Improving Trucks Management at Dumpsites through the Application of Queue Theory- The Case of Solous III Dumpsite, Igando, Lagos State

By Christopher Gbenga & Akande
Lagos State University

Abstract- Queue models are potent and veritable tools for urgent allocation of resources and basis for planning officers, resource managers and corporate organizations to respond to the demand rate with appropriate service rate. The usefulness of queue model in developing relevant policies for allocating and managing resources at the dumpsites has been emphasized in this study. Some factors (number of trucks and average waiting time in the system and in the queue etc.) were used to measure the performance of Solous III dumpsite based on the trucks’ arrival rate and corresponding service rate rendered by the managers of this dumpsite. This study used simple queue model (M/M/1) of First Come, First Served (FCFS) to evaluate trucks activities at the dumpsite. The performance measures include: numbers of trucks in the queue and in the system, waiting time of trucks in the queue and in the system as well as the probabilities associated with the trucks at the dumpsite. The queue analysis indicated that the traffic intensity was 0.96 which is close to 1 and that an average of 24 trucks are in the system while an average of 23 trucks are in the queue per hour. Average waiting time in the queue and in the system accounted for 28mins and 30mins respectively.

Keywords: queue, queue models, performance measures, trucks, dumpsites etc.

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Queue theory is an important tool used to model many supply chain problems and it is mostly applicable to situations where customers form a line and wait to be served by the service facility (Odior, 2013). A queue generally can be formed by vehicles, jobs or humans especially when the rate of arrival of items exceeds that of service required (Paul and Akpofire, 2015). Where arriving customers are being attended to by a single server, the queue system is said to be a simple or single queue system. If it is two or more, the queue system is referred to as multiple or multi-servers system (Saaty, 1983).

At any point a customer arrives and the server is busy, the customer has to join a queue to be served. The rate at which customers arrive for service is known as arrival rate per unit of time (Taylor, 1994), while the average number of completed service per unit of time is known as service rate. The service rate depends on the service system adopted by the organization’s management (single or multiple) and the service discipline which could be First Come, First Served or Last Come, First Out (Tanner, 1995). There is the probability that arriving customer at a service point has to wait for some moment before being attended to especially if on arrival, it finds one or more customers already at the service point. This process is otherwise

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1. Introduction

The need for orderliness in most human activities that could result in chaos informed the art and science of queuing. Precious hours and resources are always lost to chaotic situations that occur in our everyday life. Queues are waiting lines which are common experiences at public places such as hospitals’ drug dispensaries (Green, 2003), hospitals’ General Out-Patient Department (GOPD) (Ameh et al 2013), Bus Stops (Koko, 2018), Bus Stations, Port Terminals (Oyatoye, et al), Petrol Filling Stations (Akinnuli, 2014), Banks and ATM points (Famule, 2010), Shopping Malls (Hall, 1990), Arrival and Departure Rooms of Airports, Eateries (Onoja, et al, 2018) to mention but a few. Queues also occur in service industries – when jobs have to wait for machine process or at telecommunication centres when calls are on hold until they are mature (Sharma, 2004) and vehicles have to wait until Traffic Signal Lights (TSL) turn green at road intersections (Anoyke, et al 2013), among others. At Dumpsites, trucks have to wait in-lines to get served due to limited service facilities to prevent unnecessary traffic congestion.

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referred to as traffic intensity which lies between zero (0) and one (1) (Okoko, 2000). It must be less than 1, otherwise there will be an infinite queue and the system needs to operate for a long time to return to a steady state – where arrival rate can be easily managed by the system’s service facilities at any time.

Generally, customers may display different types of behaviour when they see a long queue on arrival (Ojo and Adebisi, 2018). Where customers decide to leave the line or queue without being served is known as Reneging or Abandonment. Where customers do not join the line but try to look for any available opportunity to enter the queue illegitimately is referred to as Baulking or Shunting. Others may move forward and backward in-between queues, looking for a fast-moving queue, thereby exhibiting a behaviour known as Jockeying.

In queue theory, a model is constructed so that queue lengths and waiting times can be predicted (Anokye, et al. 2013). This also helps in predicting the number of servers that will be needed in a system for cost minimization and profit optimization. A reduction in the average service time, E(t) through the addition of another service point will lead to a further reduction in the average queue length and waiting time as well as other queue performance measures especially in a multi-channel situations (Odior, 2013).

Queue Modeling as a mathematical one involves formulation of mathematical equations and it is useful in making some predictions about any system of study. The model is an offshoot of either probabilistic or stochastic modeling. Queue studies involve a number of systems. These consist of four sub-systems, comprising arrival pattern, a queue discipline, service facility and the outlet. The arrival pattern concerns ways through which trucks or items come into the system in a discrete manner. The queue discipline describes the arrival time of items versus when the service is performed and this follows some set of rules i.e. first come, first served or last come, first served. The service deals with the length of time a customer is served, i.e. number of servers and the service pattern. The length of time to serve a customer is known as the service rate. The queue in the system can be single or multiple and service points can be single and multiple too. However, the service rate normally has a negative exponential distribution. The outlet is the exit or departure from the system and this can influence the arrival and service rates in one way or the other (Wright and Ashford 1989; Lucey 1992).

At Solous III dumpsite in Igando, Lagos State, it is a recurrent decimal to find waste trucks forming long queues on both sides of LASU-Iba Expressway waiting for long period of time before being served. Frequent maneuverings of these trucks creates incessant traffic congestion which result in huge man-hour loss to commuters and motorists (Olorunfemi, 2003). Delays experienced by these trucks before service is rendered, affect their level of turn-around and this level of performance of the dumpsite have implications for the city’s waste management (Odewunmi, 2004; Taylor, 1994). This development informed the interest to study the likely causal factors of such long queues of trucks at the Solous III dumpsite at Igando, Lagos State towards evolving some useful way to manage trucks at the dumpsite.

a) Objectives of the study
i. Evaluate the queue system parameters such as the arrival rate, the service rate and the traffic intensity of trucks at Solous III dumpsite, Igando, Lagos State.
ii. Determine the queue systems performance measures such as the number of trucks in the system and in the queue, the waiting time of trucks in the system and in the queue as well as probabilities of events (trucks) at the dumpsites,
iii. Suggest some ways to improving dumpsites performance measures at Solous III dumpsite, Igando, Lagos State.

b) Hypothesis Testing

The study will also test the following hypothesis, as a way of validating the results of this research work.

H₀: Trucks Arrival rate does not follow a Poisson distribution at the dumpsite.
H₁: Trucks Arrival rate follows a Poisson distribution at the dumpsite.

II. METHODOLOGY: USING M/M/1 QUEUE MODEL

Data collection process for this study was based on Arrival rate and Service rate of Trucks at Solous III dumpsite situated along LASU-Iba Expressway at Igando, Lagos State. Solous III is the only functional dumpsite within Igando Area at the moment where trucks arrive in a Poisson process, discharge waste and exit the system after service has been completed (Magnus, 2015). This is synonymous to what is known as birth and death process in queue system. Birth refers to the arrival of trucks to join the existing queue at the dumpsite and death means departure or exit from the system having received service (Gross and Harris, 1985).

It is assumed that the time interval between successive arrivals and service time is independent and identically distributed especially in a simple queue formation (Anokye, et al 2013). A queue is said to be simple queue, if it is a single queue, single server and the pattern of arrival follows a Poisson type and Poisson probability distribution. On the other hand, the service time is also random, having a negative exponential distribution (Wright and Ashford 1989; Lucey 1992).

The queue system adopted at the dumpsite is a simple queue model (M/M/I) and a queue discipline of First-Come-First-Served (FCFS) and data were collected...
on first Monday of March, 2020 from 7am to 7pm. Data collected were analysed manually using some queue model formulae (see table 1) to determine the queue parameters such as the Traffic Intensity, Arrival rate and Service rate as well as other performance measures (Number of Trucks in the system, waiting time and other related probabilities associated with trucks behaviours at the dumpsites). The queue parameters are as listed in item 2.1 for clarity and the notations are frequently reflected in the analysis. Data used in determining the performance of the queue system were obtained through direct monitoring of both arrival rate and service rate. The time a truck joined and exited the system was noted using designed templates, pens and watches. The results of analysis were validated through the use of chi-square (X²) statistics and variance methods. The outcome of the model analysis was presented using tables and graphs.

Some Queue Performance Notations (Parameters)

- \( \lambda \): Average Arrival Rate
- \( \mu \): Average Service Rate
- \( R \): Traffic Intensity
- \( L_s \): Average Number of Trucks in the System
- \( L_n \): Average Number of Trucks in the Queue (when there is no queue or length of queue).
- \( L_q \): Average Number of Trucks in the Queue (when there is a queue)
- \( W_q \): Average Waiting Time in the Queue
- \( W_s \): Average Waiting Time in the System
- \( P_q \): Probability that a Truck will queue on arrival
- \( P_a \): Probability that a Truck will not queue on arrival
- \( P_n \): Probability of having exactly ‘n’ number of Truck in the system
- \( P_m \): Probability of having n or more Trucks in the system
- \( P_o \): Probability of having no Truck at all in the system

Table 1: Some already established Simple Queue model formulae in Literature

<table>
<thead>
<tr>
<th>s/n</th>
<th>Parameters/Performance measures</th>
<th>Model Formula: Single Queue Single Server</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Traffic Intensity</td>
<td>( R = \frac{\lambda}{\mu} )</td>
</tr>
<tr>
<td>2</td>
<td>Average Number of Trucks in the System</td>
<td>( n = \frac{\lambda}{\mu - \lambda} ) or ( n = \frac{R}{1-R} )</td>
</tr>
<tr>
<td>3</td>
<td>Average Number of Trucks when there is no Queue or Average length of the Queue</td>
<td>( m = \frac{\lambda}{\mu(\mu - \lambda)} ) or ( m = \frac{R^2}{1-R} )</td>
</tr>
<tr>
<td>4</td>
<td>Average Number of Trucks in the Queue when there is a Queue</td>
<td>( n = \frac{1}{1-R} )</td>
</tr>
<tr>
<td>5</td>
<td>Average Waiting Time in the Queue</td>
<td>( \frac{R - \frac{1}{1-R}}{R^{\lambda} - \frac{1}{\mu}} ) or ( \frac{\lambda}{\mu(\mu - \lambda)} )</td>
</tr>
<tr>
<td>6</td>
<td>Average Waiting Time in the System (both in the Queue and in the those receiving Service)</td>
<td>( \frac{1}{1-R} \times \frac{1}{\mu} )</td>
</tr>
<tr>
<td>7</td>
<td>Probability that a Truck will queue on arrival</td>
<td>( P_q = R )</td>
</tr>
<tr>
<td>8</td>
<td>Probability that a Truck will not queue on arrival</td>
<td>( P_a = 1 - R )</td>
</tr>
<tr>
<td>9</td>
<td>Probability of having exactly “n” number of Trucks in the System</td>
<td>( P_{(n)} = \left( \frac{\lambda}{\mu} \right)^n \left( 1 - \frac{\lambda}{\mu} \right) )</td>
</tr>
<tr>
<td>10</td>
<td>Probability of having “n” or more Trucks in the System</td>
<td>( P = R^n )</td>
</tr>
<tr>
<td>11</td>
<td>Probability of having “no” Trucks at all in the System (the percentage of time that the server will be idle)</td>
<td>( P_{(0)} = R^2(1-R) )</td>
</tr>
</tbody>
</table>
III. DATA ANALYSIS AND PRESENTATION OF RESULTS

Data obtained on Arrival rate and Service Rate were computed and their performance measures were determined accordingly. Table 2 showed the trucks’ Arrival rate and the Service rate at Solous III dumpsite against the period of the day.

The outcome of the relationship was plotted as depicted in Figure 1.

Figure 1: Interaction between Variables of Arrival rate and Service rate

Table 2: Relationship between Arrival rate and Service rate

<table>
<thead>
<tr>
<th>S/N</th>
<th>Hour</th>
<th>Arrival rate</th>
<th>Service rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>7-7.59</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>2.</td>
<td>8-8.59</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>3.</td>
<td>9-9.59</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>4.</td>
<td>10-10.59</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>5.</td>
<td>11-11.59</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>6.</td>
<td>12-12.59</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>7.</td>
<td>1-1.59</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>8.</td>
<td>2-2.59</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>9.</td>
<td>3-3.59</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>10.</td>
<td>4-4.59</td>
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<td>11.</td>
<td>5-5.59</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>12.</td>
<td>6-6.59</td>
<td>7</td>
<td>7</td>
</tr>
</tbody>
</table>

Total: 48 50

b) Time spent in the system
i. Average waiting time in the system

\[ \frac{R}{1-R} \times \frac{1}{\lambda} \] or \[ \frac{\lambda}{\mu(\mu-\lambda)} \]

\[ \frac{0.96}{1-0.96} \times \frac{1}{50} = 0.96 \times \frac{1}{50} = 24 \times 0.02 = 0.48 \text{ of an hour OR } 0.48 \times 60 = 28.8 \text{ (28min, 8sec)} \]

\[ \frac{48}{50(50-48)} = \frac{48}{2500-2400} = \frac{48}{100} = 0.48 \text{ of an hour OR } 0.48 \times 60 = 28.8 \text{ (28min, 48sec)} \]

ii. Average waiting time in the system (both queue and receiving service)

\[ \frac{1}{1-R} \times \frac{1}{\mu} \times \frac{1}{\mu} \times 0.02 = 25 \times 0.02 = 0.5 \text{ of an hour = 30mins} \]

2. The probability that a Truck will not queue on arrival

\[ = 1-R = 0.04 \text{ or 4%} \]

3. Probability of having exactly ‘n’ number of Truck in the system

34

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If \( n = 6 \) Trucks

\[
P(n) = \left( \frac{\lambda}{\mu} \right)^n (1 - \frac{\lambda}{\mu})
\]

\[
\therefore P(6) = \left( \frac{48}{50} \right)^6 \left( 1 - \frac{48}{50} \right) = \frac{221184}{250000} = 0.8847 \times 0.04 = 0.0354 \text{ or } 3.54\%
\]

4. Probability of having ‘n’ or more cars in the system

\[
P(n) = R^n = 0.7828 \text{ or } 78.28\%
\]

5. Probability of having no Truck at all in the system

\[
P(0) = (R)(1 - R) = (0.96)^2(1 - 0.96) = 0.04 \text{ or } 4\%
\]

Table 3: Summary of Model Results

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( \mu )</th>
<th>( R )</th>
<th>( \mu_s )</th>
<th>( \mu_q )</th>
<th>( W_q )</th>
<th>( W_s )</th>
<th>( P_q )</th>
<th>( P_s )</th>
<th>( P_n )</th>
<th>( P_m )</th>
<th>( P_o )</th>
</tr>
</thead>
<tbody>
<tr>
<td>48</td>
<td>50</td>
<td>0.96</td>
<td>24</td>
<td>23</td>
<td>25</td>
<td>28.8</td>
<td>30</td>
<td>0.96</td>
<td>0.04</td>
<td>0.0354</td>
<td>0.7828</td>
</tr>
</tbody>
</table>

IV. Results and Discussion

In this study, traffic intensity stood at 0.96 which was close to 1, indicating that the queue situation was getting to an infinite state in which long queue could be observable at the dumpsite throughout the day. As revealed in table 3, the average number of trucks in the system, average number of trucks in the queue (length of queue) and average number of trucks in the queue (where there is queue) were 24 trucks, 23 trucks and 25 trucks respectively. Average waiting time in the queue and in the system accounted for 28mins and 30mins respectively.

The study’s results also showed 96% probability that a truck will queue on arrival before being served while the probability that a truck will not queue on arrival is just 4%. The probability of having exactly ‘n’ number of Truck in the system stood at 3% and probability of having ‘n’ or more trucks in the system was 78%. Probability of having no Truck at all in the system (idle time) was 4%. This is so as the traffic intensity is running to 1 and queue development is bound to occur, meaning that the system will always be busy from time to time.

Again, it is clear that an average of 25 trucks will be waiting in line and each will spend at least 28 mins before receiving service. Apart from this, the 96% of traffic waiting in line is indicative that the long queue is affecting the truck drivers negatively. Consequently, some of them may want to leave without being served (balking or abandonment), or look for available space upfront to enter the queue illegitimately (shunting), thereby worsening the traffic situation on the major road due to frustration.

a) Model Validation

In this study, Chi square test \((X^2)\) was used to test whether the values of the arrival rate followed Poisson distribution or not at 5% level of significance and this was carried out in relation to its variance.

Table 4: Model Hypothesis

<table>
<thead>
<tr>
<th>Mean ((\bar{x}))</th>
<th>Variance</th>
<th>(X^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2.333</td>
<td>1.8955</td>
</tr>
</tbody>
</table>

\(H_0\): Trucks Arrival rate does not follow a Poisson distribution at the dumpsite.

\(H_1\): Trucks Arrival rate follows a Poisson distribution at the dumpsite.

The expected or predicted service rate was set at 4 trucks/hour. The critical value of \(X^2(X^2CV)\) was set at \(a = 0.05\) and 11 degree of freedom.

The table value is 19.675. Since the calculated \(P\)-value \((1.8955)\) is less than the table value \((19.675)\), the result is not significant. \(H_0\) is accepted – meaning that the truck arrival rate does not follow a Poisson distribution at the dumpsite. Although there exists a close fit in the observed arrival rate and the predicted service rate as well as \(X^2\) value \((1.8955)\) and the value of the variance \((2.333)\).

V. Conclusion

Applying queue model for trucks management at Solous III dumpsite in Igando showed that queue always exists at the dumpsite as revealed by the level of the traffic intensity but this can be reduced drastically if more service points are created by the management. The simple queue model and possibly multiple server model could be applied as monitoring or evaluation tools for either the dumpsite performance or management of the trucks that arrive the dumpsite for waste disposal. It is believed that better improvement can be achieved by using the multiple queue model to manage trucks at the dumpsite by the Lagos Waste Management Authority (LAWMA). This can go a long
way in making vital socio-economic decisions in truck scheduling to reduce incessant traffic congestion around the dumpsite’s location.

VI. Recommendation

In view of the 4% (low) probability of the system being idle, this study asserts that, there will be problem of long queues, waiting times and high utilization of the system (being always busy), this situation may not aid efficiency of the system in future. To further reduce the waiting time of trucks at the dumpsite from 30mins to somewhat 15mins, it is recommended that another service channel be opened by the dumpsite’s management to enhance the service rate and rapid turnaround of trucks. It also suggested that officials of the Lagos State Traffic Management Authority (LASTMA) be drafted into this area to control traffic.

There is need for Lagos State Government to create a slip road of at least 500meters on both side of the major road to serve as waiting area for the trucks to reduce traffic congestion at the dumpsite’s area and the access road to the dumpsite should be re-surfaced. This will consequently improve the trucks’ service time and the level of trucks’ turn-around from the dumpsite.

References Références Referencias

APPENDIX

<table>
<thead>
<tr>
<th>S/n</th>
<th>E</th>
<th>O</th>
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<th>(O – E)^2</th>
<th>(O–E)^2/E</th>
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</tr>
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<td>0</td>
<td>25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ X^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} \]

O – Observed Frequencies
E – Expected Frequencies
n – Number of Categories

Variance = \[ \frac{\sum E^2}{n} - x^2 \]

\[ = \frac{220}{12} - 4^2 \]

\[ = 18.33 - 16 \]

\[ = 2.333 \]

Standard Deviation = \[ \sqrt{2.333} \]

\[ = 1.5275 \]

Mean (x) = \[ \frac{\sum E}{n} = \frac{48}{12} = 4 \]