Global Journal of Researches in Engineering: A
Mechanical and Mechanics Engineering
Volume 20 Issue 1 Version 1.0 Year 2020
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4596 & Print ISSN: 0975-5861

Wheel Tooth Profiles of Hydraulic Machines and Mechanical Gears: Traditions and Innovations

By D.F. Baldenko & F.D. Baldenko
Russian State University

Abstract- An analysis of the advantages and disadvantages of the most common types of gears in practice, technical requirements and technological capabilities in their design and manufacture at the present stage of development shows that developments in the field of creating gears with unconventional tooth profiles are relevant. The technical solution proposed in the article refers to cylindrical gears of external and internal gearing, the shape of the teeth of the wheels of which is formed as the envelope of the original contour of the rack, and the number of teeth are assigned depending on the purpose of the mechanism, the required gear ratio and the diametric dimensions. Such mechanisms are used in various branches of mechanical engineering in the form of gear wheels of gearboxes, winches, planetary and wave gears, and also as working bodies of pumps, hydraulic motors, compressors and internal combustion engines with straight and helical teeth.

Keywords: cycloidal gearing, gear, harmonic profile, involute gearing, main circle, number of teeth, rack, running-in, tooth profile.

GJRE-A Classification: FOR Code: 091399p

WHEELTOOTHPROFILESOFHYDRAULICMACHINESANDMECHANICALGEARSTRADITIONSANDINNOVATIONS

Strictly as per the compliance and regulations of:

Wheel Tooth Profiles of Hydraulic Machines and Mechanical Gears: Traditions and Innovations

D.F. Baldenko [α] & F.D. Baldenko [σ]

Abstract- An analysis of the advantages and disadvantages of the most common types of gears in practice, technical requirements and technological capabilities in their design and manufacture at the present stage of development shows that developments in the field of creating gears with unconventional tooth profiles are relevant. The technical solution proposed in the article refers to cylindrical gears of external and internal gearing, the shape of the teeth of the wheels of which is formed as the envelope of the original contour of the rack, and the number of teeth are assigned depending on the purpose of the mechanism, the required gear ratio and the diametric dimensions. Such mechanisms are used in various branches of mechanical engineering in the form of gear wheels of gearboxes, winches, planetary and wave gears, and also as working bodies of pumps, hydraulic motors, compressors and internal combustion engines with straight and helical teeth.

Keywords: cycloidal gearing, gear, harmonic profile, involute gearing, main circle, number of teeth, rack, running-in, tooth profile.

I. Introduction

Gears as one of the main structural elements of various mechanisms and machines are known since ancient times, and gearing theory is one of the fundamental applied branches of physical and mathematical science and general engineering. The modern stage of the development of the theory and practice of gearing, based on the use of innovative computer technologies and three-dimensional modeling, allows us to solve new problems, including the development of non-traditional types of gears with optimized geometrical parameters of the teeth. To date, three main types of cylindrical gears of external and internal gearing, differing in the profile of the teeth and the technology of their manufacture, have received wide industrial application [1-3]:

- Involute;
- Cycloidal (including pinworms);
- With circular teeth (Novikov gearing).

Among the variety of gear mechanisms from a technological point of view, it is possible to distinguish involute and cycloidal gears, the face profile of the wheels of which is formed by the method of running-in the tool rack around the main circle.

Profiling from the rack is the most common and universal way to build a profile of the teeth, and in the case of a displacement of the rack relative to the main circumference, this allows you to correct the shape of the teeth, achieving optimal gearing for various operating conditions.

Wheels with an involute tooth profile - the most common in mechanical engineering - have significant technological and operational advantages compared to other types of gears [2]: a simple tool for cutting teeth; a constant position of the general normal to the profiles of the mating teeth, which reduces dynamic loads; insensitivity to a small change in center distance. At present, involute gearing is the basis for the development of the majority of transmission devices for mechanisms and machines of all industries and agriculture, and the theory is most fully developed in relation to this type of gearing [1].

The classical involute tooth profile of mechanical gear wheels and working bodies of volumetric pumps is formed as the envelope of the initial contour of the gear rack, which is an isosceles trapezoid with an angle of 20° and is generally displaced relative to the generating straight and main circle [2]. Currently, the vast majority of cylindrical gears for various purposes have a profile of the side faces of the teeth, built on the basis of standard or partially modified involute engagement. .

At the same time, despite the large number of positive operational and technological qualities of involute tooth profiles, one of their significant drawbacks is the tendency to undercuts and self-intersections with small numbers of gear teeth (as is known, for an involute profile without displacement $z_{min} = 17$, Fig. 1), which reduces the efficiency and scope of involute gears, increases their weight and size and cost indicators, and also limits the ability to design the working bodies of rotary machines. In addition, the convex contact of the surfaces in the case of external gearing of the wheels increases the contact stress and reduces the bearing capacity of the transmission.

Author α: Russian Scientific Research Institute of Drilling Techniques. e-mail: fbaldenko@mail.ru
Author σ: Russian State University of Oil and Gas named after Gubkin.

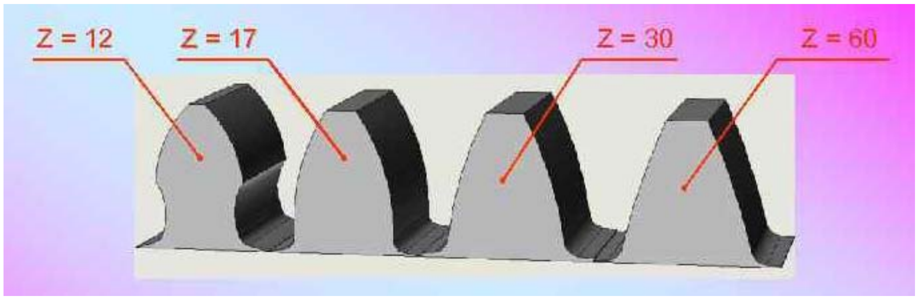


Fig. 1: The influence of the number of teeth on the shape of the profile of an involute tooth

Wheels with a cycloidal tooth profile are characterized by a smooth contour, the possibility of gearing with a minimum number of teeth (including single and double gears) without the risk of trimming the profile and creating gears of internal gearing of the wheels with the number of teeth differing by one (which is used in planetary gearboxes and rotary hydraulic working bodies machines, compressors and ICE). The cycloidal profile has the widest range of tooth shape changes, which allows the development of mechanical gears with almost any necessary combination of tooth numbers and the curvature of the mating wheels. When using cycloidal gearing, it becomes possible to create low-tooth profiles of the same sign of curvature (round, oval, triangular with curved faces), which is required for the development of highly efficient rotary machines.

The initial cycloidal profile related to the outer or inner element of the gear pair is formed from a shortened hypo- and epicycloid; their equidistant as well as the envelope of a cycloidal rack [4].

An extracentroid profile formed from a shortened cycloidal curve (when the producing point is not located on a rolling circle) is most widely used in mechanical engineering.

An equidistant profile, which is characterized by a smooth contour and a continuous change in the position of the contact point on the mating surfaces of the stator and rotor (a combination of sliding and rolling), has found widespread application in working bodies of single-screw pumps and hydraulic motors (Fig. 2).

According to the ratio of the number of teeth (lobes), cycloidal mechanisms are distinguished with arbitrary numbers of teeth of the wheels and with the numbers of teeth differing by one

Option $|z_1 - z_2| = 1$ for internal gearing is used in two cases: 1) in planetary gearboxes, which ensures maximum gear ratio and load distribution at the same time over several pairs of mating gear teeth; 2) in the working bodies of rotary volumetric machines in order to form isolated working chambers, hermetically separated from the entrance and exit.

According to the arrangement of profiles with internal gearing, the number of teeth of which differ by one, schemes can be applied in which a wheel with a smaller number of teeth is an internal or external element.

The choice of one of these schemes for a given initial profile (epi- or hypo) is determined by the design requirements of the designed machine and depends on the method of constructing the conjugate profile (as an internal or external envelope) and the ratio of the radii of the initial circles (centroid) of the initial and conjugate profiles.

An option when a larger number of teeth falls on the stator profile is characteristic of the working bodies screw pumps (kinematic ratio 1:2) and hydraulic motors (3:4 ... 9:10) with equidistant engagement (Fig. 2), while the option where the stator has one less number of teeth than the inside its rotor belongs to the well-known schemes of internal combustion engines and compressors with eccentric non-equidistant gearing and a kinematic ratio of 3:2 and 2:1 (Fig. 3).

The relative movements of the conjugate cycloidal tooth profiles distinguish between mechanisms: with variable touch conditions (when the contact point moves along both the original and the conjugate profile), with partially variable contact conditions along only one of the profiles (for example, along the stator), while the mating profile is engaged at one constant point.

Variable contact conditions of the profiles are characteristic for the engagement of equidistant cycloidal mechanisms (when both rolling speeds along the conjugate profiles are not equal to zero). The constant contact conditions of one of the profiles relate to the case of engagement of profiles formed directly from hypo- or epicycloids without equidistant procedures.

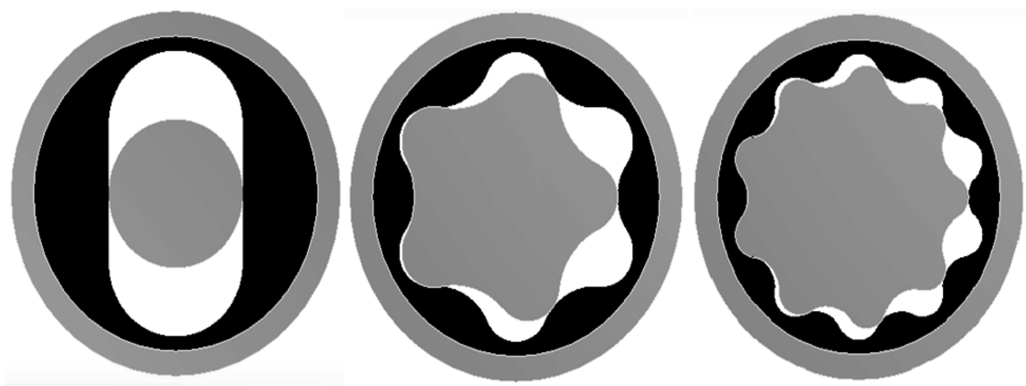
Provided that one of the profiles is constantly touched, it is possible to create a local seal assembly (moving at the profile point of the inner element) or fixed (at the profile point of the outer element), depending on the mechanism scheme, respectively, on the surface of the rotor or stator.

Examples of cycloidal mechanisms with constant conditions of contact of one of the profiles (Fig. 3) are the Wankel engine and rotary compressor (constant contact of the tips of the protrusions of the teeth of the rotor), as well as a gas engine with constant contact of the tips of the protrusions of the teeth of the three-lobe stator.

The cycloidal tooth profile has been widely used in the oil and gas industry in the development of single-screw hydraulic machines with an elastic stator lining (pumps for oil production and downhole hydraulic motors for drilling wells), the working body of which is a screw gerotor mechanism with *internal* equidistant cycloidal gearing (Fig. 2).

For multi-lobe screw gerotor mechanisms ($z_2 \geq 2$), the conjugated end profiles of the helical surfaces of the stator and rotor are formed by the method of rolling in a cycloidal rack, the initial contour of which was proposed at the Perm branch of the All-Union Scientific Research Institute of Drilling Techniques [5]. In the general case, for a given contour diameter of the gerotor mechanism, the shape of the cycloidal profile is determined by a combination of five dimensionless geometric parameters (gear ratio, gearing coefficients (epi or hypo), eccentricity, tooth shape, and rack displacement), which complicates the manufacturing technology (due to the need to perform equidistant procedures for the contour of the rack) and the choice of the optimal shape of the profiles described by complex mathematical expressions, depending on the combination of the five above dimensionless parameters.

External cycloidal gearing can be used in mechanical gears and working bodies of pumps and compressors, since it allows to obtain a large gear ratio in one stage due to the absence of restrictions on the number of teeth up to before using a single-tooth gear (the cross-section of which is a circle with a displaced center as in the classical and tested Moineau pump scheme, Fig. 2a), which was used by Russian engineers (Tomsk University) in eccentric-cycloidal gearing [6].

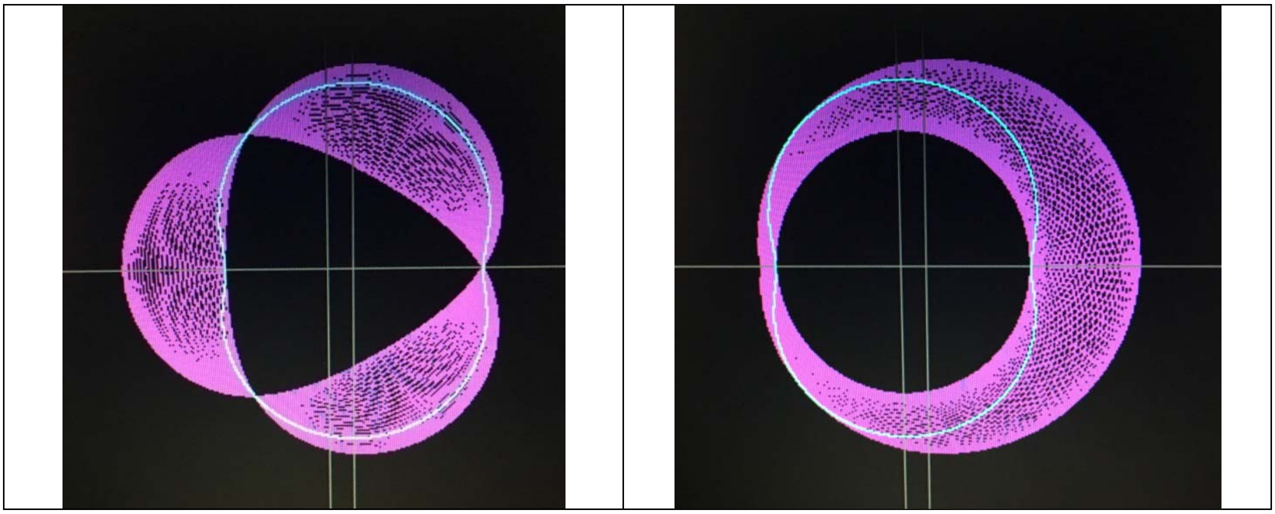


a *b* *c*

Fig. 2: Cross-sections of single-screw hydraulic machines with equidistant hypocycloidal profile and different kinematic relation [4]:

a – 1:2; *b* – 5:6; *c* – 9:10

Cycloidal gearing in various oil field mechanisms (cylindrical gears of draw works and mud pumps, top drive systems of drilling rigs and sucker-rod pumps, beam pumping units; rack and pinion lifts; planetary gears) can provide the following operational advantages in the future: reduced number the teeth of the driving gear ($z = 1 \div 6$) and, as a result, the minimum weight and size indicators and increased gear ratios; preferential rolling of profiles in the vicinity of the pole with slight slip; rational conjugation conditions (convex-concave contact in many phases of engagement); high efficiency; reduced vibration; improved technological capabilities in the manufacture and hardening of wheels. Calculations show the real possibility of creating a cycloidal planetary gearbox with a gear ratio of $i = 6 \div 16$ (Fig. 4) [7], which significantly exceeds the capabilities of a similar scheme with traditional gearing (involute or Novikov gearing) and gives a new impetus to the development of highly efficient gear turbodrills and borehole pumps for oil production. An interesting feature of the presented scheme of the cycloidal planetary gearbox is also the fact that the number of teeth of all wheels is simultaneously a multiple of 2 and 3.



a *b*

Fig. 3: Non-equidistant (skeletal) profiles of the working bodies of rotary machines, formed during the running-in of two-way shortened epicycloid:

a – on an external centroid; *b* – on the internal centroid

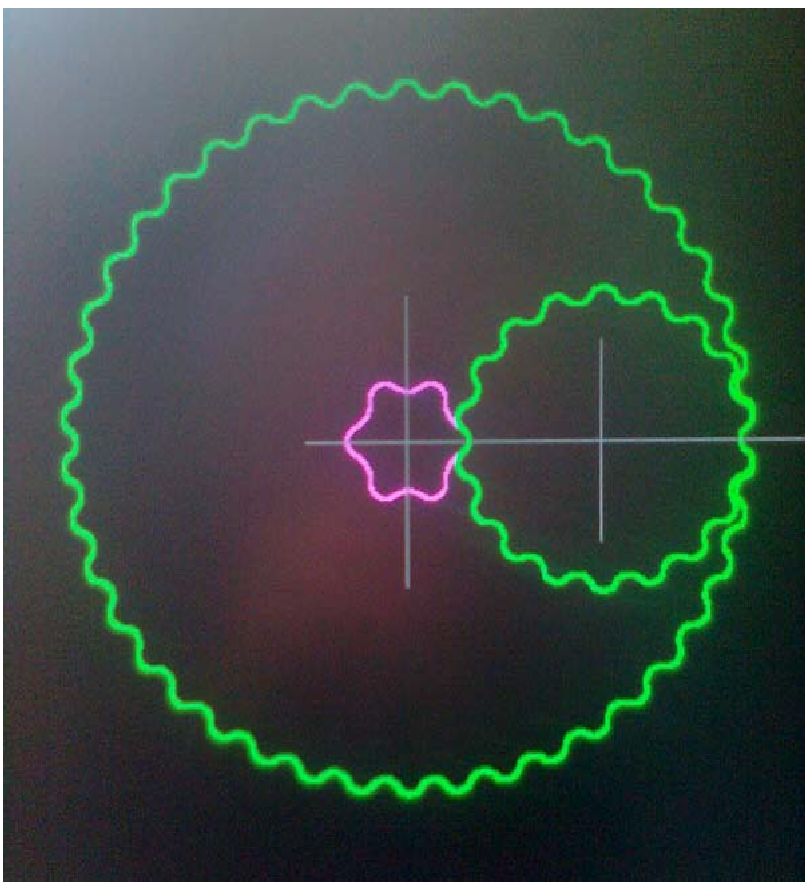


Fig. 4: Cycloidal planetary reducer according to the scheme 6-18-42 ($i = 8$):

hypocycloidal wheel; epicycloidal wheel

Improving the shape of traditional wheels and developing new types of gears is attracting more and more attention due to the influence of gear mechanisms on the basic technical characteristics of machines, as well as the general scientific and technological progress, the development of materials science, machine tool building and digital technologies.

Great prospects for improving involute and cycloidal transmissions open when modifying their nominal end profiles. There are various schemes for modifying external and internal gearing associated with high-altitude and longitudinal teeth correction, transition to an asymmetric profile shape [2, 4, 8], which allows to improve the characteristics of a rotary machine, as well as increase the load capacity, efficiency and durability of mechanical transmission. For cycloidal working bodies of down hole hydraulic motors with the execution of the teeth of one of the wheels (stator) of elastic material, the modification of gearing can be carried out proceeding from the conditions of rational redistribution of elastomer deformation and reduction of mechanical losses in the pole tooth zone due to the effect of a hydrodynamic wedge.

Among the unconventional types of gears that have appeared recently in the technical literature and patent fund, innovative wheel designs can be noted: 1) with a non-involute tooth profile, the curvature of which varies according to a certain law depending on the slip in the mate [9]; 2) with a radius profile of the teeth, consisting of tangent arcs of the circumferences of the heads and legs with their centers on the pitch circle [10]; 3) with an engagement line made in the form of a lemniscate [11]. The inventors of these inventions substantiate the technical and technological advantages of their geometry in comparison with the involute profile and indicate the areas of possible use of gears.

This article proposes a new alternative type of gearing [12], characterized in that the initial contour of the rack is made in the form of a harmonic curve (sinusoid or cosine wave), displaced in general relative to the producing straight line, and the end profile of the gear teeth is formed as an envelope families of harmonic curves when they are run in the main circle of a certain radius, selected depending on the required number of teeth of the wheel. At the same time, the number of wheel teeth can take on any value, starting from one, which creates the premise of expanding the kinematic capabilities of the gear mechanism and obtaining a high ratio in one stage.

The main difference and advantage of the harmonic curve over the cycloid is its smoothness, which does not require the execution of equidistance procedures and makes it possible to use the harmonic itself directly as the initial loop contour, rolled around the main circle of radius *rz*. Thus, when the end profile is formed from a harmonic rack, similar to the case of involute gearing, a transition is made from a three-stage scheme (rack - equidistant of the rack - offset equidistant) to a two-stage scheme (rack – the offset rack), in which the initial contour of the rack I coincides with the original circuit II (Fig. 5).

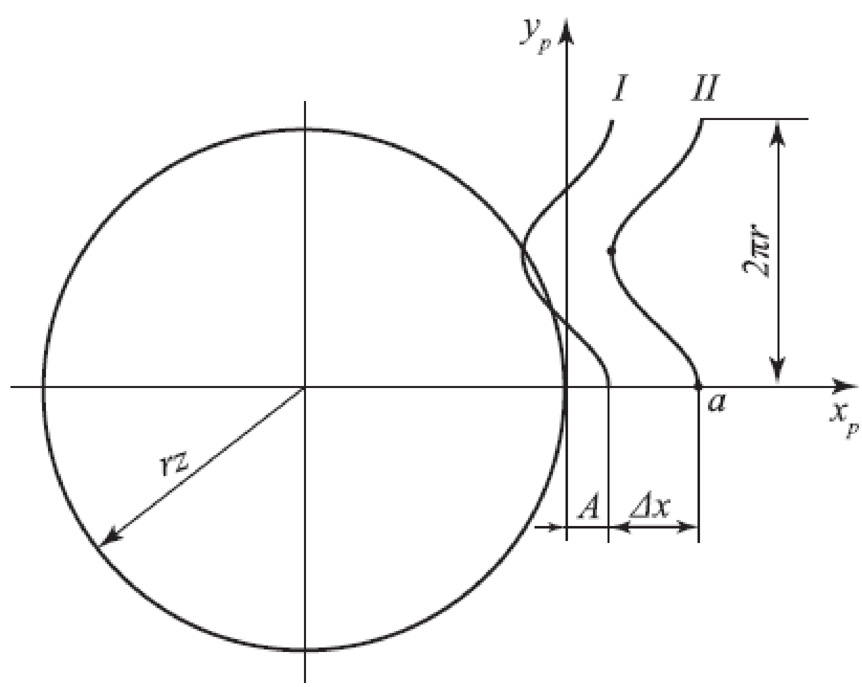


Fig. 5: The initial position of the rack during the formation of a harmonic profile

The coordinates of the initial contour of the harmonic rack relative to the generating line (tool axes $x_p y_p$) can be represented in the form:

$$x_p = x_o + \Delta x$$

$$y_p = y_o \,, \tag{1}$$

where x_o, y_o are the initial coordinates of the harmonic I symmetrically located relative to the generating line; Δx

Global Journal of Researches in Engineering (A) Volume XX Issue I Version I Year 2020

is the displacement of the initial rack contour relative to the generating line.

If we use the cosine wave as a harmonic (Fig. 5), then

$$x_o = -uA\cos\psi$$
$$y_o = r\psi \, , \qquad (2)$$

where ψ is the angular parameter varying from 0 to 2π; r is the radius of the unit circle; A – harmonic amplitude equal to half the height rack; u is a profile type coefficient: $u = 1$ when forming an epigarmonic profile (from a positive cosine, when the protrusion of the rack is directed from the center of the main circle), $u = -1$ when forming a hypogarmonic profile (from a negative cosine, when the protrusion of the rack is directed to the center main circle, Fig. 5).

In Fig. 6 shows the current positions of the rods during the formation of epi- and hypogarmonic profiles using the positive and negative cosine waves, respectively. The difference between the families of these curves from the point of view of the initial position of the staff consists in the location of the point corresponding to the maximum diameter of the profile: in the case of the formation of a hypogarmonic profile, this point (a, Fig. 5) is located on the x_p axis at the beginning of the angular pitch of the staff ($\psi = 0$), while during the formation of the epigarmonic profile, a similar point will be located in the middle of the angular step ($\psi = \pi$).

In the current position, when running along the main circle, the generating line and the rail connected with it are rotated through the angle φ_p. Since running without slip, the family of curves of the rack is described by the following parametric equations:

$$X = (x_p + rz)\cos\varphi_p - (y_p - rz\varphi_p)\sin\varphi_p$$
$$Y = (x_p + rz)\sin\varphi_p + (y_p - rz\varphi_p)\cos\varphi_p. \qquad (3)$$

The harmonic gear profile is the inner envelope of the family of run-in racks (Fig. 6). To go from (3) to the envelope equation it is necessary to establish the relationship between ψ and φ_p. Her can be obtained through general geometric constructions, given that the normal to the envelope passes through the pole - the point of contact of the generating line and the main circle.

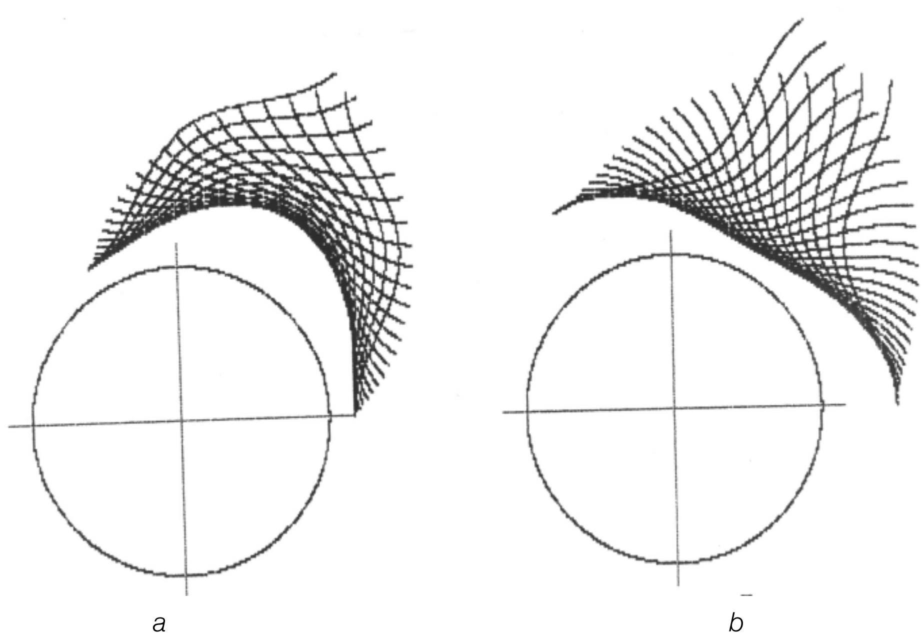


Fig. 6: The scheme of formation of one branch of a harmonic profile ($\varphi_p = 2\pi/z$) according to the method of running-in a rack ($z = 3$):

a - epigarmonic; *b* - hypogarmonic

A distinctive feature of a closed harmonic profile is the independence of its shape from the pattern of formation: epi- and hypogarmonic profiles are identical (phase-shifted by an angle π/z). Therefore, in contrast to the cycloidal one, such a profile can be called the general term "harmonic", without specifying from which curve (sine or cosine) the rail is formed and without adding an epi- or hypo prefix.

The average diameter D_a of the harmonic profile (in the pitch circle of the teeth) does not depend on the harmonic amplitude, and the height of the teeth h – from the rack offset:

$$D_a = 2(rz + \Delta x). \quad (4)$$

$$h = 2A. \quad (5)$$

The shape of the harmonic end profiles of a pair of gears engaged in meshing (the initial one is index 1 and the conjugate one is index 2) is completely determined by a combination of three dimensionless geometric parameters:

- gear ratio $i = z_2 : z_1$;
- harmonic shape coefficient $c_A = A / r$;
- displacement coefficient of the original rack contour $c_\Delta = \Delta x / r$.

The choice of the numerical values of these dimensionless geometric parameters depends on the type and required characteristics of the gear pair.

Gears with a harmonious tooth profile (Fig. 7, 8) can be used in various mechanisms of external and internal gearing with parallel axes.

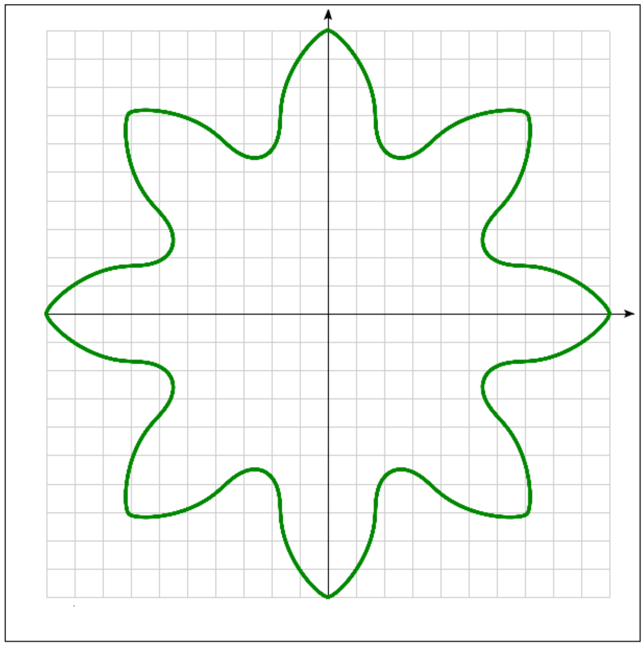


Fig. 7: Gear with nominal harmonic profile ($z = 8$; $c_A = 2$; $c_\Delta = 0$)

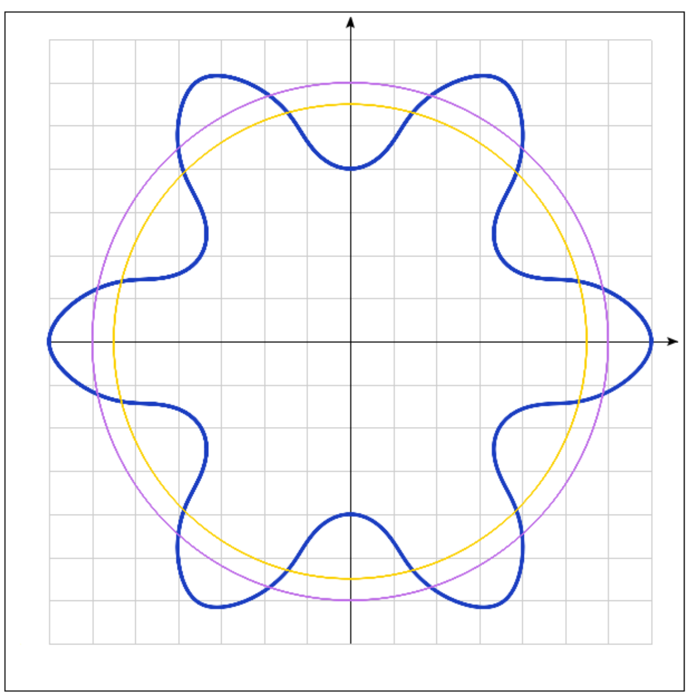


Fig. 8: Gear with a shifted harmonic profile ($z = 6$; $c_A = 1{,}5$; $c_\Delta = -0{,}5$) and drawing the main and dividing circles

In the general case, when the initial harmonic profile of the wheel is selected by the z_1 tooth (the construction scheme of which is shown in Figs. 5 and 6), the conjugate profile of the second wheel is formed as the envelope of the initial profile during the running-in of the initial circles of the wheels, the ratio of the radii of which corresponds to gear ratio z_2/z_1.

In practice, when constructing a mating profile, you can use the simplified method in which the mating profile is performed similarly to the initial one by the method of running from the general contour of the rack [4]. In relation to the proposed gearing, this means that when the profiles of the mating wheels are formed, the harmonic shape remains unchanged ($A_1=A_2=A$), and only the magnitude of the rack shift (Δx_1; Δx_2) changes [12].

As an example in Fig. 9 shows an embodiment of a gear pair of external harmonic gearing without bias with a ratio of 7:25 in the characteristic gearing phase corresponding to the contact of the tooth protrusion of the driving gear with the tooth cavity of the driven wheel.

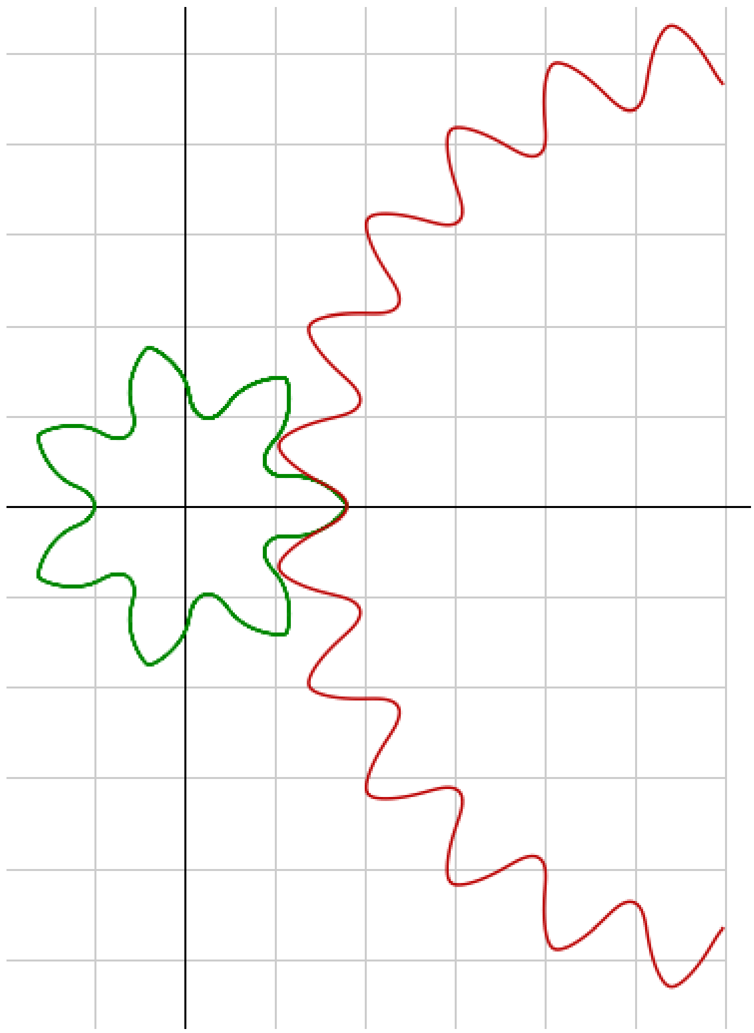


Fig. 9: Gearing of wheels with a harmonic profile ($c_A = 2$; $c_\Delta = 0$)

When designing gears with a harmonic profile, the shape of their teeth can vary widely depending on the combination between the dimensional parameters (*r; A*; Δx) or dimensionless coefficients (c_A; c_Δ), which will ensure the best gearing performance (with geometric, kinematic and power points of view), including by choosing the optimal curvature and relative sliding of the profiles for various gear ratios of the mechanism.

Thus, by changing the relationship between dimensionless profile coefficients, it is possible to give the tooth a different shape (from smooth to pointed at the top) depending on the purpose and gearing pattern (mechanical transmission or working parts of the hydraulic machine), a given characteristic and operating conditions of the mechanism.

An engagement with a harmonic tooth profile formed by the method of rolling in a rack accumulates the positive properties of similar involute and cycloidal gears (Fig. 10) and has the following advantages:

- compared to cycloidal gearing, it is possible, as in the case of involute gearing, to use the original rack contour identical to both profiles in the design and manufacture of mating gears of external gearing. This is due to the fact that the initial harmonic curve I (Fig. 5) in the interval from 0 to 2π is located symmetrically with respect to the longitudinal axis yp and has the same curvature of its vertices; therefore, the teeth of the rack are characterized by the identical shape of their protrusions and depressions. In the case of external cycloidal engagement, the wheel profiles are represented by conjugated hypo- and epicycloidal curves (Fig. 4), the profiling of which is carried out from the corresponding initial contours;

- compared to involute gearing, it is possible, due to smoothness, lack of transition curves and less tendency to undercuts and sharpenings of the sinusoidal curve, to develop wheel profiles with a minimum number of teeth, up to the limit modifications of single and double tooth designs in the form of a cam and a guitar. This opens up new kinematic, constructive and technological possibilities when creating gear transmission mechanisms and working bodies of hydraulic machines with external and internal gearing, including a decrease in their overall dimensions.

In addition to mechanisms with cylindrical wheels, a harmonic end profile of teeth can also be used in the development of bevel and worm gears.

The technical result of the proposed type of gearing is to increase the efficiency and flexibility of the process of designing and manufacturing gears (including on modern gear-processing equipment), which in the long run creates real prerequisites for further expanding the scope and improving mechanical gears and volumetric rotary machines in various branches of technology.

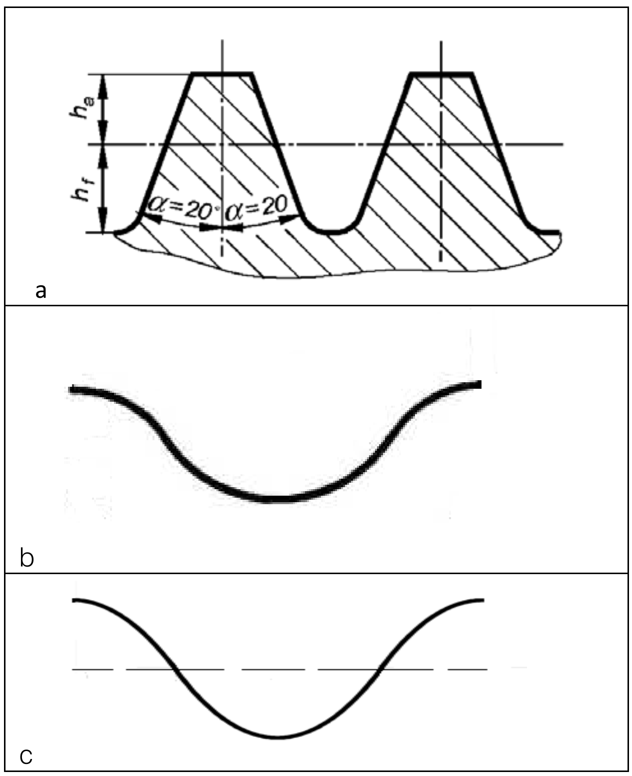


Fig. 10: The initial contours of the gear racks for cutting teeth of various profiles:

a - involute; *b* - cycloidal; *c* - harmonic

References Références Referencias

1. *Litvin F. L.* Theory of gears. Moscow, Science, 1968.
2. *Vulgakov E.B.* Theory of involute gears. Moscow, Mechanical Engineering, 1995.
3. *Korotkin V.I., Onishkov N.P., Kharitonov Yu.D.* Novikov Gearing: Achievements and Developments (Mechanical Engineering Theory and Applications). Nova Science Pub Inc, 2011.
4. *Baldenko D.F., Baldenko F.D., Gnoevykh A.N.* Single screw hydraulic machines (in two volumes). Moscow, IRC Gazprom, 2005-2007.
5. Patent 671463 of the USSR. Gerotor mechanism / *A.V. Tsepkov* et al.; 11.11.1974.
6. European Patent 2530359A1. Eccentrically cycloidal engagement of toothed profiles (variant embodiments) / *V.V. Stanovskoy* and etc..; 17.01.2011.
7. *Baldenko F.D.* Prospects for the use of cycloidal gearing in the oil and gas industry // Territory Neftegaz. 2014, No 6.
8. *Kapelevich A.* Direct Gear Design for Asymmetric Tooth Gears. AKGears, LLC. USA, 2016.

9. US Patent 5271289A. Non-involute gear / *M.L. Baxter Jr*.; 16.12.1992.
10. *Pakhomov S.N.* Radius alternative to involute gearing // Modern equipment and technologies. 2016, No 4.
11. US Patent 20070207051. Toothed member and a corresponding locusus / *A. Katz*; 11.03.2005.
12. Patent 184504 of the Russian Federation. A gear wheel with a harmonic tooth profile / *D.F. Baldenko, F.D. Baldenko*; 15.03.2018.