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Gravitomagnetics a Simpler Approach Applied to Dynamics within the Solar System

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Abstract- Galileo studied bodies falling under gravity and Tycho Brahe made extensive astronomical observations which led Kepler to formulate his three famous laws of planetary motion. All these observations were of relative motion. This led Newton to propose his theory of gravity which could just as well have been expressed in a form that does not involve the concept of force. The approach in this paper extends the Newtonian theory and the Special Theory of Relativity by including relative velocity by comparison with electromagnetic effects as shown in section 1.4 based on the Lorentz force. It is also guided from the form of measured data. This enables the non-Newtonian effects of gravity to be calculated in a simpler manner than by use of the General Theory of Relativity (GR). Application to the precession of the perihelion of Mercury and the gravitational deflection of light gives results which agree with observations and are identical to those of GR. It also gives the accepted expression for the Schwarzschild Radius. This approach could be used to determine non-Newtonian variations in the trajectories of satellites.

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An extra term is then added to the initial basic equation which acts in the direction of the relative velocity. The amended basic equation now predicts a change in the speed of light due to gravity and derives the accepted measured result for the Shapiro time delay. It also gives the accepted value for the Last Stable Orbit. Further, it shows that light passing through a gravitational field refracts in accordance with Snell's Law. It also shows that anti-gravity is possible but only when relative speeds get close to that of light.

Because the extra term is a function of $(v/c)^4$ the previously mentioned predictions are not significantly changed.

The prime action in this paper is to show the reasons for creating the form of gravitomagnetics. The applications are discussed to justify the equations. As shown in [32].

Keywords: *gravity, relativity, lorentz force, speed of light.*

I. The Basics

a) Newtonian Gravity

Galileo studied bodies falling to Earth under gravity and concluded that all bodies fell with the same acceleration independent of size and material. Tycho Brahe made extensive astronomical observations which led Kepler to formulate his three famous laws of planetary motion relative to the Sun. All of these observations were of relative motion but the mass of one body was, in each case, much greater than that of the other. These led Newton to propose his theory of gravity using the concept of force and yielding an equation which gives the acceleration of a body relative to the centre of mass. He could just as well have presented it in the form

$$a_{B/A} = -\frac{G(m_A + m_B)}{r_{B/A}^2} \tag{1}$$

without invoking the concept of force and only requiring one definition of mass. This means that the principal of equivalence does not appear.

That is, the acceleration of body B relative to A, in the radial direction, is proportional to the sum of their masses and inversely proportional to the square of their separation. *G* is the gravitational constant.

b) Gravitomagnetics

It is now proposed that equation (1) be extended to include the relative velocity. The axioms are.

a) It is assumed that in mass-free space light travels in straight lines. This defines a non-rotating frame of reference.
b) Because all motion is relative there are no other restrictions on the frame of reference.
c) Gravity propagates at the same speed as light.
d) Mass, or rest mass, is simply the quantity of matter and is regarded as constant. It could be a count of the number of basic particles.

See section I d for the similarity with the Lorentz force.

The initial proposed equation is based on comparisons with electromagnetics. This equation gives results which agree with the measured results of the precession of the perihelion of Mercury and with the deflection of light grazing the Sun. Also it gives the correct definition for the Schwarzschild Radius. However, it suggests that the speed of light is constant. As a result it does not predict the Shapiro Time Delay. An extra term is then added which gives agreement with the time delay and also generates the accepted value for the Last Stable Orbit. See equations (2a), (3a), (4a) and (5a).

The proposed equation is

$$\boldsymbol{a} = -\frac{K}{r^2}\left(1 - \frac{v^2}{c^2}\right)\boldsymbol{e}_r + \frac{2K}{r^2c^2}\boldsymbol{v}\times(\boldsymbol{v}\times\boldsymbol{e}_r) \tag{2}$$

or

$$\boldsymbol{a} = -\frac{K}{r^2}\left(1 + \frac{v^2}{c^2}\right)\boldsymbol{e}_r + \frac{2Kv_r}{r^2c^2}\boldsymbol{v} \tag{3}$$

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where $\boldsymbol{a}$ = acceleration of body B relative to body A, $\boldsymbol{v}$ = the relative velocity, r = the separation and e_r = the unit vector from body A to body B. Also c = speed of light, $K = G\ (m_A + m_B)$ and v_r is the radial component of velocity. Note that G is a constant which could be incorporated into the definition of the quantity of matter. These equations reduce to equation (1) when $v << c$.

The equation can also be written in terms of the Newtonian part plus the gravitomagnetic part

$$\mathbf{a} = \mathbf{a_N} + \mathbf{a_V} = -\frac{K}{r^2}\mathbf{e_r} + \frac{K}{r^2}\left(\frac{v}{c}\right)^2 \mathbf{e_{2\phi}} \quad (4)$$

where ϕ is the angle between the velocity and the radius.

It should be noted that the velocities of the individual bodies do not appear in these equations, only the relative velocity. For two isolated bodies the relative motion is the only measureable value.

A convenient definition of force is

$$\mathbf{P} = \mu\mathbf{a} = -\frac{Gm_A m_B}{r^2}\left(1-\frac{v^2}{c^2}\right)\mathbf{e_r} + \frac{2Gm_A m_B}{r^2c^2}\mathbf{v}\times\left(\mathbf{v}\times\mathbf{e_r}\right) \quad (5)$$

where $\mu = m_A m_B/(m_A + m_B)$, the reduced mass.

By definition of the centre of mass (or the centre of momentum) the total momentum is zero with reference to the centre of mass. It is now proposed that the motion of the centre of mass of two bodies is not affected by collision. From this it follows that for a group of particles the motion of the centre of mass is unaffected by internal impacts.

The relative acceleration is only radial when the relative velocity is either radial or tangential. In general the moment of momentum can be shown to be a function of the relative position. So, for an elliptic orbit it remains within bounds.

General inferences from equation (2).
Reverts to Newtonian form when $v << c$.
The second term of (2) is normal to the velocity.

If $v = c$ the first term of (2) vanishes so that there is no change of speed.

Moment of velocity (or moment of momentum per total quantity of matter) is shown to be a function of r.

The equivalence of inertial mass to gravitational mass does not arise.

c) Modified Equations

An extra term is added in the direction of the relative velocity. This will affect the speed of light but not its deflection. As the term is a function of c^4 it only has a very small effect on the motion of large bodies in Solar orbits.

The new equation is

$$\boxed{\mathbf{a} = \mathbf{a_N} + \mathbf{a_V} + \mathbf{a}_T = -\frac{K}{r^2}\mathbf{e_r} + \frac{K}{r^2}\left(\frac{v}{c}\right)^2\mathbf{e_{2\phi}} + \frac{2K}{r^2}\frac{v_r}{c}\left(\frac{v}{c}\right)^3\mathbf{t}} \quad (4a)$$

Where $\boldsymbol{v}$ is the relative velocity and v_r is the radial component. Also $\boldsymbol{t} = \boldsymbol{v}/|v|$ is the vector in the direction of the velocity. The angle between the velocity and the radius is ϕ. For the third term the sign of the acceleration depends only on the sign of the radial velocity. $K = G(m_A + m_B)$ and c is the speed of light in a gravity free vacuum.
This equation will be considered to be the basic for Post Newtonian Gravity. Justification will come from agreement with verified experimental data.

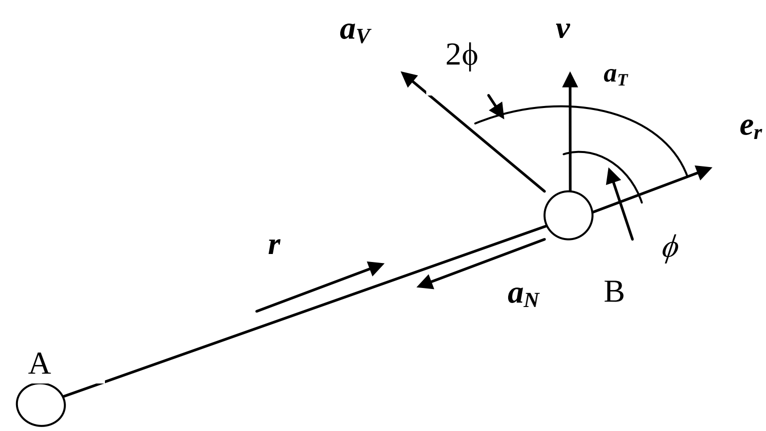

Figure 1

Equation (4a) may be re-written as

$$\mathbf{a} = -\frac{K}{r^2}\left(1 - \frac{v^2}{c^2}\right)\mathbf{e}_\mathbf{r} + \frac{2K}{r^2c^2}\mathbf{v}\times(\mathbf{v}\times\mathbf{e}_\mathbf{r}) \quad + \frac{2K}{r^2}\frac{v_r}{c}\left(\frac{v}{c}\right)^3\mathbf{t} \tag{2a}$$

or

$$\mathbf{a} = -\frac{K}{r^2}\left(1 + \frac{v^2}{c^2}\right)\mathbf{e}_r + \frac{2Kv_r}{r^2c^2}\mathbf{v} + \frac{2K}{r^2}\frac{v_r}{c}\left(\frac{v}{c}\right)^3\mathbf{t} \tag{3a}$$

From which it is seen that the additional term is negligible when *(v/c)⁴* is small compared to unity

Again, noting that $\boldsymbol{t} = \boldsymbol{v}/|v|$ means that (3a) may be written as

$$\mathbf{a} = -\frac{K}{r^2}\left(1 + \frac{v^2}{c^2}\right)\mathbf{e}_\mathbf{r} + \frac{2Kv_r}{r^2c^2}\left[1 + \left(\frac{v}{c}\right)^2\right]\mathbf{v} \quad .$$

or

$$\boldsymbol{a} = -\frac{K}{r^2}\left(1 + \frac{v^2}{c^2}\right)\boldsymbol{e}_r + \frac{2Kv_r}{r^2c^2}Q\boldsymbol{v} \quad \text{. where} \quad Q = \left[1 + \left(\frac{v}{c}\right)^2\right]$$

And

$$\mathbf{P} = \mu\mathbf{a} = -\frac{Gm_Am_B}{r^2}\left(1 - \frac{v^2}{c^2}\right)\mathbf{e}_\mathbf{r} + \frac{2Gm_Am_B}{r^2c^2}\mathbf{v}\times(\mathbf{v}\times\mathbf{e}_\mathbf{r}) + \frac{2Gm_Am_B}{r^2c^2}\frac{v_r}{c}\left(\frac{v}{c}\right)^3\mathbf{t} \tag{5a}$$

where $\mu = m_Am_B/(m_A + m_B)$, the reduced mass.

Equations (2 - 5) will account for the precession of the perihelion of Mercury and will predict the observed value for the deflection of light grazing the Sun. These results were heralded as confirmation of Einstein's General Theory of Relativity. They also give the accepted value for the Schwarzschild Radius.

Equations (2a – 5a) will generate the same results as mentioned above but will also give the accepted value for the Shapiro Time Delay because the speed of light is now affected by gravity. Light is slowed down by gravity which is the opposite of mass particles. The value generated for the Last Stable Orbit is the accepted value.

d) Lorentz force

We shall look at the standard theory of electromagnetism, but this is only to obtain some guidance as to the possible form of a gravitomagnetic theory. The electromagnetic notation is based on reference [7] for SI units.

The force on a charge q_2 in an electromagnetic field is

$$\mathbf{F}_2 = q_2\mathbf{E} + q_2\mathbf{v}_2\times\mathbf{B} \tag{1.4.1}$$

B is the magnetic field and *E* is the electrostatic field and v_2 is the velocity of the charge. This equation defines the Lorentz force. The magnetic field, due to a length of conductor dl_1 carrying a current i_1 is

$$\mathbf{B} = \frac{\mu_o}{4\pi r_{2/1}^2} i_1 d\mathbf{l}_1 \times \mathbf{e}_{2/1} \quad . \tag{1.4.2}$$

but $$i_1 d\boldsymbol{l}_1 = q_1 \boldsymbol{v}_1$$

so, for point charges, we can write for the force on charge 2 due to charge 1 is

$$\boldsymbol{F}_{2/1} = \frac{1}{4\pi\ \varepsilon_0}\frac{q_1 q_2}{r_{2/1}^2}\boldsymbol{e}_{2/1} + \frac{\mu_o\, q_1 q_2}{4\pi\ r_{2/1}^2}\boldsymbol{v}_2 \times (\boldsymbol{v}_1 \times \boldsymbol{e}_{2/1})$$

(see also page 256 of ref. [7])

More detailed analysis of the electrodynamics shows that the right hand side of the above equations should be multiplied by γ, but is usually omitted for small velocities.

The speed of light $c = \dfrac{1}{\sqrt{\varepsilon_o \mu_o}}$ so we have

$$\boldsymbol{F}_{2/1} = \frac{\gamma q_1 q_2}{4\pi\varepsilon_o r_{2/1}^2}\left[\boldsymbol{e}_{2/1} + \left(\frac{\boldsymbol{v}_2}{c}\right) \times \left(\frac{\boldsymbol{v}_1}{c} \times \boldsymbol{e}_{2/1}\right)\right] \tag{1.4.3}$$

For mass elements we assume that

$$\boldsymbol{F}_{2/1} = -\frac{\gamma G m_1 m_2}{r_{2/1}^2}\left[\boldsymbol{e}_{2/1} + \left(\frac{\boldsymbol{v}_2}{c}\right) \times \left(\frac{\boldsymbol{v}_1}{c} \times \boldsymbol{e}_{2/1}\right)\right] \tag{1.4.4}$$

Here c, the speed of gravity waves, is assumed to be the same as that of light.

The form of this equation will be taken as a guide; the problem is to choose the most appropriate values for the velocities. The assumptions that are made in the following development are:

a) That the field generated by body (1) depends on the velocity of body (1) relative to body (2) i.e. ($\boldsymbol{v}_{1/2}$) and
b) The force on body (2) depends on the velocity of body (2) relative to the field i.e. ($\boldsymbol{v}_{2/1}$). Let $\boldsymbol{v}_{2/1} = \boldsymbol{v}$, therefore $\boldsymbol{v}_{1/2} = -\boldsymbol{v}$.

It must be emphasised that force and force fields are inventions solely for the purpose of visualisation.

Substituting these values into equation (1.4.4) gives

$$\boldsymbol{F}_{2/1} = -\frac{\gamma G m_1 m_2}{r_{2/1}^2}\left[\boldsymbol{e}_{2/1} - \left(\frac{\boldsymbol{v}}{c}\right) \times \left(\frac{\boldsymbol{v}}{c} \times \boldsymbol{e}_{2/1}\right)\right] = -\boldsymbol{F}_{1/2} \tag{1.4.5}$$

For the Newtonian case the acceleration of body (2) relative to body (1) is

$$\boldsymbol{a} = \ddot{\boldsymbol{r}}_2 - \ddot{\boldsymbol{r}}_1 = \frac{\boldsymbol{F}_{2/1}}{m_2} - \frac{(-\boldsymbol{F}_{2/1})}{m_1} = \boldsymbol{F}_{2/1}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)$$

$$= \boldsymbol{F}_{2/1}\left(\frac{m_1 + m_2}{m_1 m_2}\right) = \frac{\boldsymbol{F}_{2/1}}{\mu}$$

where $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$ is known as the reduced mass.

For the relativistic case we shall define force as

$$\boldsymbol{F} = \frac{d}{dt}(\gamma\mu\boldsymbol{v}) = \left(\gamma\boldsymbol{a} + \frac{\gamma^3}{c^2}(\boldsymbol{v}\cdot\boldsymbol{a})\boldsymbol{v}\right)\mu$$

where $\gamma = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

Using this definition of force we obtain, using equation (1.4.5),

$$\gamma\ \boldsymbol{a} + \frac{\gamma^3}{c^2}(\boldsymbol{v}\cdot\boldsymbol{a})\boldsymbol{v} = -\frac{G(m_1 + m_2)\gamma}{r_{2/1}^2}\left[\boldsymbol{e}_{2/1} - \left(\frac{\boldsymbol{v}}{c}\right) \times \left(\frac{\boldsymbol{v}}{c} \times \boldsymbol{e}_{2/1}\right)\right]$$

or

$$\boldsymbol{a} + \frac{\gamma^2}{c^2}(\boldsymbol{v}\cdot\boldsymbol{a})\boldsymbol{v} = -\frac{G(m_1 + m_2)}{r_{2/1}^2}\left[\boldsymbol{e}_{2/1} + \boldsymbol{e}_{2/1}\left(\frac{v}{c}\right)^2 - \frac{\boldsymbol{v}}{c}\left(\frac{\boldsymbol{v}}{c}\cdot\boldsymbol{e}_{2/1}\right)\right] \tag{1.4.6}$$

Where m_1 and m_2, are the respective rest masses.
Change the suffix of the unit vector to r,
let $K = G(m_1 + m_2)$ and
let $r = |\boldsymbol{r}_{2/1}|$ be the separation. Equation (1.4.6) now becomes

$$\boldsymbol{a} + \frac{\gamma^2}{c^2}(\boldsymbol{v}\cdot\boldsymbol{a})\boldsymbol{v} = -\frac{K}{r^2}\left[1 + \frac{v^2}{c^2}\right]\boldsymbol{e}_r + \frac{K v_r}{r^2 c^2}\boldsymbol{v} \tag{1.4.7}$$

Note that this equation is symmetric with regard to the two bodies.

The scalar product of equation (1.4.7) with $\boldsymbol{v}$ yields

$$(\boldsymbol{a}\cdot\boldsymbol{v})\left[1 + \gamma^2\frac{v^2}{c^2}\right] = -\frac{K v_r}{r^2}$$

By definition, $\gamma^2 = 1/\left(1 - v^2/c^2\right)$ therefore

$$\gamma^2(\boldsymbol{v}\cdot\boldsymbol{a}) = -\frac{Kv_r}{r^2} \quad (1.4.8)$$

Substituting equation (1.4.8) into equation (1.4.7) gives

$$\boldsymbol{a} = -\frac{K}{r^2}\left(1-\frac{v^2}{c^2}\right)\boldsymbol{e}_r + \frac{2K}{r^2c^2}\boldsymbol{v}\times(\boldsymbol{v}\times\boldsymbol{e}_r) \quad \text{as (2)}$$

$$\boldsymbol{a} = -\frac{K}{r^2}\left(1+\frac{v^2}{c^2}\right)\boldsymbol{e}_r + \frac{2Kv_r}{r^2c^2}\boldsymbol{v} \quad \text{as (3)}$$

II. Application to Two Mass Problem

a) *Polar Coordinates*

The following development is based on the conventional treatment of the two body gravitational problem. For the dynamics of bodies in Solar orbits the modified equations are not required. Here, *r* is the separation and $\boldsymbol{e}_r$ is the unit vector in the direction of body 2 as seen from body 1. θ is the orientation of the unit vector with respect to the 'fixed' stars and $\boldsymbol{e}_\theta$ is the unit vector normal to $\boldsymbol{e}_r$ in the plane of the motion.

Now $$\boldsymbol{a} = (\ddot{r} - r\dot{\theta}^2)\boldsymbol{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\boldsymbol{e}_\theta$$

and $$\boldsymbol{v} = r\boldsymbol{e}_r + (r\dot{\theta})\boldsymbol{e}_\theta$$

Equation (3a) can now be expressed in component form

$$\ddot{r} - r\dot{\theta}^2 = -\frac{K}{r^2}\left[1+\left(\frac{v}{c}\right)^2\right] + \frac{2Kv_r^2}{r^2c^2}Q \quad (6)$$

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{1}{r}\frac{d}{dt}\left(r^2\dot{\theta}\right) = \frac{2Kv_rv_\theta}{r^2c^2}Q \quad (7)$$

but for low values of *(v/c)* the term *Q* will be taken to be unity.

Define $h = r(r\dot{\theta})$, the moment of momentum per reduced mass, and *u* = *1/r*. So that $h = \dot{\theta}/u^2$ thus

$$\dot{r} = -\frac{1}{u^2}\frac{du}{d\theta}\frac{d\theta}{dt} = -h\frac{du}{d\theta},$$

$$\ddot{r} = -u^2h^2\frac{d^2u}{d\theta^2} - \dot{h}\frac{du}{d\theta},$$

and

$$\ddot{\theta} = -\frac{2h\dot{r}}{r^3} + \frac{\dot{h}}{r^2} = 2u^3h^2\frac{du}{d\theta} + u^2\dot{h}$$

Equations (6, 7) may now be written

$$\frac{d^2u}{d\theta^2} + \frac{du}{d\theta}\frac{\dot{h}}{u^2h^2} + u = \frac{K}{h^2} + \frac{Ku^2}{c^2} - \frac{K}{c^2}\left(\frac{du}{d\theta}\right)^2 \quad (8)$$

and

$$u\dot{h} = -\frac{2K}{c^2}u^3h^2\frac{du}{d\theta} \quad (9)$$

Since $\dot{h} = \frac{dh}{d\theta}\dot{\theta} = \frac{dh}{d\theta}u^2h$ combining with equation (9) gives, $\frac{dh}{h} = -\left(\frac{2Kdu}{c^2}\right)$ (10)

Integrating equation (10) leads to

$$h = h_0e^{-2K(u-u_0)/c^2} \quad (11)$$

Therefore, for small variations

$$h^2 \approx h_0^2(1 - 4K(u-u_0)/c^2) \quad (12)$$

where the suffix 0 refers, in this case, to the position $\theta = \pi/2$ measured from the periapsis.

Substituting in equation (8) for *h*, using equation (12), we obtain

$$\frac{d^2u}{d\theta^2} + u = \frac{K}{h_0^2} + \frac{K}{c^2}\left[\frac{4K(u-u_0)}{h_0^2} + \left(\frac{du}{d\theta}\right)^2 + u^2\right] \quad (13)$$

b) *Precession of the Periapsis*

Equation (4) is very much easier to apply. This equation is equally applicable to the prediction of satellite trajectories. Because in these aces the relative speeds are not close to the speed of light.

The equation which was developed in reference [19] for calculating the precession of the perihelion of Mercury per orbit is

$$\theta_P = \frac{6\pi G(m_1+m_2)}{c^2a(1-e^2)} \quad (14)$$

where *a* is the semi-major axis and *e* is the eccentricity. This generates 42.89 arcsec/century.

For the binary pulsar PSR 1913+16, which was discovered by Hulse and Taylor in 1974, (see reference [22]), the accepted data is that the masses of the two stars are 1.441 and 1.387 times the mass of the Sun, the semi-major axis is 1,950,100 km, the eccentricity is 0.617131 and the orbital period is 7.751939106 hr. Using equation (14) we obtain the result 4.22 deg/yr, which is in agreement with the measured value and that predicted by General Relativity. The orbital decay, or inward spiralling, of binary pulsars is said to be simply due to energy loss caused by gravitational wave emission. This may be the case but energy loss alone will not account for the phenomenon. The loss of mass alone would cause outward spiralling as do most cases of tidal drag.

c) Moment of Momentum

If the additional term is negligible then it can be shown that the moment of momentum is

$$h_2 = h_1 \exp\left(\frac{2K}{c^2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)\right)$$

which depends on separation but is constant when c = infinity.

d) Schwarzschild Radius

For a constant radius $v_r = 0$ and $v = v_\theta$ so equation (3) or (3a) becomes

$$-\frac{K}{r^2}\left(1+\frac{v_\theta^2}{c^2}\right)\boldsymbol{e_r} = -\frac{v_\theta^2}{r}\boldsymbol{e_r} \text{ so if } v_\theta = c \text{ then } r = \frac{2K}{c^2} = \frac{2G(M+0)}{c^2} = r_g$$

which is known as the Schwarzschild Radius.

e) Last Stable Orbit

Numerical integration of equation (3a) shows that the Last Stable Orbit occurs when the radius of the orbit is 3 times the Schwarzschild Radius, which is the accepted result based on General Relativity. If equation (3) is used then a value of 2.62 r_g may be calculated algebraically. However if Q is not unity, as shown in equation (3a), then equation (12), with Q included is,

$$h^2 \approx h_0^2(1 - 2r_g(u-u_0)Q) \qquad (12a)$$

where the suffix 0 refers to circular motion when $u = u_o$

Substituting in equation (8) for h, using equation (12a), equation (13) becomes

$$\frac{d^2u}{d\theta^2} + u = \frac{K}{h_0^2} + \frac{K}{c^2}\left[\frac{4K(u-u_0)Q}{h_0^2} + \left(\frac{du}{d\theta}\right)^2 + u^2 - \left(\frac{du}{d\theta}\right)2(Q-1)\right] \qquad (13a)$$

If $u = u_o + \varepsilon$ then, for small variations

$$\varepsilon'' + \varepsilon\left(1 - r_g u_o(2Q+1) + (r_g u_o)^2 Q\right) = 0 \qquad (13b)$$

For circular motion it can be shown that

$$\left(\frac{v}{c}\right)^2 = \frac{u_o r_g}{2-u_o r_g} \quad \text{so} \quad Q = \left[1 + \left(\frac{u_o r_g}{2-u_o r_g}\right)\right]$$

For a stable near circular orbit then

$\left(1 - r_g u_o(2Q+1) + (r_g u_o)^2 Q\right) > 0$ so when the factor of ε in equation (13b) is zero, algebraic manipulation of (13b) gives $r_o/r_g = 3$, which is the accepted value. It also gives a value of 0.5.

f) Deflection of Light

In equation (3a) terms 2 and 3 are parallel to the velocity so the component normal to the velocity is

$$\boldsymbol{a}\cdot\boldsymbol{n} = -\frac{K}{r^2}\left(1+\left(\frac{v}{c}\right)^2\right)\boldsymbol{e_r}\cdot\boldsymbol{n} = v\frac{d\psi}{dt}$$

For small variation of the speed of light assume that $v = c$. Also, for small deflections the scalar product of $\boldsymbol{e_r}$ and $\boldsymbol{n}$ can be seen from Figure (2) to be R_s / r. Therefore

$$\boldsymbol{a}\cdot\boldsymbol{n} = -\frac{2K}{r^2}\frac{R_s}{r} \qquad \text{now } r = \sqrt{x^2+R_s^2} \text{ so}$$

$$\boldsymbol{a}\cdot\boldsymbol{n} = -\frac{2KR_s}{\left(x^2+R_s^2\right)^{3/2}} = c\frac{d\psi}{dt}c\frac{d\psi}{dx}\frac{dx}{dt}$$

or, as dx/dt is approximately c

$$d\psi = -\frac{2K}{c^2}R_s\frac{1}{\left(x^2+R_s^2\right)^{3/2}}dx \qquad \text{integrating gives}$$

$$\psi = \frac{2K}{c^2R_s}\left(\frac{x}{\sqrt{\left(x^2+R_s^2\right)}}\right).$$

When $x = 0$ $\psi = 0$ and when x goes to infinity

$$\psi = -\frac{2K}{c^2R_s}$$

Therefore the total deflection $\delta = 2\psi$ so

$$\delta = \frac{4K}{c^2R_s} \qquad (15)$$

With $K = M_{sun}G$ and R_s being the radius of the Sun the deflection is 1.75 arcsec. This value agrees with the measured value and with General Relativity. This confirms the assumption that the deflection of light grazing the Sun is small.

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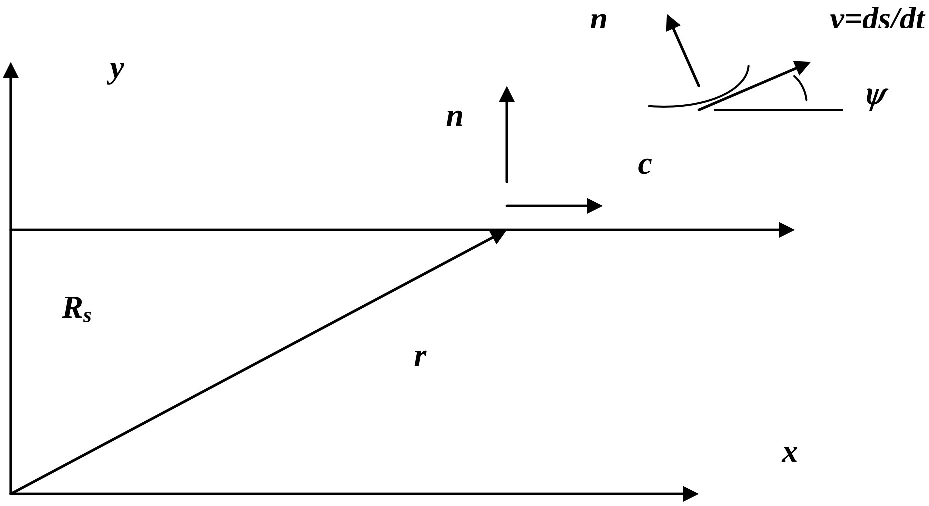

Figure 2

g) Shapiro Time Delay

When $v = c$ equation (2a) gives $\mathbf{a} \cdot \mathbf{t} = \frac{2K}{r^2}\frac{v_r}{c} = \frac{\mathrm{dc}}{\mathrm{dt}}$

Therefore $\frac{2K}{r^2}\frac{\dot{r}}{c} = \frac{dc}{dr}\frac{dr}{dt}$ or $\frac{2K}{r^2 c}dr = dc$

Integration leads to $[c]_i^\infty = \left[-\frac{2K}{rc}\right]_i^\infty$, $c - c_i = 0 + \frac{2K}{r_i c}$

where c is now the speed of light where r tends to infinity.

Or $c_i = c\left(1 - \frac{2GM}{rc^2}\right)$ (16)

Consider the case of light grazing the Sun at a radius R_s and calculate the journey time. As an approximation assume the path to be a straight line.

Then, since $c_i = \frac{dx}{dt}$

$$c\ dt = \frac{dx}{1 - 2GM/(rc^2)} \quad \text{where} \quad r = \sqrt{x^2 + R_s^2} \quad \text{also as} \quad \frac{2GM}{rc^2} << 1$$

$$\int c\ dt = \int\left(1 + \frac{2GM/c^2}{\sqrt{x^2 + R_s{}^2}}\right)dx = x + \frac{2GM}{c^2}\ln\left[2\left(\sqrt{x^2 + R_s{}^2} + x\right)\right]$$

Between the limits x = 0 to x and t = 0 to t we have the total time

$$t = \frac{x}{c} + \frac{2GM}{c^3}\ln\left(2\sqrt{x^2 + R_s^2} + 2x\right) - \frac{2GM}{c^3}\ln(2R_s)$$

so the additional time due to the gravitational effect is

$$\Delta t = \frac{2GM}{c^3}\ln\left[\frac{\sqrt{x^2 + R_s^2} + x}{R_s}\right] \quad (17)$$

This gives a time delay of 232μs for the double transit time from Earth to Venus. This is exactly the same as quoted by Bertotti, reference made to C. M. Will. The value for Mars is 247μs which is as quoted by Reasenberg, Shapiro et. al., is also given by the above equation.

This shows that the speed of light is reduced by gravity. In the case of light grazing the Sun the reduction in speed/c_o is $\frac{2GM_s}{c^2 R_s} \approx 4 \times 10^{-6}$, that is, only four parts per million. The variable speed of light has been incorporated into the basic equation however this would not have any major effect on the motion of material bodies. The valuation of the deflection of light is not changed. Because the light path is slightly curved the increase in path length will add to the delay but as the speed change is so small the additional time is less than 1%.

h) Gravity and the refraction of light

The form of the extended equation is based on the known observations or deductions. The extra term is required to be in the direction of the relative velocity. Also, because the speed of light is at its maximum then passing though empty space it must reduce when moving into a gravitational field so assume that it depends on the magnitude of the radial speed. From this it follows that the acceleration will be repulsive. Also, for circular orbits the speed remains constant, which is true when $v_r = 0$.

The additional term could be

$\boldsymbol{a_t} = \lambda \frac{K}{r^2}\left(\frac{v_r}{c}\right)\frac{\boldsymbol{v}}{c}$, where λ is a constant depending on application.

When applied to the Last Stable Orbit with $\lambda = 2\left(\frac{v}{c}\right)^2$ the value agrees with the generally accepted value. The constant seems reasonable, so

$$\boldsymbol{a_t} = \frac{2K}{r^2}\left(\frac{v}{c}\right)^2\left(\frac{v_r}{c}\right)\frac{\boldsymbol{v}}{c}$$

This is the form shown in equation (2a) which leads to predicting the speed of light due to gravity.

Consider the case when speed is very close to the speed of light in a gravity free vacuum.

Equation (2a) gives the acceleration parallel to the velocity

$$\boldsymbol{a}\cdot\boldsymbol{t} = \left(\frac{2K}{r^2c}v_r\right)$$

From equation (3a) the acceleration normal to the velocity

$$\boldsymbol{a}\cdot\boldsymbol{n} = -\frac{2K}{r^2}\boldsymbol{e_r}\cdot\boldsymbol{n}$$

With ∅ being the angle between the radius from the gravity source and the velocity

$$v_r = c\cos\emptyset \qquad \text{and} \qquad \boldsymbol{e_r}\cdot\boldsymbol{n} = -\sin\emptyset$$

therefore

$$a_t = \boldsymbol{a}\cdot\boldsymbol{t} = \left(\frac{2K}{r^2}\right)\cos\emptyset \qquad \text{and}$$

$$a_n = \boldsymbol{a}\cdot\boldsymbol{n} = \left(\frac{2K}{r^2}\right)\sin\emptyset$$

So $$a_n = a_t\tan\emptyset \qquad \text{(a)}$$

The refractive index n is defined as speed of light in a gravity free vacuum/ speed of light in a transparent media.

For light passing through different media Snell's Law states that

$$\frac{n_1}{n_2} = \frac{c/c_1}{c/c_2} = \frac{\sin\emptyset_2}{\sin\emptyset_1} = \frac{c_2}{c_1}, \text{ or } c_1/\sin\emptyset_1 = c_2/\sin\emptyset_2 = constant$$

With $c/\sin\emptyset = B\ (a\ constant)$, then $c = B\sin\emptyset$, differentiating with respect to time gives

$\frac{dc}{dt} = B\cos\emptyset\frac{d\emptyset}{dt} = \frac{c}{\tan\emptyset}\frac{d\emptyset}{dt}$, using path coordinates

$a_t = \frac{dc}{dt}$ and $a_n = c\frac{d\emptyset}{dt}$

Thus $a_n = a_t\tan\emptyset$ as equation (a).

This is applicable to the passage of light through the Earth's atmosphere. Hence light passing through a strong gravity field will be affected in the same way as light passing through the atmosphere.

This result gives more confirmation of the applicability of the basic equation of the paper.

III. Gravitomagnetics Applied to Rotating Bodies

a) Basic Equations

When equation (2) is applied to two body systems the equation generated is identical to the de Sitter form and agrees with the measurement of precession of the perihelion of Mercury and of the Binary Pulsar PSR 1913+16. The equation is equally applicable if one spherical body is large and non-rotating. Note that the additional term, which is a function of c^3, is negligible for Solar dynamics.

$$\boldsymbol{a} = -\frac{K}{r^2}\left(1-\frac{v^2}{c^2}\right)\boldsymbol{e_r} + \frac{2K}{r^2c^2}\boldsymbol{v}\times\left(\boldsymbol{v}\times\boldsymbol{e_r}\right)$$

(2rpt) (18a)

where c is the speed of light, $\boldsymbol{a}$ is relative acceleration, $\boldsymbol{v}$ is relative velocity and r is relative position. Also $\boldsymbol{e_r}$ is the unit vector from body A to body B.

$K = G(m_A + m_B)$ where G is the gravitational constant

The calculations are made easier for multi-body systems by the use of a defined force as shown in equation (5).

$$\boldsymbol{P} = \mu\boldsymbol{a} = -\frac{Gm_Am_B}{r^2}\left(1-\frac{v^2}{c^2}\right)\boldsymbol{e_r} + \frac{2Gm_Am_B}{r^2c^2}\boldsymbol{v}\times\left(\boldsymbol{v}\times\boldsymbol{e_r}\right) \qquad \text{(5rpt) (18b)}$$

μ is the reduced mass $m_Am_B/(m_A+m_B)$

b) Gravity Probe B

Gravity Probe B is the study of the precession of a gyroscope in a polar orbit about the Earth. The spin axis of the gyroscopes is perpendicular to the spin axis of the Earth. After over 3 decades of study and design at Stanford University the final results of the experiment were published in 2011 [27].

When the new theory is applied to Gravity Probe B the following equations are derived algebraically using equation (5). The modified equation is not required because the relative velocities are not close to the speed of light.

$$\dot{\phi}_x = \frac{Gm_E\dot{\alpha}}{c^2R_c} \quad \text{Geodesic or de Sitter. (North to South)}$$

$$\dot{\phi}_z = \frac{GI_E\Omega}{2c^2R_c^3}\left(2-3\cos^2\alpha\right) \quad \text{frame dragging or Lense - Thirring. (West to East)}$$

where R_E = Radius of the Earth, m_E = Mass of the Earth, I_E = Moment of inertia of the Earth, R_c = Radius of orbit, G = Gravitational constant and c = Speed of light.

Also, α is the location of the satellite in its orbit and Ω is the angular velocity of the Earth. These equations yield,

$$\dot{\phi}_x \; 4.4 \; \pm 0.0 \quad \frac{\text{arcsec}}{\text{yr}} \quad (\; 6.6 \;\; \pm 0.0 \text{ based on GP} - \text{B update })$$

$$\dot{\phi}_z = 0.02 \;\; \pm 0.06 \, \frac{\text{arcsec}}{\text{yr}} \quad (0.037 \; \pm \;\; 0.007 \text{ based on GP-B update})$$

Gravity Probe B Status Update 2011 Ref [27}. Note that one arc second equals 1/3600 degrees of angle. Also, one year is a long time. All four of the gyros give results for de Sitter value which are very close to each other. However two of the gyros give results for the Lense -Thirring value which are very close to the value quoted in this paper.

c) Precession of the Periapsis of a small body orbiting a large rotating mass

This problem is similar to the discussion of the precession of the perihelion of Mercury except that now the rotation of the Earth is taken into account. For Mercury the rotation of the Sun has negligible effect. The LAGEOS satellites yield results for the so called frame dragging, or Lense Thirring effect, which results from the rotation of the Earth.

The Earth is regarded as a uniform spherical body which can be regarded as a set of uniform spherical shells. The sphere, of mass *M* and moment of inertia *I,* rotates about the Z axis at a constant angular speed Ω.

Consider a test body in orbit around the Earth performing an elliptical orbit where *e* is the eccentricity and *a* is the semi-major axis and a period of *T*. The plane has an inclination (inc) relative to the equatorial XY plane of the Earth.

Again, based on equation (5).the rate of precession of the periapsis, as seen from the plane of the orbit, in radians per orbit, is

$$\Delta\phi_P = \frac{6\pi GM}{c^2a\left(1-e^2\right)} - \frac{2GI\Omega}{c^2a^3\left(1-e^2\right)^{3/2}}\cos(inc)T \quad (19)$$

The first term, the de Sitter precession, has been derived algebraically from equation (2). It agrees exactly with the generally accepted form and agrees with the measured results for the precession of the perihelion of Mercury and for the binary pulsar PSR 1913+16. However, the second term, the Lense-Thirring term, justified by numerical integration, is only half of the generally accepted value.

d) Anti-Gravity

Weight loss of a rotating ring.

Consider a ring rotating about a horizontal axis above a large body, such as the Earth. Using equation (3a) evaluate the component of the acceleration in the radial direction.

$$\mathbf{a} = -\frac{K}{r^2}\left(1+\frac{v^2}{c^2}\right)\mathbf{e}_r + \frac{2Kv_r}{r^2c^2}\mathbf{v} + \frac{2K}{r^2}\frac{v_r}{c}\left(\frac{v}{c}\right)^3\mathbf{t} \quad (3a)$$

Noting that $\boldsymbol{t} = \boldsymbol{v}/|v|$ means that (3a) may be written as

$$\mathbf{a} = -\frac{K}{r^2}\left(1+\frac{v^2}{c^2}\right)\mathbf{e_r} + \frac{2Kv_r}{r^2c^2}\left[1+\left(\frac{v}{c}\right)^2\right]\mathbf{v} \quad \text{or}$$

$$\frac{\boldsymbol{a}}{K/r^2} = -\left(1+\frac{v^2}{c^2}\right)\boldsymbol{e_r} + \left(1+\frac{v^2}{c^2}\right)\frac{2v_r}{c}\frac{\boldsymbol{v}}{c}$$

Also as *K* is *G* times the total mass therefore as the mass of the ring is very small compared to the large body its mass is negligible. So, *K* = G (mass of the large body).

Now $v_r = v\cos\theta$ also $\boldsymbol{v}\cdot\boldsymbol{e_r} = v\cos\theta$, where θ is the angle between the velocity and the radius from the large mass.

$$\frac{\boldsymbol{a}\cdot\boldsymbol{e_r}}{K/r^2} = \left(1+\frac{v^2}{c^2}\right)\left[-1+2\left(\frac{v}{c}\right)^2\cos^2\theta\right]$$

This equation is the radial acceleration divided by the magnitude of the Newtonian gravity acceleration. If *v/c* is negligible then the right hand side is -1, as expected. This equation is applicable to a point mass so for the whole ring we need to integrate for point mass *md/2π* divided by total mass of the ring *m.*

$$\int_{\mathbf{0}}^{\mathbf{2\pi}}\left(1+\frac{v^2}{c^2}\right)\left[-1+2\left(\frac{v}{c}\right)^{\mathbf{2}}cos^2\theta\right]\frac{\boldsymbol{d\theta}}{\mathbf{2\pi}}$$

Note that the integral of $cos^2\theta$ between *0 and 2π* equals *π*.

The second term in the square bracket is always positive therefore this term of the acceleration will be radially outwards for all parts of the ring. Integration for all parts of the ring leads to

$$\frac{\boldsymbol{a}\cdot\boldsymbol{e_r}}{K/r^2}=(-1+\beta^4),\text{ where }\beta={}^{v}/_{c}.$$

The value runs from -1, which is the Newtonian, to 0, which means weightless. When the spinning speed is small compared that of light then the chance of a measurable change in gravitational acceleration, or weight, is very unlikely since it depends of the forth power of *v/c*.

If negative gravity ever existed then start with a pool of particles with mixed gravity and electrical charge. Electrical charge is stronger than gravity so the positive electrical charged particles will attract the negative ones and will have zero electrical charge. The particles with gravitational positive charge will repel the negatively charges ones so the groups will separate.

IV. Discussion

Equation (4a) is easier to apply than the theory of General Relativity (GR) and therefore leaves less room for misinterpretation. That force is a secondary quantity was strongly advocated by H. R. Hertz who regarded force as “a sleeping partner”. Force is to dynamics as money is to commerce. Once force has been demoted to a defined quantity then force fields and inertia are also defined quantities, similarly for work and energy. Equation (2) is loosely modelled on the Lorentz force but this relationship is for guidance only in the same way that Maxwell used a mechanical model to form his equations. However, he abandoned the reference in his final paper on the subject once he had established that his equations predicted the then known observations.

As shown above, when the new approach is applied to two body systems it agrees with the well verified observations of the precession of the perihelion of Mercury, deflection of light passing the Sun and the definition of the Schwarzschild Radius. All agree with the results obtained from the General Theory of Relativity. The third term in equation (4a) was added as it agrees with the measurements of the Shapiro Time Delay and generates a value equal to the accepted value for the Last Stable Orbit.

The Gravity Probe B experiment testing the precession of gyroscopes in Earth orbit displays two equations, one for the geodesic term and one for the frame-dragging effect. The geodesic term does not involve the rotation of the Earth but the frame-dragging term does. The same form of equations have been generated algebraically using equation (5). The frame-dragging term is half of the published value, however, the geodesic term is about two thirds of the published value.

The de Sitter effect agrees with the accepted results of analysis whether algebraically or by numerical integration for two body systems or large non-rotating bodies. This is true whether using equation (2) or equation (5). However, for the Lense-Thirring terms there is an unresolved factor which affects the periapsis precession. The published nodal precession test on the Earth satellites LAGEOS I & II, see reference [28], appear to agree with the accepted theory. The inclination of the satellites is approximately 90^0 +/- 20^0. The reason for this is that the accepted Lense-Thirring term does not depend on the inclination but all other effects do and therefore can be cancelled out. See also references [10] and [29].

The gravitational effect on the speed of light is still discussed but apart from the Shapiro Time Delay the effect is negligible when dealing with the motion of bodies. The decrease of the speed of light grazing the Sun is only 4 parts per million. Gravitational Redshift it is sometimes regarded as a proof of GR, however, it can be derived from other fundamental theories. As shown, light passing through a gravitational field refracts in accordance with Snell’s Law.

It has proved to be impossible, so far, to find any modification to equation (4) such that it gives the generally accepted value for the Lense-Thirring effect without changing the de Sitter effect applications. The de Sitter results have been obtained by several observations but the Lense-Thirring effect is very small compared to other effects. In the LAGEOS experiments for the precession of the periapsis the Lense-Thirring effect is less than 1% of the de Sitter effect, which makes it more difficult to evaluate. The GP-B test results have recently been published, reference [27]. There are four gyroscopes, two of which have original frame-dragging results which are close to that predicted by the new theory. The geodesic results are, on average, close to those of the accepted value. Nevertheless, over a one month period two of the gyroscopes precess at a rate close to the new theory predictions.

It is widely stated that the inward spiralling of a binary star system is due to gravitational radiation. The loss of energy alone is not the cause of this effect. Energy loss can be related to outward spiralling, as is the case for the Earth Moon system. However, radiation pressure could be the cause.

When general relativity is applied to multiple body systems several authors have produced slightly different results. Some results even do not return to the

Newtonian form when the velocities are zero but only if the speed of light is taken to be infinite. This new approach does not undermine the General Theory of Relativity but because it is a simpler method it leaves less room for misinterpretation. Many of the extensions of GR are very complex mathematically, making errors more likely.

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