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Vibration Study of a Cutting Tool By The Finite Element Method

M. Rahou^a, A. Cheikh^Ω, F. Sebaa^β, A. Youcef^ψ

Abstract - The problem of vibration cutting tools for machining has an important influence on the state of surface and manufacturing tolerances. In this paper, we presented a model of the dynamic behavior of a cutting tool; the goal is to be determined by the finite element method hierarchical trigonometric frequencies own a tool from the kinetic energy and energy deformation. A programme has been developed in FOLTRAN 77 can generate automatically calculate the frequencies.

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I. INTRODUCTION

The phenomenon of the vibration of the cutting tools is a main issue which appears during the process of cut of metals. This problem with an influence very import on the qualitative level such as tolerances of manufacture, surface quality....Several studies were presented. Bourdim et al [1] illustrates an approach of the study of the dynamic stability of the system part-tool in the course of machining .In work of (Arfaoui et al) (Bourdim et al) (Wu D.W., Liu C.R) [2,3,4], the authors present a vibratory modeling of the cutting tool in turning, they showed that the vibratory behavior of the tool depends primarily on the angle of attack, angle strip, advance, depth of cut and the cutting speed . the work of Younes R. [5] a study was carried out on the influence cutting speed on the vibrations of chattering of the tool with an aim of proposing an analytical study by analogy with the model of Vander pole. The work of N. Ouelaa et al [6] presents an experimental semi study of the vibratory behavior of the cutting tool goods of the operation of slide-lathing, fact object of showing that it is possible to consider the roughness average of the part machined starting from displacement resulting from the nozzle of the tool and for the work of Vincent et al [7] a study was presented on the influence of the position of the tool on dynamic behavior in milling of thin walls.

In the last 20 years, the p-version finite element method has generated a lot of interest [8, 9]. The main reason for this interest is the remarkable convergence properties that the method has [10].The p version gets these convergence properties from the fact that it converges to the solution by increasing the order of element shape functions instead of increasing the number of elements as the h - version finite element

approach does.

In our work a theoretical study on the vibrations of the cutting tools by the finite element method which makes it possible to choose a good system design starting from the Eigen frequencies.

II. VIBRATORY BEHAVIOR OF THE CUTTING TOOL MODELING

In general, we can consider that a structure or an element of structure will be of beam type if one of its dimensions (length) is large in front of the two others. The experiments [11] watch which we can make a known simplifying assumption under the name of assumption of Bernoulli-Euler who expresses himself as follows:

"Any normal plane section with average fiber before deformation remains plane and normal with this average fiber after deformation (effect of null shearing)" [11,13].In our case, we can modelize the cutting tool by a beam Clamped-Free (Figure1).

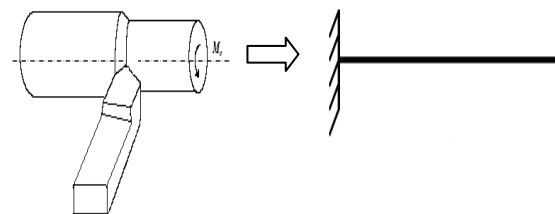


Figure 1 Modeling of the cutting tool

a) Beam element displacement field

The displacement of the point P of the cross-section (Figure2) is given by the system (1):

$$\begin{cases} U(x, y, z) = U_0(x) - y\theta \\ V(x, y, z) = V_0(x) \\ W(x, y, z) = 0 \end{cases} \quad (1)$$

With:

$U_0(x) = U(x, 0, 0)$: Longitudinal displacement
 $V_0(x) = V(x, 0, 0)$: Transverse displacement
 $-y\theta$: Displacement of with the rotation of the cross-section.

There: Position of a point M of the cross-section.

b) Deformation tensor

The components ε_{ij} of the tensor of deformation are given by the relation (2):

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$$\varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \quad i, j = 1, 2, 3 \quad (2)$$

For ε_{xx} , ε_{xy} , ε_{yy} we have the matrix (3) following:

$$\varepsilon = \begin{bmatrix} \frac{\partial U_0}{\partial x} - y \frac{\partial \theta}{\partial x} & \frac{1}{2} \left(\frac{\partial V_0}{\partial x} - \theta \right) \\ \frac{1}{2} \left(\frac{\partial V_0}{\partial x} - \theta \right) & 0 \end{bmatrix} \quad (3)$$

In the form of the tensor, while noting:

ε_0 : it is the generalized relative deformation of the average line.

$$\varepsilon_0 = \frac{\partial U_0}{\partial x} \quad (4)$$

γ : it is the shearing strain.

$$\gamma = \frac{\partial V_0}{\partial x} - \theta \quad (5)$$

χ : it is the curve of the average line, with ρ radius of the curvature.

$$\chi = \frac{\partial \theta}{\partial x} = \frac{1}{\rho} \quad (6)$$

c) Shearing assumption

In the majority of the cases, the deflections longitudinal, corresponding has ε_{xx} due to the traction and compression and/or with the bending, are definitely more important than the deformations due to shearing. Noun let us can thus very often neglect the deformations due to shearing γ in the case of the long beams ($L/h > 20$).

Therefore, by supposing $\gamma=0$ systematically, the following relation is obtained:

$$\frac{\partial V_0}{\partial x} - \theta = 0 \implies \frac{\partial V_0}{\partial x} = \theta \quad (7)$$

Lead:

$$\varepsilon = \begin{bmatrix} \frac{\partial U_0}{\partial x} - y \frac{\partial^2 V_0}{\partial x^2} & 0 \\ 0 & 0 \end{bmatrix} \quad (8)$$

d) Stress Deformations Relations

The relation (9) is obtained starting from the generalization of the law of Hooke:

$$\sigma = D \varepsilon \quad (9)$$

e) Deformation energy

The deformation energy of the system, equation (10), it is the sum of deformation energies of each element.

$$U_{\Sigma} = \sum_{e=1}^n U_e \quad (10)$$

The general form of the deformation energy will be given by the relation (11):

$$U_e = \frac{1}{2} \int_V \sigma_{ij} \cdot \varepsilon_{ij} dv \quad (11)$$

f) Kinetic energy

The kinetic energy of the system, equation (12), it is the kinetic sum of energy of each element:

$$E_{\Sigma} = \sum_{e=1}^n E_{c_e} \quad (12)$$

The kinetic energy of an element moving, formula (13), it is the sum of the energy of the beam and the energy of the mass of the Mj engine:

$$E_{c_e} = E_{c_p} + E_{c_{Mj}} \quad (13)$$

With

E_{c_p} : Kinetic energy of the beam, equation (14).

$E_{c_{Mj}}$: Kinetic energy of the concentrated mass equation (15).

$$\begin{aligned} E_{c_p} &= \frac{1}{2} \rho \int_V \dot{h}^T \cdot \dot{h} dv + \frac{1}{2} \rho \int_V 2 \cdot \dot{h}^T \Omega \cdot h dv \\ &+ \frac{1}{2} \rho \int_V h^T \cdot \Omega^2 \cdot h dv + \frac{1}{2} \rho \int_V 2 \dot{h}^T (A^T \dot{r}_i + \Omega x) dv \\ &+ \frac{1}{2} \rho \int_V 2 h^T \Omega^T (A^T \dot{r}_i + \Omega x) dv \\ &+ \frac{1}{2} \rho \int_V (\dot{r}_i^T \dot{r}_i + 2 \dot{r}_i^T A \Omega x + x^T \Omega^T \Omega x) dv \\ E_{c_{Mj}} &= \frac{1}{2} M_j \left[\dot{h}^T \dot{h} + 2 \dot{h}^T \Omega \cdot h \right. \\ &+ h^T \cdot \Omega^2 \cdot h + 2 h^T \Omega^T (A^T \dot{r}_i + \Omega x) \\ &+ 2 \dot{h}^T (A^T \dot{r}_i + \Omega x) + (\dot{r}_i^T \dot{r}_i \\ &\left. + 2 \dot{r}_i^T A \Omega x + x^T \Omega^T \Omega x) \right] \Big|_{x=l} \end{aligned} \quad (14)$$

g) Motion equations

The equations of Lagrange make it possible to obtain the equations of the movement of an element of the structure (element beam with mass concentrated at the end) starting from the expression of the kinetic and potential energy elementary, equation (16).

$$L(q_i, \dot{q}_i, t) = E_c - U \quad (16)$$

III. FORMULATION BY THE FINITE ELEMENT METHOD HIERARCHICAL

The particular characteristic of the finite element method is that the field, in which an approximate solution is required is divided into under field called finite elements. The unknown quantity, such as displacement is represented in each element by polynomial functions simple. Its functions are called elementary functions of forms.

For a given formulation the choice of the subdivision (mesh) and functions of forms determines the precision of the approximation. The cutting tool is modeled by only one element, called hierarchical finite element, the field of displacement describe the element is given by (17):

$$\begin{Bmatrix} U \\ V \end{Bmatrix} = [N]^e \{q\}^e \quad (17)$$

With:

$$\{q\}^T = \{X_1, \dots, X_p, Y_1, \dots, Y_p\}^T, \quad n=1, \dots, P$$

P: Many functions of form.

$$[N]^e = \begin{bmatrix} [N_u] & 0 \\ 0 & [N_v] \end{bmatrix}$$

Such as:

$$\begin{aligned} [N_u] &= [f_1(\xi), f_2(\xi) \dots f_p(\xi)] \\ [N_v] &= [g_1(\xi), g_2(\xi) \dots g_p(\xi)] \end{aligned}$$

The components of the vector $\{E\}$ necessary to the determination of rigidity are given in the system of coordinates Cartesian, relation (18).

$$\{e\} = [A]^e \begin{Bmatrix} U \\ V \end{Bmatrix} \quad (18)$$

With:

$$[A]^e = \begin{bmatrix} \frac{1}{L} \frac{\partial}{\partial \xi} & -\frac{y}{L^2} \frac{\partial}{\partial \xi^2} \end{bmatrix}$$

The vector of deformation $\{E\}$ is given according to $\{Q\}$ in the matrix form (19) following:

$$\begin{aligned} \{e\} &= [A]^e [N]^e \{q\} \\ \{e\} &= [B]^e \{q\} \end{aligned} \quad (19)$$

With:

$$[B]^e = [A]^e [N]^e$$

a) Energy in the matrix form

The kinetic energy and of elementary deformation in the matrix form is given by the relation (20):

$$\begin{aligned} Ec_p = & \frac{1}{2} \rho S L \int_0^1 \left\{ \dot{q} \right\}^T [N]^T [N] \left\{ \dot{q} \right\} d\xi \\ & + \frac{1}{2} \rho S L 2 \int_0^1 \left\{ \dot{q} \right\}^T [N]^T \Omega [N] \{q\} d\xi \\ & + \frac{1}{2} \rho S L \Omega^2 \int_0^1 \{q\}^T [N]^T [N] \{q\} d\xi + \\ & \frac{1}{2} \rho S L 2 \int_0^1 \{q\}^T [N]^T \Omega^T (A^T \dot{r}_i + \Omega x) d\xi \end{aligned} \quad (20)$$

$$\begin{aligned} & + \frac{1}{2} \rho S L 2 \int_0^1 \left\{ \dot{q} \right\}^T [N]^T (A^T \dot{r}_i + \Omega x) d\xi + \\ & \frac{1}{2} \rho S L \int_0^1 (\dot{r}_i^T \dot{r}_i + 2 \dot{r}_i^T A \Omega x + x^T \Omega^T \Omega x) d\xi \end{aligned}$$

The kinetic energy of the mass is given in the matrix form by the relation (21):

$$\begin{aligned} Ec_{MJ} = & \frac{1}{2} M_j \left(\left\{ \dot{q} \right\}^T [N]^T [N] \left\{ \dot{q} \right\} \right. \\ & + 2 \left\{ \dot{q} \right\}^T [N]^T \Omega [N] \{q\} \\ & + \{q\}^T [N]^T \Omega^T \Omega [N] \{q\} \\ & + 2 \left\{ \dot{q} \right\}^T [N]^T \Omega I + 2 \left\{ \dot{q} \right\}^T [N]^T A^T \dot{r}_i \\ & + 2 \{q\}^T [N]^T \Omega^T \Omega I \\ & \left. + \dot{r}_i^T \dot{r}_i + 2 \{q\}^T [N]^T \Omega^T A^T \dot{r}_i \right) \end{aligned} \quad (21)$$

The relation (22) presents the deformation energy in the matrix form.

$$\begin{aligned} U_e = & \frac{1}{2} \frac{E S}{L} \int_0^1 \{q\}^T [N_{u,\xi}]^T [N_{u,\xi}] \{q\} d\xi + \\ & \frac{1}{2} \frac{E I}{L^3} \int_0^1 \{q\}^T [N_{v,\xi\xi}]^T [N_{v,\xi\xi}] \{q\} d\xi \end{aligned} \quad (22)$$

The equations of Lagrange are written in the matrix form, equation (23).

$$\begin{aligned} [M]^e \left\{ \ddot{q} \right\} + [G]^e \left\{ \dot{q} \right\} \\ + ([K]^e + [R]^e + [P]^e) \{q\} = \{r\} + \{F_{ext}\} \end{aligned} \quad (23)$$

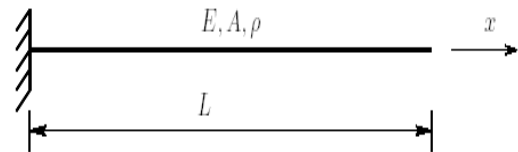


Figure 2: Cleanliness of the Clamped-Free beam

The matrix of rigidity is given by (24):

$$[K]^e = E \int_V [B e]^T [B e] dV \quad (24)$$

The centrifugal matrix of rigidification is given par.

$$[R]_p = \rho \omega_s^2 S L \int_0^1 [N]^T [N] d\xi \quad (25)$$

$$[R]_{MJ} = M_j \omega_s^2 [N(1)]^T [N(1)] \quad (26)$$

The matrix of rigidification of angular acceleration $[P]$ is given by:

$$[P]^e = [P]_p + [P]_{MJ} \quad (27)$$

$$[P]_p = \rho S L \int_0^1 [N]^T \dot{\Omega} [N] d\xi \quad (28)$$

$$[P]_{MJ} = M_j [N(1)]^T \dot{\Omega} [N(1)] \quad (29)$$

According to the kinetic energy the matrix mass is given by

$$[M] = [M]_p + [M]_{MJ} \quad (30)$$

By identification.

$$[M]_p = \rho S L \int_0^1 [N]^T [N] d\xi \quad (31)$$

$$[M]_{MJ} = M_j [N(1)]^T [N(1)] \quad (32)$$

According to the kinetic energy, the gyroscopic matrix is given by:

$$[G] = [G]_p + [G]_{MJ} \quad (33)$$

$$[G]_p = 2 \rho S L \int_0^1 [N]^T \Omega [N] d\xi \quad (34)$$

$$[G]_{MJ} = 2 M_j [N(1)]^T \Omega [N(1)] \quad (35)$$

b) Numerical example

The FORTRAN program in 77 allowing determining the dynamic characteristics of the structure beam (cutting tool) The first four pulsations are given in the following tables for various cases of boundary conditions. The resulting ones calculated are compared with the reference (M Geradin ,D Rixen), which is illustrated in table 1 in the case of a beam in rest or. The parameter of frequency and other parameters used for the validation of the model are given by

$$\omega^* = \sqrt{\frac{\rho A L^4}{E I_z}} \omega, \quad \alpha = \sqrt{\frac{A L^2}{I_z}}$$

$$\Omega = \sqrt{\frac{\rho A L^4}{E I_z}} \omega_3$$

or ω^* , α and Ω are respectively the parameter of frequency, parameter speed and the parameter defining the type of beam used (Bernoulli or Timoshenko type).

- Clamped-Free Beam:

Table 1 illustrates the pulsation values for the solution of the hierarchical finite element method as well as the values of the exact solution, for a Clamped-Free beam

Table 1: Comparison of the values of pulsations for a beam (C-F):

Modes	Solution HFEM	Exact solution	% error
1	3.5160	3.1560	0,114
2	22.0348	22.0344	0,181E-04
3	61.7036	61.6972	0,103 E-03
4	120.9762	120.9019	0,614E-03

With:

$$\% = \frac{HFEM - EXACT.S}{EXACT.S}$$

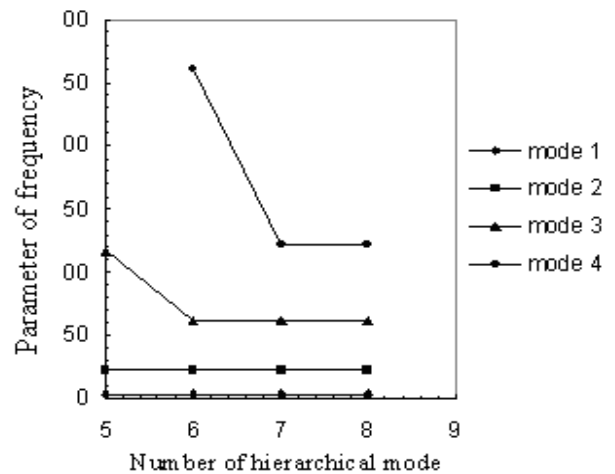


Figure 3: Chart of the convergence of the pulsation values w^* according to NHM in the case of beam (C-F).

In the graph, represented by Figure 3, in the case of give the variation of the first three parameters of frequency according to the number of hierarchical modes NMH a clamped-free beam (C-F), we notice that convergence is ensured starting from a number of modes equal to 8.

C. Influence speed Ω on the values of the pulsations w^* :

The variation of the parameters of pulsation w^* of the first four modes for variations rotational speeds Ω are given by table 2.

Table 2 : Variation of the values of pulsations ω^* according to rotational speed Ω .

Ω	ω_1^*	ω_2^*	ω_3^*	ω_4^*
0	3,51601	22,0348	61,7036	109,958
0,1	3,51743	22,0350	61,7037	109,958
0,2	3,52167	22,0357	61,7039	109,959
0,3	3,52874	22,0368	61,7043	109,960
0,4	3,53860	22,0383	61,7048	109,962
1	3,65489	22,0568	61,7112	109,981
1,1	3,68338	22,0615	61,7128	109,986
1,2	3,71433	22,0665	61,7145	109,991
1,3	3,74767	22,0721	61,7164	109,996
1,4	3,78335	22,0780	61,7185	110,003
2	4,04256	22,1229	61,7339	110,049
2,1	4,09263	22,1319	61,7370	110,058
2,2	4,14445	22,1414	61,7402	110,068
2,3	4,19806	22,1512	61,7436	110,078
3	4,61556	22,2325	61,7717	110,162
3,1	4,68056	22,2458	61,7763	110,176
3,2	4,74673	22,2596	61,7811	110,190
3,3	4,81402	22,2738	61,7860	110,205
4	5,31259	22,3850	61,8246	110,320
5	6,08914	22,5793	61,8925	110,523
10	10,44095	24,1317	62,4549	112,208

Figure 4 presents the variation of the parameter of frequency according to the rotational speed for a variation from 0 to 110, we notice that the second, third and the fourth mode vary linearly for the values of Ω superior to 20, whereas the first mode varies almost exponentially.

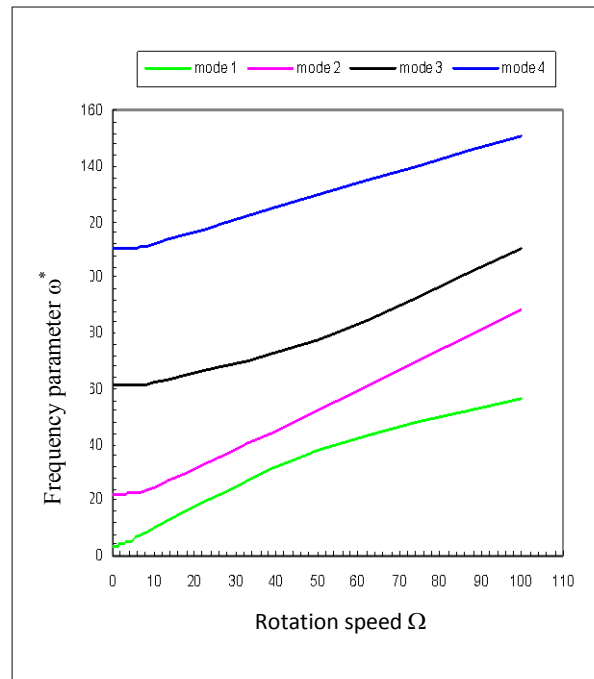


Figure 4 : Chart of the influence speed Ω on the parameter of frequencies ω^* in the band $[0;110]$

IV. CONCLUSION

In this work, a modeling of the cutting tool by the hierarchical finite element method was the object to determine the Eigen frequencies. The latter has an important influence on the tolerances of manufacture and the surface quality. The found results ensure convergence in the stationary case (rest) and moving, the compared errors are almost the same ones with [13]. In all the cases, the Eigen frequencies vary almost linearly in the beach of $[0,10]$ and increases with the increase rotational speed. These frequencies enabled us to pose a good design of system of assembly of the cutting tool.

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