

Geometrical Characteristics of the Surfaces on Trapezium-Curved Plans

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Abstract

At the article "Orthogonal curved coordinate system and forming the surfaces on trapezium plans" [1] there is given the method of forming of the orthogonal curved coordinate system at the plane and the methodic of forming of the new forms of the surfaces on the given trapezium curved plans. At the article there are given many pictures of the trapezium curved plans on the base of the different directrix curves and the figures of the surfaces on the given trapezium curved plans and the combinations of the surfaces with conjugated different directrix. The given methodic of the forming of the surfaces may be used in architecture and building for development of thin-walled space constructions in urban and industry building. But for calculation of stress-strain state of thin shell usually there are used the geometrical characteristics of the middle surface of the shell. At this state on the base of the vector equation of the surfaces on the trapezium curved plans there are received the formulas of the coefficients of the fundamental forms and of the curvatures of the surfaces. There are given the examples of the surfaces and there are received the formulas of the coefficients of the fundamental forms and curvatures of the surfaces with concrete directrix and functions of vertical coordinates of the surface. Keywords: plane curve, orthogonal curved coordinate system at the plane, trapezium-curved plan, vector equation of the surface at the trapezium curved plans, geometrical coefficients of the fundamental forms of the surface, curvatures of the surface.

Index terms— plane curve, orthogonal curved coordinate system at the plane, trapezium-curved plan,

1 Introduction

quation of the surface on the trapezium-curved plan, coefficients of quadratic forms of the surface.

The orthogonal curved system of coordinates at the plane there is formed by the system of the straight lines orthogonal to the plane base curve $(\gamma)(\gamma)(\gamma)jiruyxu + = 0$ (Fig. 1).

So, the curved-orthogonal coordinates there are organized by the system of the equidistant curves parallel to the based curve and the system of the straight lines orthogonal to the system of the equidistant curves.

The equation of the curved coordinate system $(\gamma)(\gamma)vuv, u?? = 0rr, (1)$

$?$ is a normal to the base curve, v is the coordinate of the generating curves along the normal to the base curve.

The positive direction of the coordinate of straight lines there is taken to the side of the convexity of the base curve, because in the direction of the concavity the straight lines may to cross.

Assigning some function of vertical coordinate $z(u,v)$, we receive the vector equation of the surface $(\gamma)v, u? on the base curved-orthogonal coordinate system at the plane $(\gamma)(\gamma)(\gamma)krv, uzvu v, u + ? = ? 0? . (2)$$

2 COEFFICIENTS OF THE FUNDAMENTAL FORMS:

For deduction the formulas of the coefficients it's necessary to receive the derivatives of the vector equation and to use the formulas of the classic differential geometry [2];

$$\begin{aligned} \dot{s} &= 0; \quad \dot{r} = s; \quad \dot{s} \cdot k \cdot s = 0; \\ k \cdot s &= -s \cdot k. \end{aligned} \quad (3)$$

Then receive:

() The coefficients of the first fundamental forms: $k u s u z v k s + + ? = ? ? ; k v v z + ? = ? ? . (4)() ()$
 $2 2 u s u z v k s E + + ? = ? ? ; () 2 1 v v v z G + = ? ? ; () v u v v z z F = = ? ? . (5)$

The unit normal vector of the surface $(\mathbf{u}, \mathbf{v}, \mathbf{z})$ is $\mathbf{n} = \frac{1}{\sqrt{1 + u^2 + v^2}}(-u, -v, 1)$. The discriminant of the surface is $\Delta = 4(1 + u^2 + v^2)^3$.

The second derivatives of the equation: () () k u u s s s u u z k v k s k v s + + ? + ? + ? = ? ? ? ; k u v s u v z k + = ? ? ; k v v v v z = ? . (7)

The coefficients of the second fundamental form: $(\)(\)(\)(\)? + ? ? + ? + ? + ? ? = u u s v s s u s u u z v k s z k v k s z k v s L 2 m ? ; (\)(\)? + ? = v v s v v z v k s N m ? ; (\)(\)? + ? ? ? = u v s s u u v z v k s k z M m ? .$ (8)

The curvatures of the surface: $(\)(\)(\)(\)[\]222u s u u s v s s u s u z v k s z v k s z k v k s z k v s E L k +$
 $+ ? ? + ? ? + ? + ? ? = ; (\)(\)21 v v v s v z z v k s G N k + ? + ? = ; (\)(\)\2221 v u s$
 $v v s s u u v z z v k s z v k s k z E G M k + + + ? ? + ? ? ? = . \text{ (9)}$

The coordinate system of the investigated surfaces isn't orthogonal and isn't conjugated in common, as the coefficients 0 $\neq$ M , F and the coordinate system of the surfaces isn't the lines of principle curvatures of the surface.

The investigated system of the surfaces is related to the class of normal surfaces [4][5][6] -the surfaces with the system of plane coordinate lines (generating curves) at the normal plane of the directrix curve. At the works [4,5] there was shown, that only for two kinds of normal surfaces the system of generating curves is the system of principle curvatures: 1-surfaces of rotation -directrix is a straight line, generating lines are circles; 2 -normal surfaces with the system of nonchanged generating curve. This type of surfaces is related to the Monge's surfaces [5.[7][8][9][10].

If $z=z(v)$ -the generating curve doesn't change during moving in normal plane of the directrix ($z_u = z_{uu} = 0$), there will be received the Monge's surfaces: $()^2 s v k s E + ? = ; ()^2 1 v v v z G + = ? ? ; 0 = F ; ()^2 1 v s z v k s + + ? = ? ; ()^2 1 v v s s z z k v k s L + + ? = ; 2 1 v v v z z N + = ; 0 = M ; ()^2 1 1 v s v s z v k s z k k + + ? = = ; ()^2 3 2 2 1 / v v v z z k + = .$ **(10)**

The coordinate system of the Monge's surfaces is lines of principle curvatures of the surface.

If the generating curve will be a straight line $\varphi = \text{vtg } z$ (φ is an angle of slope of generating strait line to the plane of base curve), then there will be received the torus surface of constant slope [10][11][12]. Then we'll receive: $\varphi = \text{tg } z \text{ v}$; $0 = \text{vv } z$; $\varphi = + 2 \text{ } 2 \text{ } 1 \text{ } 1 \text{ } \cos z \text{ v}$; $\varphi + \varphi = \varphi \cos \text{vk s s}$; $(\) \text{ } 2 \text{ s vk s E} + \varphi =$; $\varphi = 2 \text{ } 1 \text{ } \cos \text{G}$; $(\) \text{ } \varphi + \varphi = \sin k \text{ vk s L s s}$; $\text{N}=0$; $\text{s v s vk s sin z k k} + \varphi \varphi == 1$; $\text{k } 2=0$. **(11)**

If the angle of slope of the generating strait line $\varphi=0$, $z=0$, then will be received the trapezium-curved plate: (12)

The geometric characteristics of surfaces with concrete directrix and generating curves will be received on the base of the common formulas of coefficient of the surfaces on trapezium-curved plans (3)(4)(5)(6)(7)(8)(9)(10)(11). On the fig. 2 there is shown the surface with ellipse as directrix and generating sine with linier change of its amplitude: $(\)(\)(\)j i r u Y u X u + = 0$; $(\)u \cos a u X =$; $(\)u \sin b u Y =$; $(\)d v \sin u c v , u z ? ? = 2$; $? \div = 2 0 u : d v \div = 0$

, c is maximum amplitude of sine curve; d is the width of trapezium curved plan.

Determine parameters of the directrix ellipse and derivatives of generative curve: $\varphi = \varphi + \varphi = \varphi$ a Y X s 2 2 ; u cos u sin 2 2 2 $\varphi + \varphi$; a b = φ ; $\varphi \varphi = \varphi \varphi$ 2 a s ; () u sin 2 1 2 $\varphi + \varphi \varphi$; 2 3 3 / a s Y X Y X k $\varphi \varphi = \varphi \varphi \varphi \varphi \varphi \varphi \varphi =$; $\varphi \varphi = \varphi = k s k s$; () 2 2 2 2 1 $\varphi \varphi + \varphi \varphi = \varphi \varphi \varphi \varphi \varphi = \varphi$ u sin k s ; d v sin c z u $\varphi \varphi = 2$: 0 = uu z ; d v cos d c z uv $\varphi = 2$; d v cos u d c z v $\varphi = 2$; d v sin u d c z vv $\varphi \varphi \varphi = 2$.

2 Coefficients of the fundamental forms:

$$\begin{aligned} & v \sin c v a E ? ? + ? ? ? ? ? ? ? ? + ? = 2 2 2 2 4 ; d v \cos u d c G ? + = 2 2 2 2 4 1 ; d v \sin u d c F ? ? \\ & = 2 8 2 ; d v \sin c d v \cos u d c v a ? ? + ? ? ? ? ? ? ? ? + ? ? ? ? ? ? ? ? + ? = ? 2 2 2 2 2 2 2 2 4 4 \\ & 1 - () ? ? \mu ? ? ? ? ? ? ? ? ? ? + ? + ? ? ? ? ? ? ? ? ? ? + ? ? + = 2 2 2 2 1 2 2 2 d v \cos u d v a d \\ & v \sin u \sin v u \sin a c L s u u m ? ? ; ? ? ? ? ? ? ? ? ? ? + ? ? ? = d v \sin u v a d c N 2 2 ; ? ? ? ? ? ? ? ? \\ & ? ? ? ? ? ? + ? + ? ? ? ? ? ? = 2 1 1 d v \cos v a d d v \sin c M . \quad (12) \end{aligned}$$

On the fig. 2 there is shown the Monge's surface with evolvent of the circle as directrix:

$$\begin{aligned} & \left(\right) \left(\right) u \sin u u \cos a u X + = ; \left(\right) \left(\right) u \cos u u \sin a u Y ? = d v \cos d b G v v 2 2 1 + = ? ? ; \left(\right) d v \\ & \cos d b v a u 2 2 1 + + = ? ; \left(\right) d v \cos d b d d v \cos v a u b L 2 2 1 + + = ; d v \cos d b d d v \sin b N 2 2 \\ & 2 1 + ? = ; \left(\right) d v \cos d b v a u d d v \cos b k 2 2 1 1 + + == ; 2 3 2 2 2 2 1 / d v \cos d b d d v \sin b k ? ? \\ & ? ? ? ? ? + ? = . \quad (13) \end{aligned}$$
[illegible]

The coefficients of fundamental forms and the curvatures of the surface: () 2 3 2 ? ? ? ? ? ? ? ? + = v u R

$a E ; ? = 2 1 \cos G ; () ? ? ? ? ? ? ? ? + = \sin v u R a L 3 2 3 ; () () v u R a \sin u R k 3 2 3 1 + ? = ; 0 2$
 $= k . (14)$
 Let us consider the linier surfaces which aren't surfaces of constant slope. On fig. ?? there are shone wavy
 linear surfaces with different directrix curves. $? + + ? ? ? ? ? ? ? + ? ? ? ? ? ? + = ? () () () ? ? ? ? ?$
 $? ? ? ? ? ? ? ? + ? + ? ? ? ? ? ? + = 1 2 2 2 2 2 2$

3 Conclusion

The surfaces on the trapezium curved plans are formed by the moving of some generating curve at the normal
 plane of directrix curve. The generative curve may change its form when it is moving along the directrix, but has
 the constant wide of the plan. At the article there is received the vector equation of the surfaces on trapezium
 curved plans. On the base of the vector equation there are received the coefficients of the fundamental forms
 and the curvatures of the surfaces. If the function of vertical coordinates depends on coordinate parameter of
 the directrix (the form of generating curve changes at moving along the directrix), then the coordinate system of
 the surface isn't orthogonal and isn't conjugated. If along directrix there moving unchangeable curve then the
 coordinate lines of the surface are lines of principle curvatures and this type of surfaces is applied to the class
 of Monge's surfaces [5],[7][8][9]. On the base of common formulas there are received the formulas of geometric
 characteristics of the Monge's surfaces. If at the normal plane of the directrix there is moving a straight line with
 the constant slope to the directrix plane, then there will be received the torus surface of constant slope. Those
 type of surfaces belong to the class of Monge's surfaces as well.
 On the base of common formulas of investigated class there are received the formulas of the surfaces and their
 geometric characteristics of the surfaces with concrete directrix and generating curves, as for surfaces of common
 type and so for Monge's and surfaces of constant slope. The using of common formulas made more simple the
 proses for receiving formulas for concrete surfaces. For every investigated surface there are given their figures.
 Also there was investigated the type of wavy surfaces formed by the generating straight line which make
 oscillations at the normal plane of directrix. There are received the formulas of the geometric characteristics of
 this type of surfaces and given the figures of wavy line surfaces with some directrix lines.
 The figures of the surfaces were made with using of vector equations of the surfaces in the "MathCad" system
 [5,13]

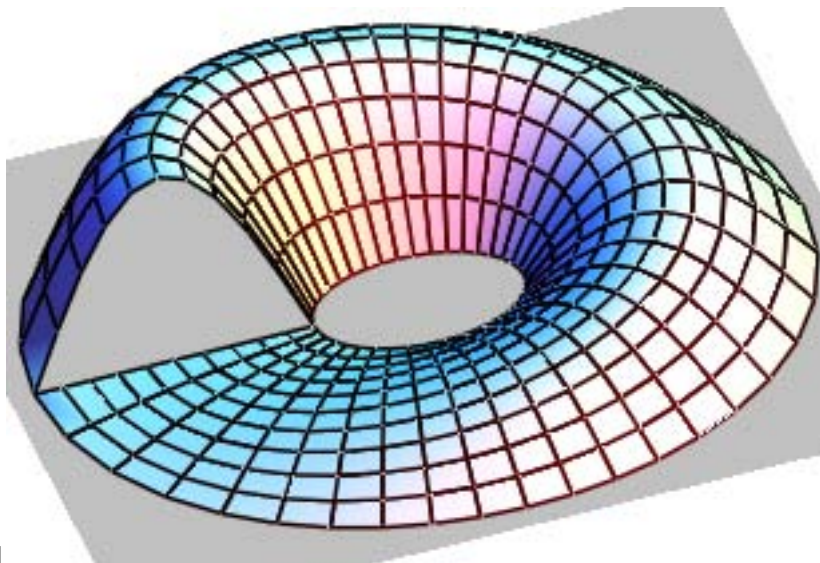
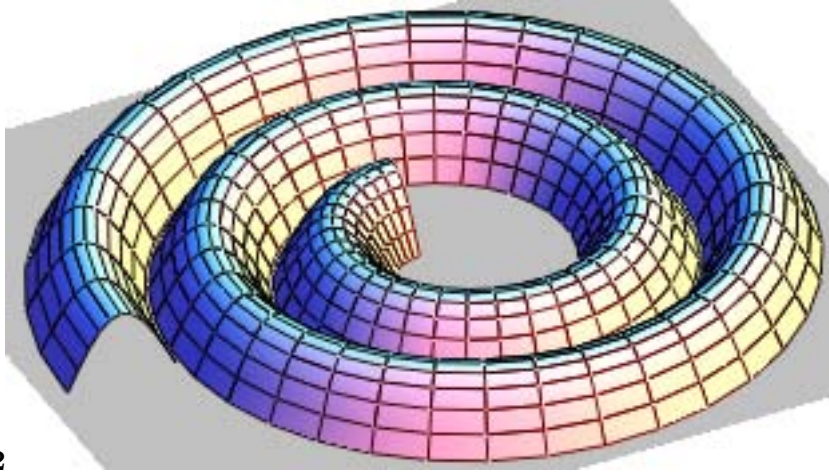


Figure 1: Fig. 1 :

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Figure 2: Fig. 2 :

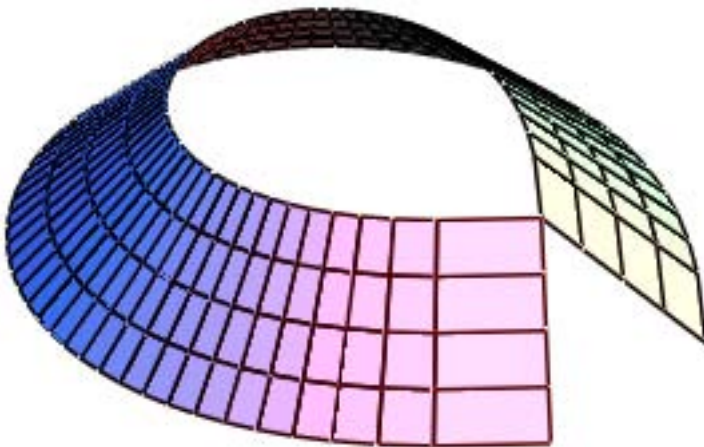


Figure 3:

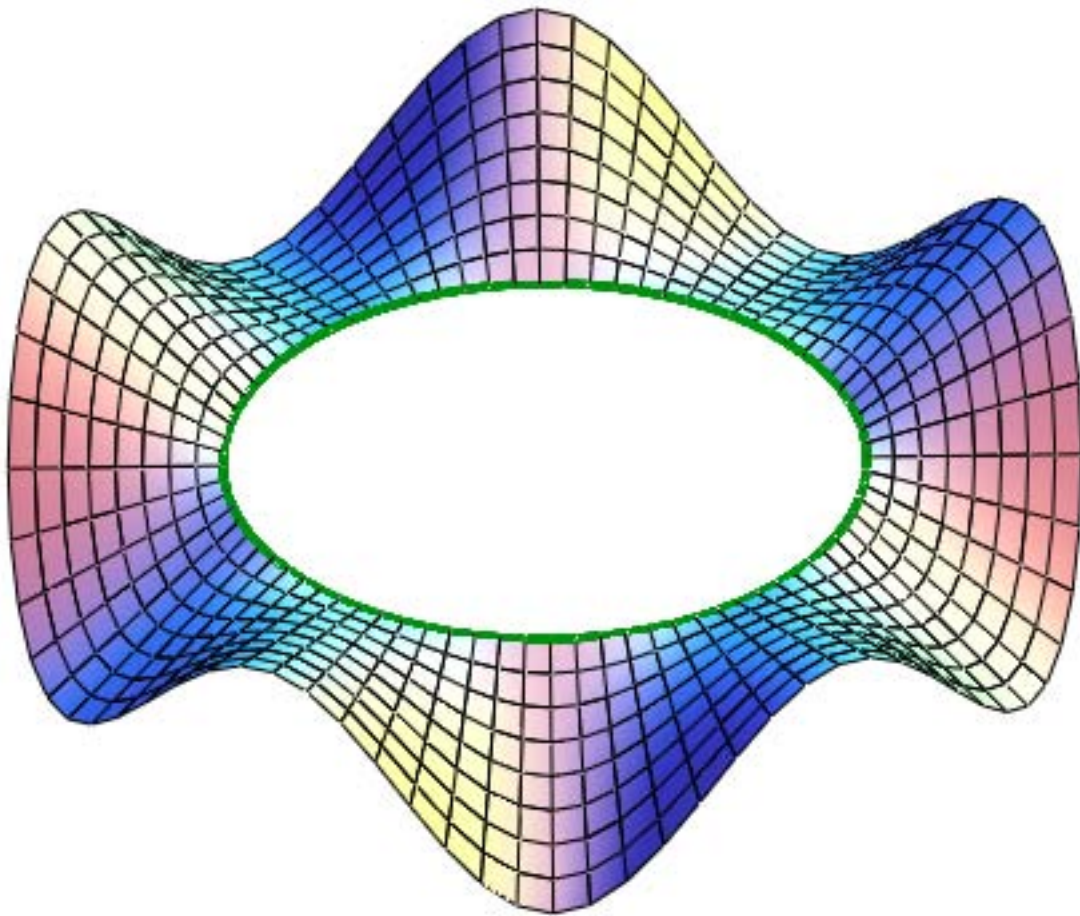


Figure 4: b

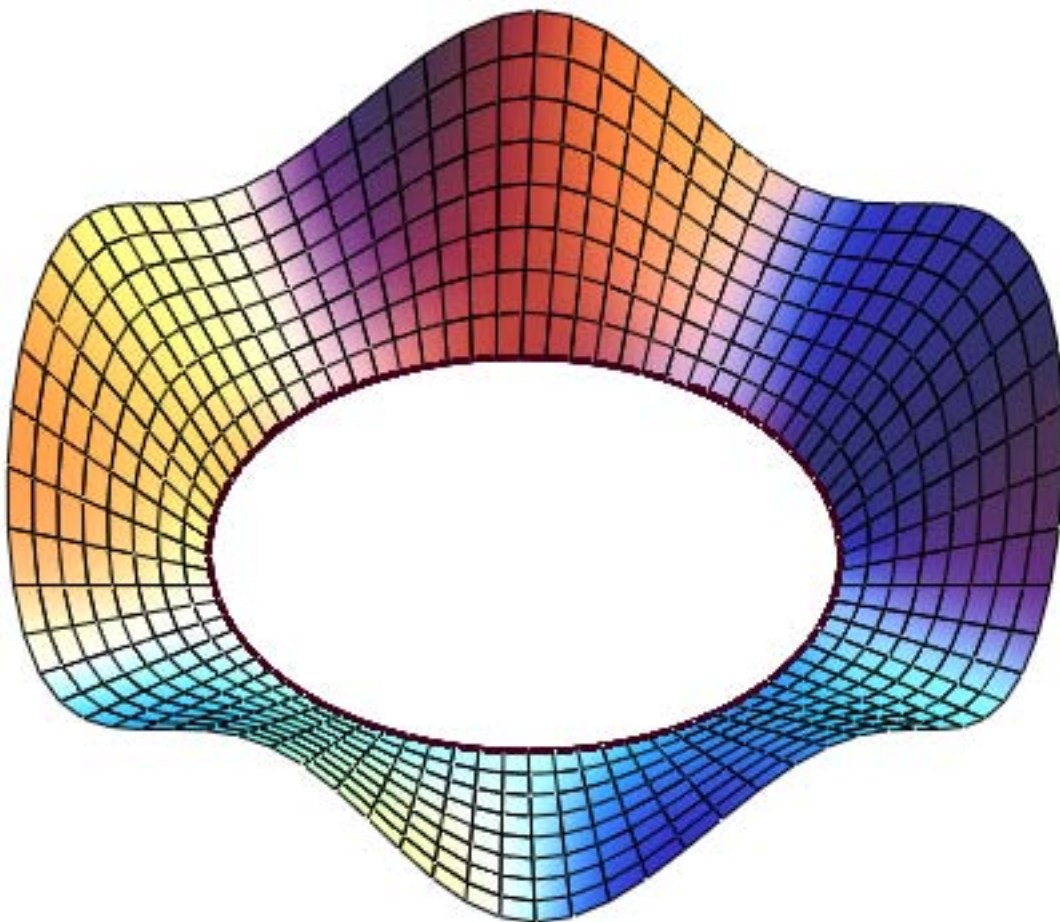
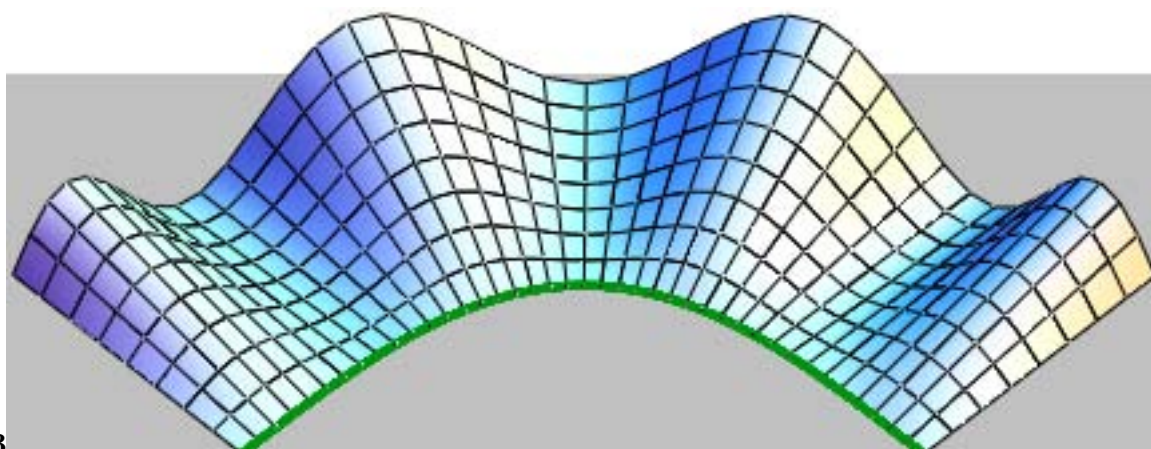


Figure 5: .



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Figure 6: Fig. 3 :

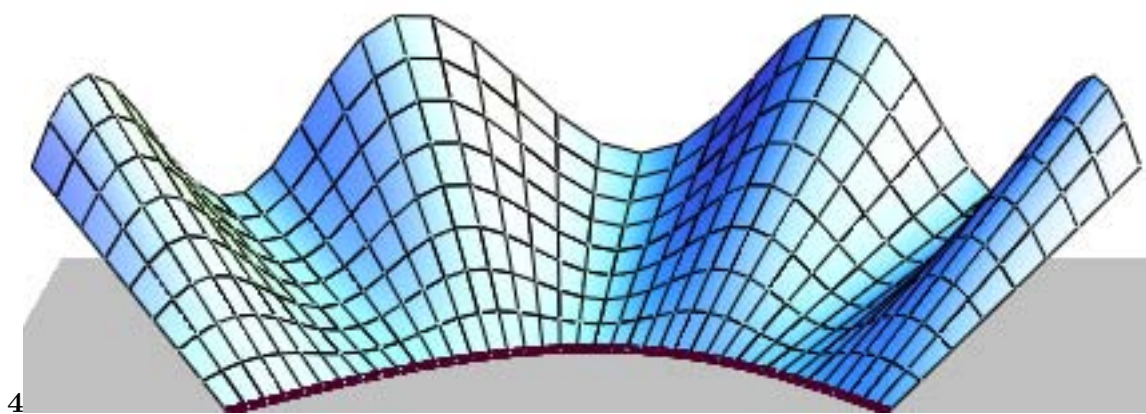


Figure 7: Fig. 4 :

