with the electromagnetic first slow harmonic as shown in Fig.1 can be expressed as,

$$\begin{bmatrix} D_{-1-1} & D_{-10} \\ D_{0-1} & D_{00} \end{bmatrix} = 0 \quad (1)$$

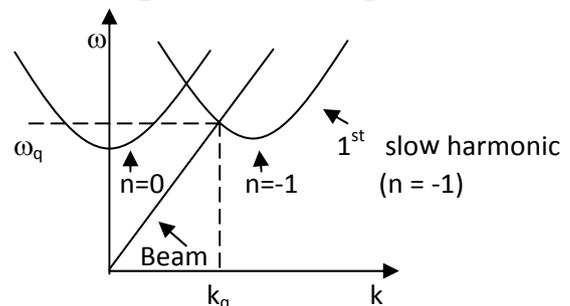


Figure 1 : Resonance of the zeroth beam and electromagnetic first slow harmonic.

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The matrix elements of the above relation are:

$$\left. \begin{aligned} D_{-1-1} &= J_0(X_{-1}) \\ D_{00} &= J_0(X_0) \\ D_{-10} &= \left( 1 + \frac{k_0 k}{\frac{\omega^2}{c^2} - k^2} \right) \frac{a}{2} X_0 J'_0(X_0) \\ D_{0-1} &= \left( 1 - \frac{k_0 k_{-1}}{\frac{\omega^2}{c^2} - k_{-1}^2} \right) \frac{a}{2} X_{-1} J'_0(X_{-1}) \end{aligned} \right\} \quad (2)$$

Here,

$$\frac{X_n^2}{R_0^2} = \left( \frac{\omega^2}{c^2} - k_n^2 \right) \left( 1 - \frac{\omega_p^2}{\omega^2} \right) - \left[ \left( \frac{\omega^2}{c^2} - k^2 \right) \frac{\omega_b^2}{\gamma^3 (\omega - k_n v_b)^2} \right] \delta_{n,0} \quad (3)$$

where,  $c$ ,  $\omega_p$ ,  $\omega_b$  and  $v_b$  are light velocity, plasma frequency, beam frequency and beam velocity respectively, and  $\delta_{n,0} = 0$  unless  $n = -1$ ;  $k_n = (k + nk_0)$ .

The oscillation frequency  $\omega_q$  and hence the wave number  $k_q$  can be obtained by solving eq.(1) with  $\omega_b = 0$ . The cubic equation describing the frequency and wave number perturbations of the three waves involved in the resonance interaction is obtained from the dispersion relation stated in eq. (1) as,

$$(\delta\omega - v_b \delta k)^2 (\delta\omega - v_g \delta k) = \Delta \quad (4)$$

$$\text{where, } \Delta = - \frac{\omega_b^2 \gamma^{-3} \beta_1 a^2 \lambda_{-1}^2 \left( \frac{\omega_q^2}{c^2} - k_q^2 \right)}{8 \lambda_0 \left[ \frac{\omega_q}{c^2} - \frac{\omega_p^2 (k_q - k_0)^2}{\omega_q^3} \right]} \times \frac{J'_0(\lambda_{-1})}{J_0(\lambda_0)} \left[ 1 + \lambda_0 \frac{J''_0(\lambda_0)}{J'_0(\lambda_0)} \right]$$

$$\beta_1 = \left[ 1 + \frac{k_0 k_q}{\frac{\omega_q^2}{c^2} - k_q^2} \right] \left[ 1 - \frac{k_0 (k_q - k_0)}{\frac{\omega_q^2}{c^2} - (k_q - k_0)^2} \right]; \quad a = \frac{h}{R_0}$$

$$\lambda_{-1}^2 = \left[ \frac{\omega_q^2}{c^2} - (k_q - k_0)^2 \right] \left[ 1 - \frac{\omega_p^2}{\omega_q^2} \right] R_0^2$$

$$\frac{J'_0(\lambda_{-1})}{J_0(\lambda_0)} \left[ 1 + \lambda_0 \frac{J''_0(\lambda_0)}{J'_0(\lambda_0)} \right] = \begin{cases} 1 & \text{for } \lambda_0 \text{ is imaginary} \\ -1 & \text{for } \lambda_0 \text{ is real} \end{cases}$$

$$\lambda_n = X_n(\omega_q, k_q, X_b = 0)$$

Expression of group velocity can be stated as,

$$v_g = \frac{\left[ \left( 1 - \frac{\omega_p^2}{\omega_q^2} \right) (k_q - k_0) \right]}{\left[ \frac{\omega_q}{c^2} - \frac{\omega_p^2}{\omega_q^3} (k_q - k_0)^2 \right]} \quad (5)$$

At the moment of absolute instability there exists a saddle point in the complex  $k$ -plane, where one finds two equal roots of complex wavenumber  $k$  for some value of complex frequency  $\omega$  with  $\omega_i > 0$ . The imaginary parts of these frequency and wavenumber represent temporal and spatial growth rates respectively. Using these values of complex  $\omega$  and  $k$  within the range of linear analysis, arbitrary value of signal strength can be calculated.

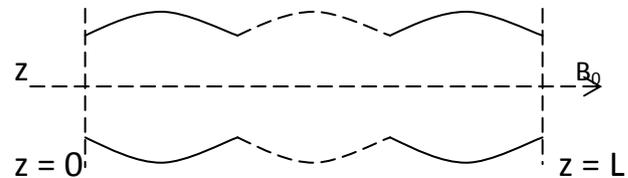


Figure 2: BWO structure showing the position ( $z = 0$ ), where arbitrary signal strength is calculated. The signal is assumed to originate at  $z = L$ .

The expression of signal strength can be written as,

$$f \propto e^{i \left( ik_i L - i\omega_i \frac{L}{v_g} \right)} \quad (6)$$

Here,  $L$  is the distance traversed by the wave in time  $t = L/v_g$ , and is equal to the axial length of the structure. It is noted that, in calculating  $t$  the group velocity  $v_g$  is used, because the energy transport velocity of a composite wave in a loss-less waveguide is equal to be the group velocity of the wave.

### III. ANALYTICAL RESULTS

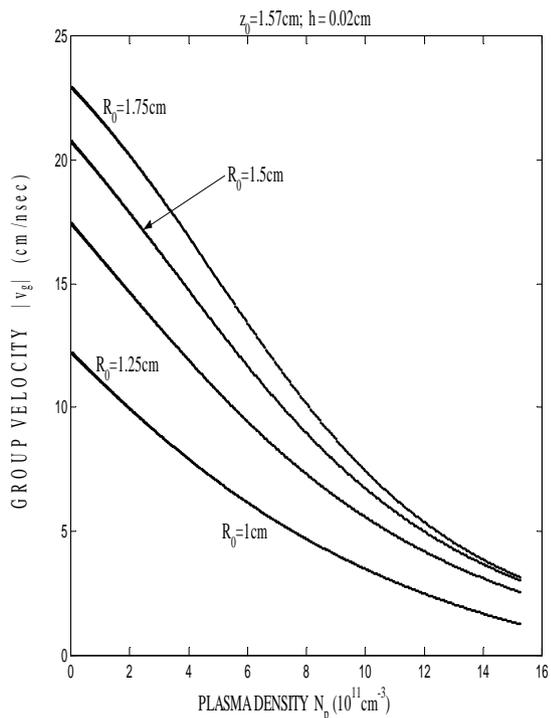


Figure 3 : Group velocity versus plasma density characteristics for different structure average radii.

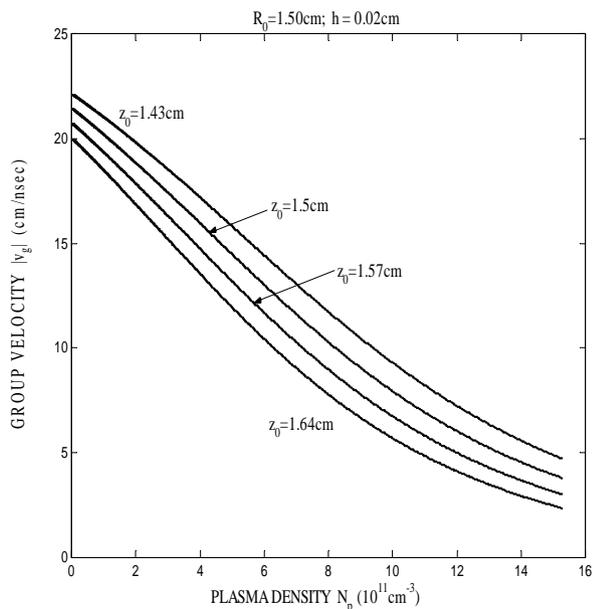


Figure 4 : Group velocity versus plasma density characteristics for different structure corrugation periods.

The effect of plasma density,  $N_p$  and structure average radius,  $R_0$  on the group velocity,  $v_g$  is shown in Fig. 3. In this observation, the variation of group velocity for various structure average radius are plotted keeping the corrugation period,  $z_0$  and the corrugation depth,  $h$  constant. From the figure it is seen that group velocity decreases with plasma density and increases with structure average radius. This figure also depicts that the rate of decrease of group velocity with the increase in plasma density is greater for the larger values of structure average radius. In Fig. 4 the effect of plasma density,  $N_p$  and structure corrugation period,  $z_0$  on the group velocity,  $v_g$  are shown. In this observation, the variation of group velocity for various structure corrugation periods are plotted keeping the structure average radius,  $R_0$  and the corrugation depth,  $h$  constant. This figure reveals that group velocity decreases with the increase in plasma density and structure corrugation period.

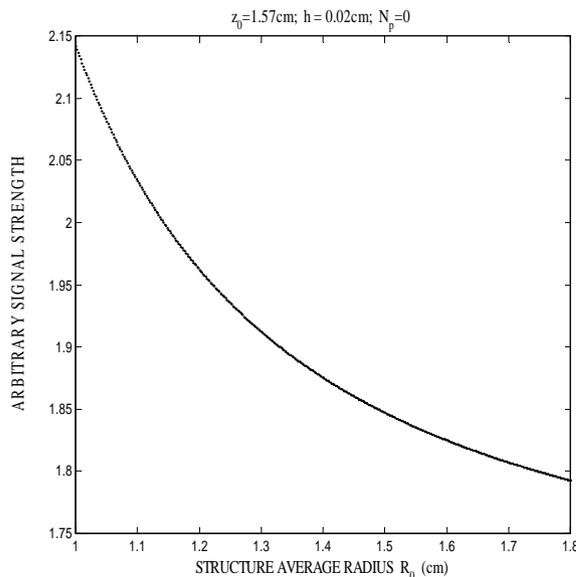


Figure 5 : Structure average radius versus arbitrary signal strength.

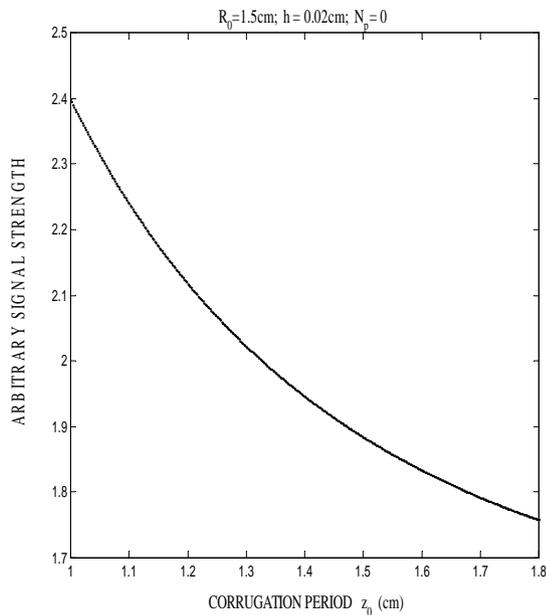


Figure 6 : Structure corrugation period versus arbitrary signal strength.

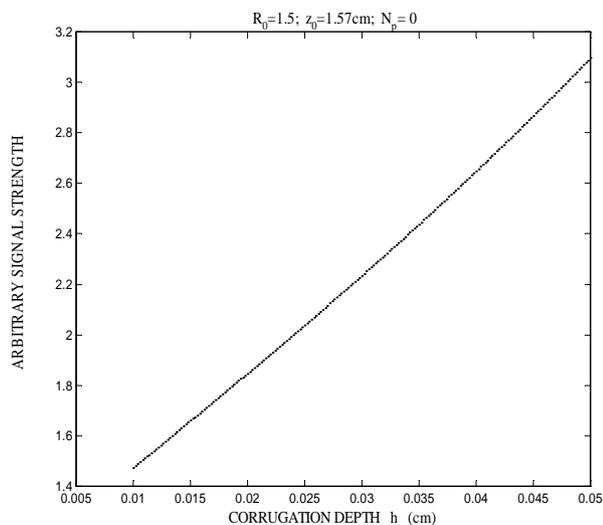


Figure 7 : Structure corrugation depth versus arbitrary signal strength.

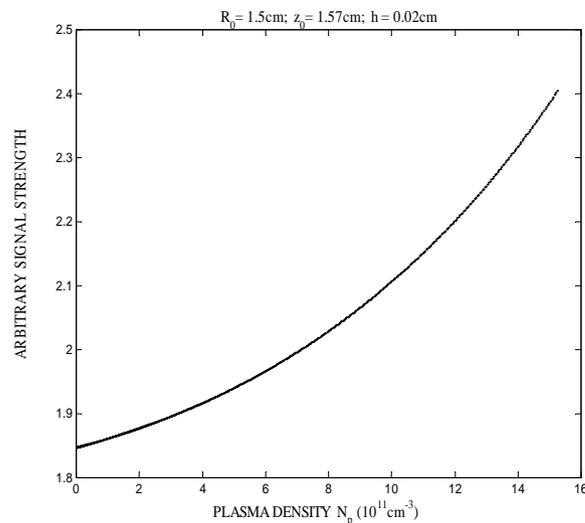


Figure 8 : Plasma density versus arbitrary signal strength.

The variation of the temporal and spatial growth rates and the group velocity result a change in the signal strength of a BWO. The simultaneous effects of these three factors on the arbitrary signal strength, due to the variation of structure parameters are presented in the Figs. 5, 6 and 7 respectively. The figures reveal that the increase in structure average radius and corrugation period cause a decrease in signal strength and the increase in structure corrugation depth causes an increase in signal strength. Fig. 8 depicts that with the increase in plasma density signal strength is also increased. The arbitrary strength of the signal is calculated at  $z = 0$ , as the signal is assumed to originate at  $z = L$  as shown in Fig.2, where the arbitrary value of  $L$  is taken as 20cms.

#### IV. SUMMARY AND DISCUSSIONS

The instability phenomena comprising a plasma-loaded BWO consisting of a sinusoidally corrugated slow wave structure having very smaller corrugation depth driven by a solid intense relativistic electron beam, has been analyzed for investigating the effects of waveguide parameters on the group velocity and signal strength. In this analytical study, the modified theory of instability of three wave interaction for a solid beam-driven plasma-loaded BWO [1] has been employed. The effect of structure-size parameters and plasma density on the group velocity and signal strength have been carefully investigated here using the analytical results of temporal and spatial growth rates[1]. From this study one can get information about the parametric and background plasma effects on the signal strength of a backward wave oscillator operating in the X-band frequency range and it may be helpful in future for further study on BWO.

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