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I. Introduction

Recently, interconnect circuit attract increasing attracting in a wide area of scientist researchers such as circuit theory design, microwave application, integrated circuit and physical model. The circuit networks analysis and design have been addressed by many research's regarding several architectures with finite or infinite size [1-4]. Many competitive methods have been developed to improving the design and modelling of the several lattice. Among them we cited, the Green function lattice [5], the Laplacian matrix [6] and the Recursion Transform method [7].

However, despite the accuracy and efficiency of these methods, there are limited to the analysis of the equivalent resistance or impedance for homogenous circuits excited an exterior source.

Latterly, a new approach of the Wave Concept Iterative Process method (WCIP) [8] is for computing the effective impedance and the current distribution of an RLC electrical circuit with triangular or hexagonal lattice.

The WCIP method is successfully used, near two decades, in the analysis of planar micro strip microwave circuits [9], [10] and it is demonstrating its power for solving the radiation and scattering electromagnetic problems [11-13]. The method is also reformulated to *analyze quasi-periodic lumped circuits with rectangular grid [14].These periodic lumped circuits can be considered as good equivalent representations to* accurately model continuous mediums if the cell's length is much smaller than the lattice.

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The main keys of the proposed method was summarised as follows: (1) the incoming and outgoing wave's concept definition from the electrical entities (voltage and current).

(2) The introduction of the auxiliary sources techniques instead of the circuit components. (3) the use of the Fourier transforms appropriate to hexagonal and triangular lattice named HFFT (hexagonal fast Fourier Transform) and the resolution of the alternative (spectral-spatial) equations by an iterative process.

Therefore, the mathematical formulation is developed into two definition domains; a spectral-domain in which periodicity and coupling between components of the circuit was defined and a spatial one describing the topology and values of network elements, and imposing the continuities conditions (Kirchhoff's laws). The above relations represent a recursive system, which is resolved by an iterative process; the transition between one domain to another is guaranteed by the HFFT and its inverse.

In the first part of this paper, we develop the mathematical formalism describing the new WCIP method approach. In the second part, we show the design of the proposed circuit results, such as the spatial variation of the electric field inside the resonator and the frequency response of the transmission coefficient.

II. Theoretical Formulation

a) Waves definition

The WCIP principle is described in many papers; it is founded on the introduction of the incident (A) and reflected (B) waves tangential to each edge of the network. These waves are defined from the voltage and the current by the following equation:

$$\begin{cases} A = \dfrac{1}{\sqrt{Z_0}}(V + Z_0 I) \\ B = \dfrac{1}{\sqrt{Z_0}}(V - Z_0 I) \end{cases} \qquad (1)$$

Where, Z_0 is an arbitrary chosen impedance.

The electric current (I) and the voltage (V) can be calculated as follows (6):

$$\begin{cases} V = \sqrt{Z_0}(A+B) \\ I = \dfrac{1}{\sqrt{Z_0}}(A-B) \end{cases} \tag{2}$$

b) Spectral-domain analysis

This domain characterizes the physical relations (periodicity and Kirchhoff laws) established between the electrical components and written in waves term.

Fig.1 shows an electrical circuit network, the electrical schema considers a capacitor connected to a resistor and distributed according to an equilateral triangular grid. The circuit is excited by a lumped source located in the center (n=0, m=0) at a vertical edge.

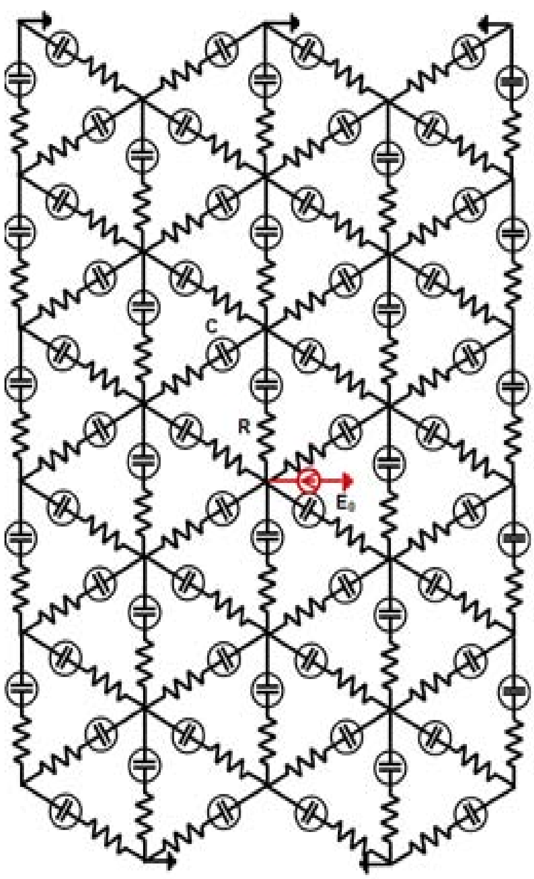


Fig. 1: A capacitor triangular circuit lattice

The potential difference across each lumped components of the circuits (capacitor, open circuit, shorted circuit and, source) is represented by an auxiliary sources.

The unit cell of the studied network is represented by the Fig.2; it considers three horizontal branches connected to a vertical one at the nodes. The shift phased the one to the other to 60 degrees. The electrical lattice is generated from the translation of the unit cell according to two of the three directional vectors ($\vec{e}_\alpha$, $\vec{e}_\beta$ and $\vec{e}_\gamma$) collinear to the horizontal branches.

We denote by (E_α, E_β, E_γ and E_z)the auxiliary sources representing the potential difference across the electrical components.

For ensuring the unit cells periodicity, periodic walls are placed perpendicular to the three directional vectors.

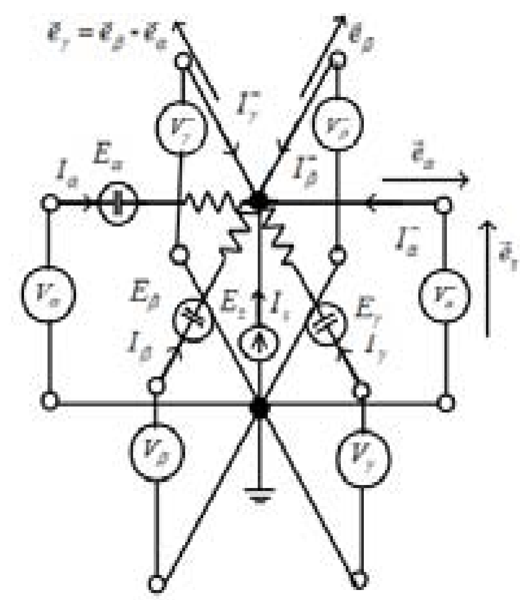


Fig. 2: Triangular unit cell

From Fig. 2, the periodicity laws permit to write

$$\begin{cases} E_z = V_\alpha e^{j\alpha} = V_\beta e^{j\beta} = V_\gamma e^{j\gamma} \\ \begin{pmatrix} I_\alpha^- \\ I_\beta^- \\ I_\gamma^- \end{pmatrix} = \begin{pmatrix} e^{j\alpha} & 0 & 0 \\ 0 & e^{j\beta} & 0 \\ 0 & 0 & e^{j\gamma} \end{pmatrix} \begin{pmatrix} I_\alpha \\ I_\beta \\ I_\gamma \end{pmatrix} \end{cases} \tag{3}$$

Establishing the Kirchhoff's laws to the unit cell, in considering relations given by Eq. 3, we obtain a spectral equation relating the electric current to the voltage

$$\begin{pmatrix} \tilde{I}_\alpha(n,m) \\ \tilde{I}_\beta(n,m) \\ \tilde{I}_\gamma(n,m) \\ \tilde{I}_z(n,m) \end{pmatrix} = \tilde{Y}_{n.m} \begin{pmatrix} \tilde{E}_\alpha(n,m) \\ \tilde{E}_\beta(n,m) \\ \tilde{E}_\gamma(n,m) \\ \tilde{E}_z(n,m) \end{pmatrix} \tag{4}$$

With

$$\tilde{Y}_{n.m} = \frac{1}{R}\begin{pmatrix} 1 & 0 & 0 & a_n \\ 0 & 1 & 0 & b_m \\ 0 & 0 & 1 & c_{n.m} \\ a_n^* & b_m^* & c_{n.m}^* & d_{n.m} \end{pmatrix} \quad (5)$$

Where $a_n = \left(1 + e^{-j\alpha_n}\right)$, $b_n = \left(1 + e^{-j\beta_n}\right)$, $c_{n.m} = \left(1 + e^{-j\gamma_{n.m}}\right)$, $d_{n.m} = \left(|a_n|^2 + |b_m|^2 + |c_{n.m}|^2\right)$ and

$$\begin{cases} \alpha_n = \dfrac{2\pi n}{N} \\ \beta_m = \dfrac{2\pi m}{M} \\ \gamma_{n.m} = \beta_m - \alpha_n \end{cases} \quad (6)$$

The subscript (*) denote the conjugate of a complex number.

Substituting Eq. (2) in Eq. (3), a spectral equation relating the incident to the reflected wave is

$$\begin{pmatrix} \tilde{B}_\alpha(n,m) \\ \tilde{B}_\beta(n,m) \\ \tilde{B}_\gamma(n,m) \\ \tilde{B}_z(n,m) \end{pmatrix} = \tilde{\Gamma}_{n.m} \begin{pmatrix} \tilde{A}_\alpha(n,m) \\ \tilde{A}_\beta(n,m) \\ \tilde{A}_\gamma(n,m) \\ \tilde{A}_z(n,m) \end{pmatrix} \quad (7)$$

With $\tilde{\Gamma}_{n.m} = (II - Z_0 Y_{n.m})(II + Z_0 Y_{n.m})^{-1}$

c) Spatial domain analysis

In the spatial domain, every auxiliary source replace by its corresponding impedance (capacitor, inductor or resistor), then the spatial reflexion operator is given by

$$S = (Z - Z_0)(Z + Z_0)^{-1} \quad (8)$$

For the open and shorted circuit, the spatial reflexion operator is given by

$$S = \begin{cases} -1 \ \textit{For the short circuit} \\ 1 \ \textit{For the open circuit} \end{cases} \quad (9)$$

In considering the excitation source, the reflected waves are related to the incidents ones by the following relationship

$$A = SB + A_0 \quad (10)$$

With A_0 represents the feeding source in wave term.

d) Iterative process

Collecting (7) and (10), the iterative process is governed by a set of two equations describing, the boundaries condition (Kirchhoff's laws) in the spatial-domain and the periodicity laws in the spectral-domain. The Hexagonal Fast Fourier Transform (HFFT) and its inverse (HFFT)$^{-1}$, ensure the transition between the two domains (Fig.3)[8]. Electrical quantities: current and voltage are determined from the incident and reflected waves at each iteration. The iterative process is halted when the voltage (or current) converges.

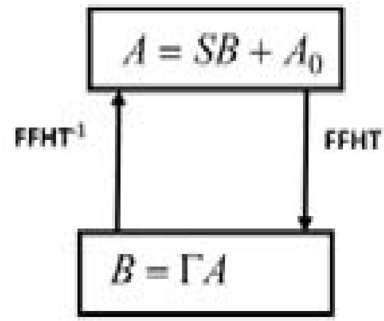


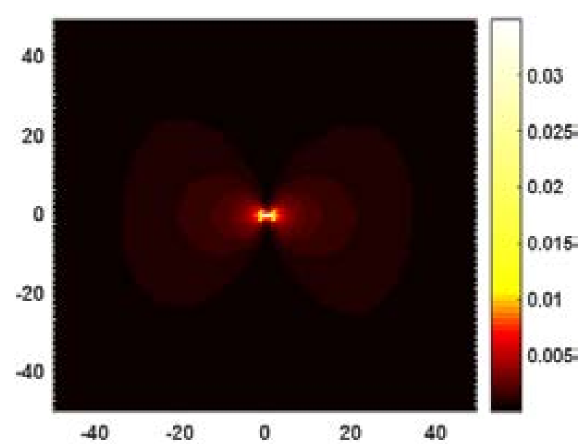
Fig. 3: Principle of the iterative scheme

III. Numerical Results

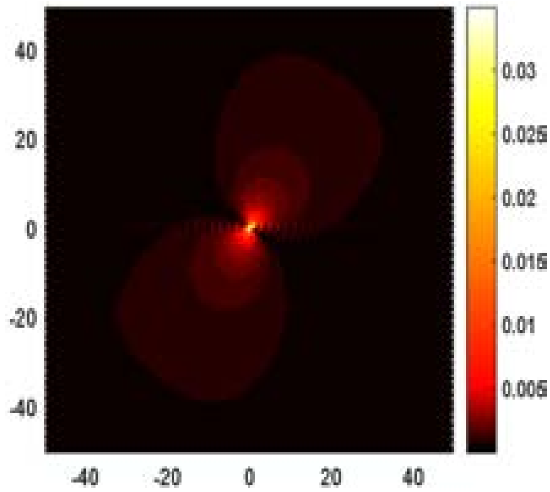
The above formulation is employed for calculated the electrical current components on the horizontal edges of the lattice and the potential difference between the nodes and ground in a first step, then the method is also used to investigating the so-called perturbed lattice.

In the numerical example, the total cells number are fixed to N= 100 and M=116, and we take C=2.3pF and R=0.4 Ω. The circuit is excited by a voltage source E_0= 1V, the source is located at the middle of the circuit in (N =0, M =0).

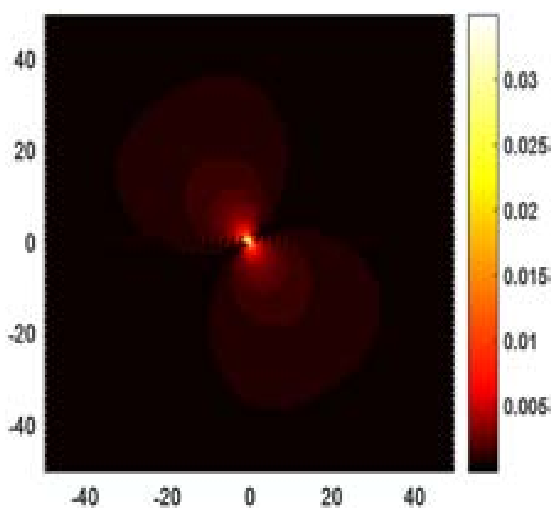
Fig.4 shows the electrical current propagation for the three horizontal components I_α, I_β and I_γ. It is observed that the distributions of each current component is oriented according its principal axis.



a) I_α component



b) I_β component



c) I_γ component

Fig. 4: Electric currents components distribution

We can also see that the components present a rotational symmetry of angle of 60 degrees. The currents figures display that the intensities are maximum at the lattice center and show a central and axial symmetry.

Fig. 5 shows the vertical voltage propagation; we note that the dispersion is considerable in the proximity of the feeding source.

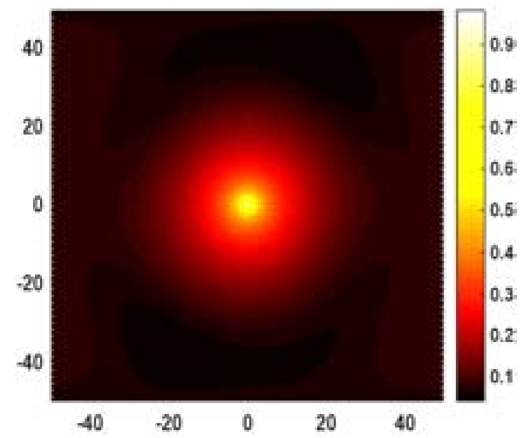


Fig. 5: Vertical voltage repartition

In the next, we analysis the so called perturbed lattice. A perturbed lattice is defined, in many types of research that interested in computing the equivalent resistance or impedance between two arbitrary nodes, by the network wherein we remove one or two bonds from the regular circuit. Herein, we extend this description and we define a perturbed architecture by the lattice that we remove one or many arbitrary part.

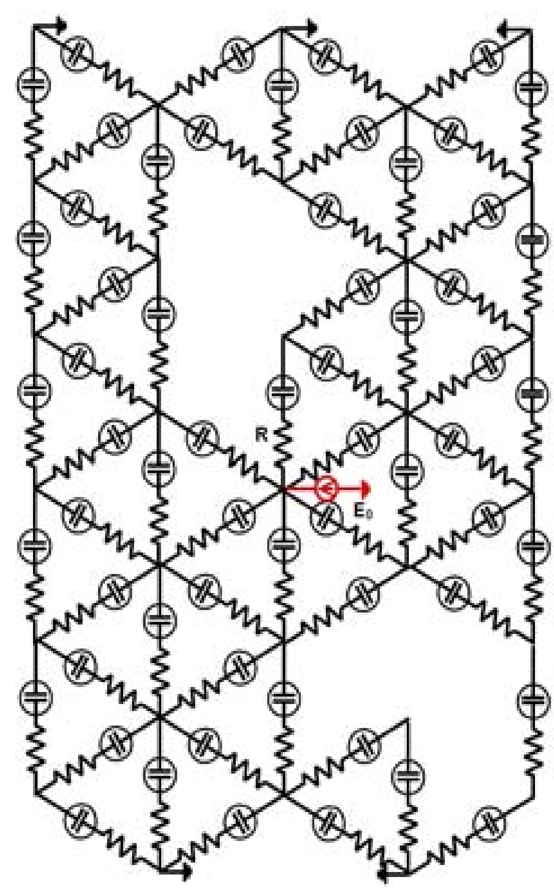
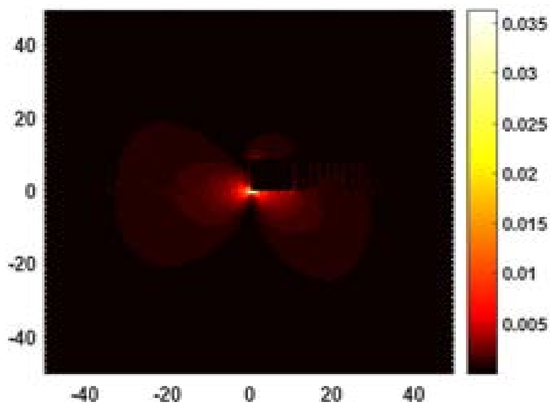
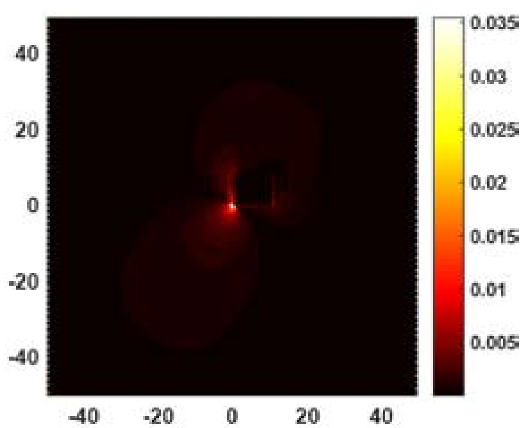


Fig. 6: Example of perturbed circuit with triangular lattice

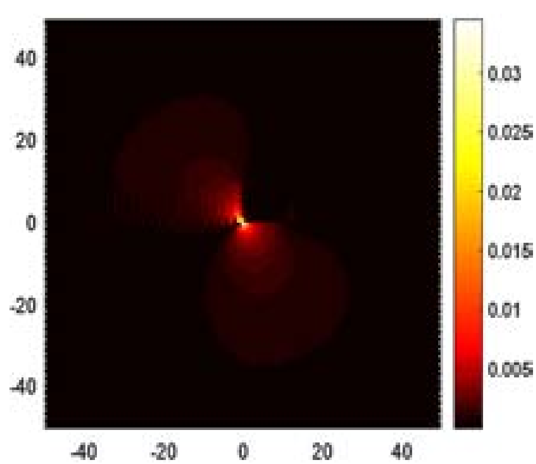
The figures (7) and (8) display the current and vertical voltage distributions for a removed band located for ($1 \leq n \leq 10$) and ($1 \leq m \leq 10$). With n and m represent the cell numbers according $\vec{e}_\alpha$ and $\vec{e}_\beta$ directions, respectively.



a) Perturbed Iα component



b) Perturbed I_β component



c) Perturbed $I_\gamma$component

Fig. 7: Electric currents components distribution for a perturbed triangular lattice: one band removed

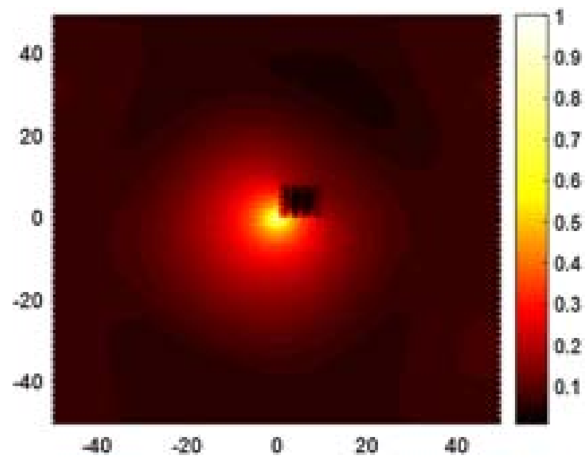
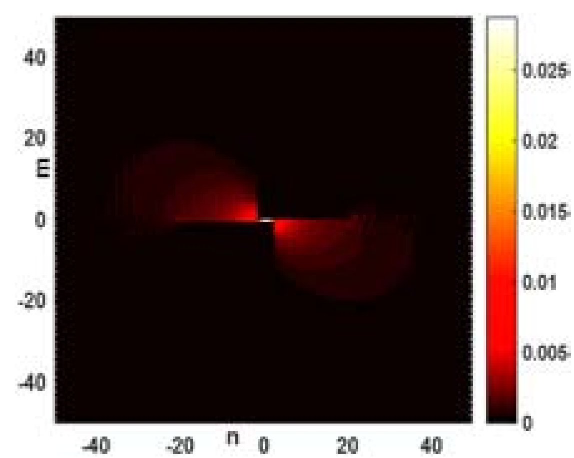


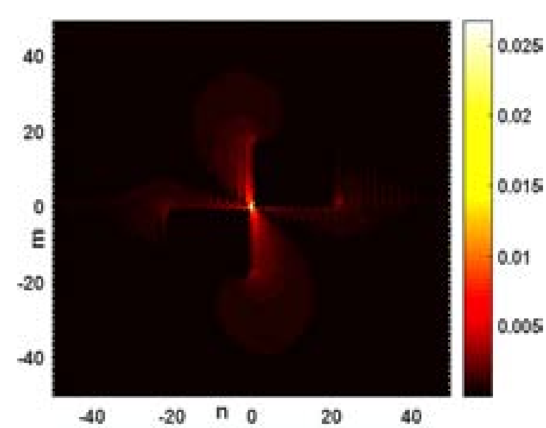
Fig. 8: Vertical electric voltage distribution for a perturbed triangular lattice

We clearly observe the perturbation effecton the graph, infact, the currents and voltage are null in the removed bands and in the irproximities.

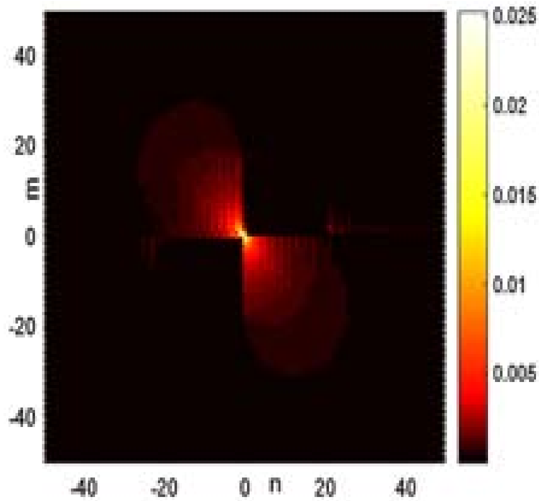
In the next example, we remove two electrical bands for a wide eliminated area. The suppressed surfaces are situated on ($1 \leq n \leq 20$), ($1 \leq m \leq 20$) and ($-20 \leq n \leq -1$),($-20 \leq m \leq -1$).



a) Perturbed I_α component for two removed bands



b) Perturbed I_β comonent for two removed bands



c) Perturbed I_y component for two removed bands

Fig. 9: Electrical current components for two removed bands

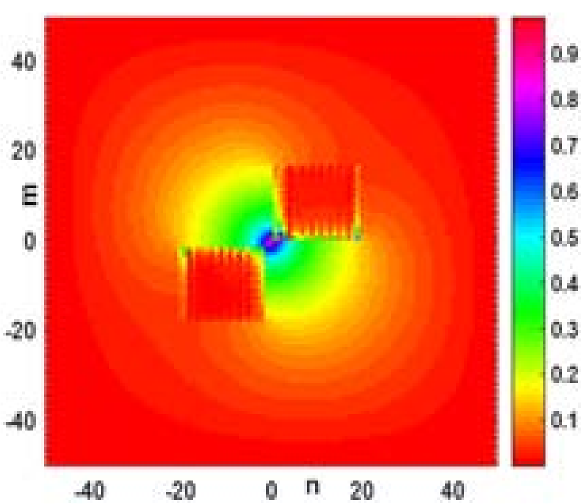


Fig. 10: Perturbed vertical voltage for two removed bands

The visualization of the electrical current and voltage depicted by figures (9) and (10) demonstrates that the propagation becomes more degenerate when the surfaces and number of the removed bans increases. It is worth noting that the problems of the electrical perturbed circuits become more interesting for several physical difficulties analysis, notably for in the modelling of semiconductors with electrical default.

IV. Conclusion

In this paper, a full-wave concept was formulated to investigate an RC circuit with triangular lattice. The method is defined in two definition domains: a spectral-domain describing the periodicity laws and a spatial-domain in the design of the circuit is defined and the Kirchhoff's laws are imposed. The auxiliary sources was introduced for characterizing the potential difference across each electrical element.

In numerical results, the electrical current and vertical potential difference distribution are visualized for a planar capacitor-resistor circuit with triangular architecture. The perturbed RC circuit is also defined and investigated, we observe a deformation of the electrical current and voltage.

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