

Gravitomagnetism a Simpler Approach Applied to Dynamics within the Solar System

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Abstract

Galileo studied bodies falling under gravity and Tycho Brahe made extensive astronomical observations which led Kepler to formulate his three famous laws of planetary motion. All these observations were of relative motion. This led Newton to propose his theory of gravity which could just as well have been expressed in a form that does not involve the concept of force. The approach in this paper extends the Newtonian theory and the Special Theory of Relativity by including relative velocity by comparison with electromagnetic effects as shown in section 1.4 based on the Lorentz force. It is also guided from the form of measured data. This enables the non-Newtonian effects of gravity to be calculated in a simpler manner than by use of the General Theory of Relativity (GR). Application to the precession of the perihelion of Mercury and the gravitational deflection of light gives results which agree with observations and are identical to those of GR. It also gives the accepted expression for the Schwarzschild Radius. This approach could be used to determine non-Newtonian variations in the trajectories of satellites.

Index terms— gravity, relativity, lorentz force, speed of light.

1 a) Newtonian Gravity

Galileo studied bodies falling to Earth under gravity and concluded that all bodies fell with the same acceleration independent of size and material. Tycho Brahe made extensive astronomical observations which led Kepler to formulate his three famous laws of planetary motion relative to the Sun. All of these observations were of relative motion but the mass of one body was, in each case, much greater than that of the other. These led Newton to propose his theory of gravity using the concept of force and yielding an equation which gives the acceleration of a body relative to the centre of mass. He could just as well have presented it in the form

$$\frac{d^2 \mathbf{r}}{dt^2} = -\frac{GM}{r^2} \hat{\mathbf{r}} \quad (1)$$

without invoking the concept of force and only requiring one definition of mass. This means that the principle of equivalence does not appear.

That is, the acceleration of body B relative to A, in the radial direction, is proportional to the sum of their masses and inversely proportional to the square of their separation. G is the gravitational constant.

2 b) Gravitomagnetism

It is now proposed that equation (1) be extended to include the relative velocity. The axioms are.

a) It is assumed that in mass-free space light travels in straight lines. This defines a non-rotating frame of reference. b) Because all motion is relative there are no other restrictions on the frame of reference. c) Gravity propagates at the same speed as light. d) Mass, or rest mass, is simply the quantity of matter and is regarded as constant. It could be a count of the number of basic particles.

5 D) LORENTZ FORCE

The initial proposed equation is based on comparisons with electromagnetics. This equation gives results which agree with the measured results of the precession of the perihelion of Mercury and with the deflection of light grazing the Sun. Also it gives the correct definition for the Schwarzschild Radius. However, it suggests that the speed of light is constant. As a result it does not predict the Shapiro Time Delay. An extra term is then added which gives agreement with the time delay and also generates the accepted value for the Last Stable Orbit. See equations (2a), (3a), (4a) and (5a). The proposed equation is () where a = acceleration of body B relative to body A, v = the relative velocity, r = the separation and \hat{r} = the unit vector from body A to body B. Also c = speed of light, $K = G(m_A + m_B)$ and v_r is the radial component of velocity. Note that G is a constant which could be incorporated into the definition of the quantity of matter. These equations reduce to equation (1) when $v \ll c$. (4)

where θ is the angle between the velocity and the radius.

It should be noted that the velocities of the individual bodies do not appear in these equations, only the relative velocity. For two isolated bodies the relative motion is the only measurable value.

A convenient definition of force is () $P \times \dots = c v_r K r$ (5)

where $\mu = \frac{m_A m_B}{m_A + m_B}$, the reduced mass.

By definition of the centre of mass (or the centre of momentum) the total momentum is zero with reference to the centre of mass. It is now proposed that the motion of the centre of mass of two bodies is not affected by collision. From this it follows that for a group of particles the motion of the centre of mass is unaffected by internal impacts.

The relative acceleration is only radial when the relative velocity is either radial or tangential. In general the moment of momentum can be shown to be a function of the relative position. So, for an elliptic orbit it remains within bounds.

3 General inferences from equation (2).

Reverts to Newtonian form when $v \ll c$. The second term of (2) is normal to the velocity.

Moment of velocity (or moment of momentum per total quantity of matter) is shown to be a function of r .

The equivalence of inertial mass to gravitational mass does not arise.

4 c) Modified Equations

An extra term is added in the direction of the relative velocity. This will affect the speed of light but not its deflection. As the term is a function of c^4 it only has a very small effect on the motion of large bodies in Solar orbits.

The new equation is (4a)

Where v is the relative velocity and v_r is the radial component. Also $\hat{v} = v/|v|$ is the vector in the direction of the velocity. The angle between the velocity and the radius is θ . For the third term the sign of the acceleration depends only on the sign of the radial velocity. $K = G(m_A + m_B)$ and c is the speed of light in a gravity free vacuum. This equation will be considered to be the basic for Post Newtonian Gravity. Justification will come from agreement with verified experimental data. Figure ?? Equation (4a) may be re-written as

(2a) or (3a)

From which it is seen that the additional term is negligible when $(v/c)^4$ is small compared to unity. Again, noting that $v/v_t = \dots$ means that (3a) may be written as $v_e a r \dots + \dots = c v_c v_r K c v_r K c v_r K r$ (2a) or $2 2 1 3 2 2 2 2 2 t v e a \dots + \dots = c v_c v_r K c r K v c v_r K r r r$ (3a).
where $\dots = 2 2 2 2 2 2 1 2 1 c v_c r K v c v_r K r$. or $v_e a r Q c r K v c v_r K r 2 2 2 2 2 1 + \dots + \dots = 2 1 c v Q$. And () $2 2 1 3 2 2 2 2 2 2 t e v v e a P r r \dots = c v_c v_r m G m c r m G m c v_r m G m r B A B A B A \mu$ (5a) where $\mu = \frac{m_A m_B}{m_A + m_B}$, the reduced mass.

Equations (2 -5) will account for the precession of the perihelion of Mercury and will predict the observed value for the deflection of light grazing the Sun. These results were heralded as confirmation of Einstein's General Theory of Relativity. They also give the accepted value for the Schwarzschild Radius.

Equations (2a -5a) will generate the same results as mentioned above but will also give the accepted value for the Shapiro Time Delay because the speed of light is now affected by gravity. Light is slowed down by gravity which is the opposite of mass particles. The value generated for the Last Stable Orbit is the accepted value.

5 d) Lorentz force

We shall look at the standard theory of electromagnetism, but this is only to obtain some guidance as to the possible form of a gravitomagnetic theory. The electromagnetic notation is based on reference [7] for SI units.

The force on a charge q in an electromagnetic field is $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ (1.4.1)

\mathbf{B} is the magnetic field and \mathbf{E} is the electrostatic field and \mathbf{v} is the velocity of the charge. This equation defines the Lorentz force. The magnetic field, due to a length of conductor dl carrying a current i is $d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{i d\mathbf{l} \times \mathbf{r}}{r^3}$

100 r a N ? ? A B a T Global Journal of Researches in Engineering (A) Volume Xx XII Issue I V ersion I 1 / 2 1
101 1 2 1 / 2 4 e l B × = d i r o ? μ . (1.4.2) but i d q 1 1 1 1 1 v =

102 so, for point charges, we can write for the force on charge 2 due to charge 1 is $F_{evve} = \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}$
103 $\frac{2}{2} \frac{1}{2} \frac{2}{2} \frac{1}{2} \frac{1}{2} \frac{4}{4} // // // () = + \times \times ? ? \mu ?$

104 q q r q q r o (see also page 256 of ref. [7]) More detailed analysis of the electrodynamics shows that the right
105 hand side of the above equations should be multiplied by γ , but is usually omitted for small velocities.

106 The speed of light $c_0 = 1 ? \mu$ so we have $?? ? ? ? ? ? ? ? ? ? ? \times \times ? ? ? ? ? ? + = 1 / 2 1 2 1 / 2$
107 $2 1 / 2 2 1 1 / 2 4 e v v e F c c r q q o ? ? ? (1.4.3)$

108 For mass elements we assume that $?? ? ? ? ? ? ? ? ? ? ? \times \times ? ? ? ? ? ? + ? = 1 / 2 1 2 1 / 2 2 1 /$
109 $2 2 1 1 / 2 e v v e F c c r m G m ? (1.4.4)$

110 Here c , the speed of gravity waves, is assumed to be the same as that of light.

111 The form of this equation will be taken as a guide; the problem is to choose the most appropriate values for
112 the velocities. The assumptions that are made in the following development are: a) That the field generated by
113 body (1) depends on the velocity of body (1) relative to body (2) i.e. $(v_{1/2})$ and b) The force on body (2)
114 depends on the velocity of body (2) relative to the field i.e. $(v_{2/1})$. Let $v_{2/1} = v$, therefore $v_{1/2} = -v$.

115 It must be emphasised that force and force fields are inventions solely for the purpose of visualisation.

116 Substituting these values into equation $??1.4.4$ gives $?? 2/1 = ? ???? ? ? 1 ? ? 2 ? ? 2/1 2 ??? ? ? / ? ? ? ?$
117 $?? ? \times ? ? ? ? \times ? ? ? ? / ? ? ? = ??? 1/2 (1.4.5)$

118 For the Newtonian case the acceleration of body (2) relative to body (1) is $1 1 (? ?) (/ 2 1 2 1 / 2 1 / 1 1 / 2 2$
119 $1 / 2 1 2 \mu 1 2 1 2 1 2 F F F F F r r a = ? ? ? ? ? ? ? ? + = ? ? ? ? ? ? ? ? + = ? ? = ? = m m m m m m$
120 $m m ? ? ? ?$ where $2 1 2 1 m m m m + = \mu$

121 is known as the reduced mass.

122 For the relativistic case we shall define force as $() () 1 1$ where $2 2 c v c dt d ? = ? ? ? ? ? ? ? ? ? + = =$
123 $? \mu ? ? ? \mu v a v a v F 2 3$

124 Using this definition of force we obtain, using equation (1.4.5), $() ? ? ? ? ? ? ? ? ? ? ? ? ? ? \times \times ? ? ? ?$
125 $?? ? ? ? + ? = ? + 1 / 2 1 / 2 2 1 / 2 2 1 2 3) (e v v e v a v a c c r m m G c ? ? ? ? or () ? ? ? ? ? ? ? ? ? ?$
126 $?? ? ? ? ? ? ? ? ? ? ? + + ? = ? + 1 / 2 2 1 / 2 1 / 2 2 1 / 2 2 1 2) (e v v e e v a v a 2 c c c v r m m G c ?$
127 (1.4.6)

128 Where m_1 and m_2 , are the respective rest masses. Change the suffix of the unit vector to r , let $K = G(m_1$
129 $+ m_2)$ and let $r = |r_{2/1}|$ be the separation. Equation (1.4.6) now becomes $() v e v a v a 2 2 2 2 2 2 1 c r$
130 $K v c v r K c r r + ? ? ? ? ? ? + ? = ? + ? (1.4.7)$

131 Note that this equation is symmetric with regard to the two bodies.

132 The scalar product of equation (1.4.7) with v yields $2 2 2 2 1) (r K v c v r ? = ? ? ? ? ? ? + ? ? v$
133 a By definition, $() ? 2 2 2 1 1 = ? / v c$ therefore © 2022 Global Journals Global Journal of Researches in
134 Engineering (A) Volume Xx XII Issue I V ersion I 2) $(r K v r ? = ? a v ? (1.4.8)$ Substituting equation (1.4.8)
135 into equation (1.4.7) gives $() 2 1 2 2 2 2 2 r r e v v e a \times \times + ? ? ? ? ? ? ? ? ? = c r K c v r K$ as $(2) 2 1 2$
136 $2 2 2 2 v e a c r K v c v r K r r + ? ? ? ? ? ? ? ? + ? = as (3)$

137 6 II. Application to Two Mass Problem a) Polar Coordinates

138 The following development is based on the conventional treatment of the two body gravitational problem. For
139 the dynamics of bodies in Solar orbits the modified equations are not required. Here, r is the separation and e
140 r is the unit vector in the direction of body 2 as seen from body 1. \hat{e}_θ is the orientation of the unit vector with
141 respect to the 'fixed' stars and \hat{e}_ϕ is the unit vector normal to \hat{e}_r in the plane of the motion. Now $?? ? ? ? e e a$
142 $) 2 () (2 ? ? ? ? ? ? ? r r r r + + ? = and () ? ? e e v ? r r r + =$ Equation (3a) can now be expressed in
143 component form $Q c r K v c v r K r r r 2 2 2 2 2 2 1 + ? ? ? ? ? ? ? ? ? ? ? ? + ? = ? ? ? ? ? (6) () Q$
144 $c r v K v r dt d r r r ? ? ? d du h dt d d du u r ? = ? = 2 1 ? , ? ? d du h d u d h u r ? ? ? ? ? = 2 2 2 2$
145 $, and h u d du h u r h r r h ? ? ? ? 2 2 3 2 3 2 2 + = + ? = ? ?$

146 Equations (6, 7) may now be written $2 2 2 2 2 2 2 2 ? ? ? ? ? ? ? ? + = + + ? ? ? d du c K c K u h K u h u$
147 $h d du d u d ? (8)$

148 and $2 2 3 2 ? d du h u c K h u ? = ? (9)$ Since $?? h dh d dh d u h = = ? ? ? 2$ combining with equation (9)
149 gives, (10)

150 Integrating equation (10) leads to $2 0 /) (2 0 c u u K e h h ? ? = (11)$

151 Therefore, for small variations $/) (4 1 (2 0 2 0 2 c u u K h h ? ? ? (12)$

152 where the suffix 0 refers, in this case, to the position $? = ?/2$ measured from the periapsis.

153 Substituting in equation (8) for h , using equation (??2), we obtain $(4 2 2 2 0 0 2 2 0 2 2 ? ? ? ? ? ? ? ? ?$
154 $+ ? ? ? ? ? ? + ? + = + u d du h u u K c K h K u d u d ? ? (13)$

155 7 b) Precession of the Periapsis

156 Equation (??) is very much easier to apply. This equation is equally applicable to the prediction of satellite
157 trajectories. Because in these cases the relative speeds are not close to the speed of light.

158 The equation which was developed in reference [19] for calculating the precession of the perihelion of Mercury
159 per orbit is $() () 2 2 2 1 1 6 e a c m m G P ? + = ? ? (14)$

160 where a is the semi-major axis and e is the eccentricity. This generates 42.89 arcsec/century.

161 For the binary pulsar PSR 1913+16, which was discovered by Hulse and Taylor in 1974, (see reference [22]), the
 162 accepted data is that the masses of the two stars are 1.441 and 1.387 times the mass of the Sun, the semi-major
 163 axis is 1,950,100 km, the eccentricity is 0.617131 and the orbital period is 7.751939106 hr. Using equation (??4)
 164 we obtain the result 4.22 deg/yr, which is in agreement with the measured value and that predicted by General
 165 Relativity. The orbital decay, or inward spiralling, of binary pulsars is said to be simply due to energy loss caused
 166 by gravitational wave emission. This may be the case but energy loss alone will not account for the phenomenon.
 167 The loss of mass

168 8 Global

169 9 c) Moment of Momentum

170 If the additional term is negligible then it can be shown that the moment of momentum is $h = m v r$ if
 171 $v = \frac{2\pi r}{T} = \frac{2\pi r}{2\pi r/c} = c$ which depends on separation but is constant when $c = \infty$.

173 10 d) Schwarzschild Radius

174 For a constant radius $v = 0$ and $v = c$ so equation (??) or (3a) becomes $g = \frac{GM}{r^2} = \frac{GM}{(2r_s)^2} = \frac{GM}{4r_s^2}$ so if
 175 $v = c$ then $g = \frac{GM}{r^2} = \frac{GM}{(2r_s)^2} = \frac{GM}{4r_s^2}$
 176 which is known as the Schwarzschild Radius.

177 11 e) Last Stable Orbit

178 Numerical integration of equation (3a) shows that the Last Stable Orbit occurs when the radius of the orbit is
 179 3 times the Schwarzschild Radius, which is the accepted result based on General Relativity. If equation (??) is
 180 used then a value of 2.62 r_g may be calculated algebraically. However if Q is not unity, as shown in equation
 181 (3a), then equation (12), with Q included is, $\frac{d^2r}{dt^2} = -\frac{GM}{r^2} + \frac{v^2}{r} - \frac{GM}{r^3} Q$ (12a)
 182 where the suffix 0 refers to circular motion when $u = v$. Substituting in equation (8) for h , using equation
 183 (12a), equation (??3) becomes $\frac{d^2r}{dt^2} = -\frac{GM}{r^2} + \frac{v^2}{r} - \frac{GM}{r^3} Q$ (13a) If $v = v_0$ then, for small variations δr
 184 $\frac{d^2\delta r}{dt^2} = -\frac{GM}{r^3} + \frac{2v_0^2}{r^3} - \frac{3GM}{r^4} Q$ (13b)
 185 For circular motion it can be shown that $\frac{GM}{r^2} = \frac{v_0^2}{r}$ so $\frac{d^2\delta r}{dt^2} = -\frac{GM}{r^3} + \frac{2v_0^2}{r^3} - \frac{3GM}{r^4} Q$
 186 $\frac{d^2\delta r}{dt^2} = -\frac{GM}{r^3} + \frac{2GM}{r^3} - \frac{3GM}{r^4} Q$
 187 For a stable near circular orbit then $\frac{d^2\delta r}{dt^2} > 0$ so $1 - 3Q > 0$ so $Q < \frac{1}{3}$
 188 so when the factor of Q in equation (13b) is zero, algebraic manipulation of (13b) gives $r_0/r_g = 3$, which is
 189 the accepted value. It also gives a value of 0.5.

191 12 f) Deflection of Light

192 In equation (3a) terms 2 and 3 are parallel to the velocity so the component normal to the velocity is $W = \frac{GM}{r^2}$
 193 $= \frac{GM}{R_s^2}$ and R_s being the radius of the Sun the deflection is 1.75 arcsec. This value agrees with the measured
 194 value and with General Relativity. This confirms the assumption that the deflection of light grazing the Sun is
 195 small. For small variation of the speed of light assume that $v = c$. Also, for small deflections the scalar product
 196 of e_r and n can be seen from Figure (??2) to be $\frac{R_s}{r} \cdot \frac{d^2r}{dt^2} = \frac{GM}{r^2} = \frac{GM}{R_s^2}$
 197 $= \frac{GM}{R_s^2} = \frac{GM}{R_s^2}$

198 13 Therefore

199 Integration leads to $\frac{1}{r} = \frac{GM}{c^2 r^2} + \frac{1}{r}$
 200 where c is now the speed of light where r tends to infinity. Or $\frac{1}{r} = \frac{GM}{c^2 r^2} + \frac{1}{r}$ (16)
 201 Consider the case of light grazing the Sun at a radius R_s and calculate the journey time. As an approximation
 202 assume the path to be a straight line.
 203 Then, since $dx = \frac{GM}{c^2} \frac{dx}{dt} = \frac{GM}{c^2} \frac{dx}{dt}$ where $\frac{dx}{dt} = c$ also as $\frac{GM}{c^2} \ll 1$ $\frac{dx}{R_s} = \frac{GM}{c^2} \frac{dx}{R_s}$
 204 $\frac{GM}{dt} = \frac{GM}{R_s} = \frac{GM}{R_s}$

205 14 ln 2

206 Between the limits $x = 0$ to x and $t = 0$ to t we have the total time t speed change is so small the additional
 207 time is less than 1%. $t = \frac{R_s}{c} \ln 2$

208 15 h) Gravity and the refraction of light

209 The form of the extended equation is based on the known observations or deductions. The extra term is required
 210 to be in the direction of the relative velocity. Also, because the speed of light is at its maximum then passing
 211 through empty space it must reduce when moving into a gravitational field so assume that it depends on the

212 magnitude of the radial speed. From this it follows that the acceleration will be repulsive. Also, for circular
213 orbits the speed remains constant, which is true when $v_r = 0$.

214 The additional term could be $\frac{1}{2} \frac{v^2}{c^2} \frac{GM}{r}$, where k is a constant depending on application.

215
216 When applied to the Last Stable Orbit with $\frac{1}{2} \frac{v^2}{c^2} \frac{GM}{r} = 2 \frac{GM}{r}$
217 the value agrees with the generally accepted value. The constant seems reasonable, so $\frac{1}{2} \frac{v^2}{c^2} \frac{GM}{r} = 2 \frac{GM}{r}$
218 $\frac{1}{2} \frac{v^2}{c^2} \frac{GM}{r} = 2 \frac{GM}{r}$

219 This is the form shown in equation (2a) which leads to predicting the speed of light due to gravity.
220 Consider the case when speed is very close to the speed of light in a gravity free vacuum.

221 16 Equation (2a) gives the acceleration parallel to the velocity

222 $\frac{dv_{\parallel}}{dt} = \frac{GM}{r^2} \cos \theta$

223 From equation (3a) the acceleration normal to the velocity $\frac{dv_{\perp}}{dt} = \frac{GM}{r^2} \sin \theta$

224 With θ being the angle between the radius from the gravity source and the velocity $\frac{dv_{\parallel}}{dt} = \frac{GM}{r^2} \cos \theta$ and $\frac{dv_{\perp}}{dt} = \frac{GM}{r^2} \sin \theta$ therefore $\frac{dv_{\parallel}}{dt} = \frac{GM}{r^2} \cos \theta$ and $\frac{dv_{\perp}}{dt} = \frac{GM}{r^2} \sin \theta$ So $\frac{dv_{\parallel}}{dt} = \frac{GM}{r^2} \tan \theta$ (a)

225
226
227 The refractive index n is defined as speed of light in a gravity free vacuum / speed of light in a transparent
228 media.

229 For light passing through different media Snell's Law states that $n_1 \sin \theta_1 = n_2 \sin \theta_2$
230 $n_1 \sin \theta_1 = n_2 \sin \theta_2$, or $n_1 / \sin \theta_1 = n_2 / \sin \theta_2 = \text{constant}$

231 17 Thus

232 $\frac{dv_{\parallel}}{dt} = \frac{GM}{r^2} \tan \theta$ as equation (a). This is applicable to the passage of light through the Earth's atmosphere.
233 Hence light passing through a strong gravity field will be affected in the same way as light passing through the
234 atmosphere.

235 This result gives more confirmation of the applicability of the basic equation of the paper.

236 18 III. Gravitomagnetism Applied to

237 Rotating Bodies a) Basic Equations When equation (1) is applied to two body systems the equation generated
238 is identical to the de Sitter form and agrees with the measurement of precession of the perihelion of Mercury
239 and of the Binary Pulsar PSR 1913+16. The equation is equally applicable if one spherical body is large and
240 non-rotating. Note that the additional term, which is a function of $\frac{v^2}{c^2}$, is negligible for Solar dynamics.

241 $\frac{d^2 \mathbf{r}}{dt^2} = -\frac{GM}{r^3} \mathbf{r} + \frac{4G}{3c^3} \frac{d^3 \mathbf{r}}{dt^3} + \frac{2G}{3c^3} \frac{d^2 \mathbf{v}}{dt^2} + \frac{G}{3c^3} \frac{d \mathbf{v}}{dt} \times \mathbf{v} + \frac{G}{3c^3} \mathbf{v} \times \frac{d \mathbf{v}}{dt}$ (2rpt) (18a)

242 where c is the speed of light, a is relative acceleration, v is relative velocity and r is relative position. Also \mathbf{e}_r
243 is the unit vector from body A to body B.

244 where G is the gravitational constant

245 The calculations are made easier for multi-body systems by the use of a defined force as shown in equation (1).
246 $\mathbf{F} = -\frac{GM}{r^2} \mathbf{e}_r + \frac{4G}{3c^3} \frac{d^3 \mathbf{r}}{dt^3} + \frac{2G}{3c^3} \frac{d^2 \mathbf{v}}{dt^2} + \frac{G}{3c^3} \frac{d \mathbf{v}}{dt} \times \mathbf{v} + \frac{G}{3c^3} \mathbf{v} \times \frac{d \mathbf{v}}{dt}$

247 μ is the reduced mass $\mu = \frac{m_A m_B}{m_A + m_B}$ / b) Gravity Probe B

248
249 Gravity Probe B is the study of the precession of a gyroscope in a polar orbit about the Earth. The spin axis
250 of the gyroscopes is perpendicular to the spin axis of the Earth. After over 3 decades of study and design at
251 Stanford University the final results of the experiment were published in 2011 [27]. $K = G(m_A + m_B)$

252 When the new theory is applied to Gravity Probe B the following equations are derived algebraically using
253 equation (5). The modified equation is not required because the relative velocities are not close to the speed of
254 light.

255 19 c) Precession of the Periapsis of a small body orbiting a large 256 rotating mass

257 This problem is similar to the discussion of the precession of the perihelion of Mercury except that now the
258 rotation of the Earth is taken into account. For Mercury the rotation of the Sun has negligible effect. The
259 LAGEOS satellites yield results for the so called frame dragging, or Lense Thirring effect, which results from the
260 rotation of the Earth.

261 The Earth is regarded as a uniform spherical body which can be regarded as a set of uniform spherical shells.
262 The sphere, of mass M and moment of inertia I , rotates about the Z axis at a constant angular speed ω .

263 Consider a test body in orbit around the Earth performing an elliptical orbit where e is the eccentricity and a
264 is the semi-major axis and a period of T . The plane has an inclination (inc) relative to the equatorial XY plane
265 of the Earth.

266 Again, based on equation (5), the rate of precession of the periapsis, as seen from the plane of the orbit, in
267 radians per orbit, is $\frac{d\omega_p}{dt} = \frac{2G}{3c^3} \frac{I \omega}{a^3} \cos(\text{inc}) \frac{e}{1 - e^2} \frac{GM}{a^3} \cos^2(\text{inc})$ (19)

327 from other fundamental theories. As shown, light passing through a gravitational field refracts in accordance
328 with Snell's Law.

329 It has proved to be impossible, so far, to find any modification to equation (??) such that it gives the generally
330 accepted value for the Lense-Thirring effect without changing the de Sitter effect applications. The de Sitter
331 results have been obtained by several observations but the Lense-Thirring effect is very small compared to other
332 effects. In the LAGEOS experiments for the precession of the periapsis the Lense-Thirring effect is less than
333 1% of the de Sitter effect, which makes it more difficult to evaluate. The GP-B test results have recently been
334 published, reference [27]. There are four gyroscopes, two of which have original framedragging results which are
335 close to that predicted by the new theory. The geodesic results are, on average, close to those of the accepted
336 value. Nevertheless, over a one month period two of the gyroscopes precess at a rate close to the new theory
337 predictions.

338 It is widely stated that the inward spiralling of a binary star system is due to gravitational radiation. The loss
339 of energy alone is not the cause of this effect. Energy loss can be related to outward spiralling, as is the case for
340 the Earth Moon system. However, radiation pressure could be the cause.

341 When general relativity is applied to multiple body systems several authors have produced slightly different
342 results. Some results even do not return to the ^{1 2 3}

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343 Newtonian form when the velocities are zero but only if the speed of light is taken to be infinite. This new
344 approach does not undermine the General Theory of Relativity but because it is a simpler method it leaves less
345 room for misinterpretation. Many of the extensions of GR are very complex mathematically, making errors more
346 likely.

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