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Estimation of Uplift Capacity of Horizontal Plate Anchor in Sand

Prof. D. M. Dewaikar [α] & Prof.V. B. Deshmukh [σ]

Abstract- In this paper, a detailed analysis of breakout resistance of a horizontally laid anchor plate in sandy soil is presented. To compute the distribution of soil reactive pressure on the failure surface, Kötter's equation is employed. The failure surface is assumed to be in the form of a cone. An analytical expression for the breakout resistance is derived. Results are reported in terms of the breakout factors and net breakout resistance. A comparison is made with the available experimental data and theoretical solutions.

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I. Introduction

The shapes of earth anchors are square, circular or rectangular and generally they are employed as foundation elements for structures requiring resistance against breakout i.e., transmission towers, sheet pile walls and offshore floating structures. This requires an analysis of behaviour of the anchors.

Several researchers (Mors, 1959; Balla, 1961; Baker and Konder, 1966; Meyerhof and Adams, 1968; Vesic, 1971; Clemence and Veesaert, 1977; Sutherland et al., 1982; Saeedy, 1987; Murray and Geddes, 1987; Ghaly et al.,1991; Tom, 2012)analysed the breakout resistance of earth anchors using limit equilibrium method. Tagaya et al. (1988) introduced the theoretical formulae for the computation of the anchor pullout resistance based on elostoplastic finite element method, whereas analyses presented by Merifield and Sloan (2006) and Kumar and Kouzer (2008), Tang et al. (2014), Hao et al. (2014) and Bhattacharya and Kumar (2016) were based on the limit analysis coupled with finite element method.

In respect to a dense soil, Balla (1961) studied model and field results and found that, for circular anchors which are shallow laid, the failure surface was closely approximated to an arc of a circle. From theoretical considerations, the angle of failure surface with the horizontal was taken as 45° - $\phi/2$. The net breakout resistance, P_{un} which is the summation of soil weight contained in the failure zone and resistance to shearing developed on the failure surface was calculated as

$$P_{un} = H^3\gamma\left[F_1\left(\phi,\frac{H}{D}\right)+F_3\left(\phi,\frac{H}{D}\right)\right] \tag{1}$$

where, D is the diameter of circular anchor plate, H is the height of circular anchor,γ is the soil unit weight and F_1 $(\phi, H/D)$, F_3 $(\phi, H/D)$ are the functions developed by Balla (1961).

Balla's (1961) analysis showed a good agreement for the dense sand up-to the embedment ratio of 5. But, in respect to anchors laid in loose and medium sand, the analysis overestimated the net breakout resistance. For embedment ratio greater than 5 even in dense sand, the analysis overestimated the breakout resistance due to deep anchor effects wherein the failure zone did not reach the ground level.

Baker and Konder (1966) conducted several laboratory model tests and used dimensional analysis to predict the ultimate uplift capacity, P_u as given by the following expressions.

For shallow circular anchors

$$P_u = C_1 HD^3 r + C_2 H^3\gamma \tag{2}$$

For deep circular anchors

$$P_u = 170D^3\gamma + C_3 D^2 t\, r + C_4 HD + \gamma \tag{3}$$

where, r and t are radius and the thickness of anchor plate respectively and H is the depth of embedment. C_1, C_2, C_3 and C_4 are the constants which are functions of angle of soil internal friction and relative density of compaction. For shallow anchors, the model test results of Baker and Konder (1966) agreed well with the predictions based on Balla's (1961) theory.

Meyerhof and Adams (1968) reported a semi-theoretical expression for breakout resistance on the basis of laboratory tests data. For the actual failure surface, simplified geometry was assumed. The failure surface makes an angle, α with the horizontal in the range, 90° − ϕ/3 to 90° − 2ϕ/3. An average value of 90° − ϕ/2 was considered. With the force equilibrium in vertical direction, the net breakout resistance, P_{un} was estimated as

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$$P_{un} = W + \frac{\pi}{2} S_F \gamma H^2 K_u \tan\phi \quad (4)$$

where, W is the weight of cylindrical soil mass above the circular anchor and S_F is the shape factor. The breakout coefficient, K_u depends on soil friction angle, ϕ and was taken equal to 0.95 for ϕ varying from 30° to 48°. The net breakout resistance, P_{un} was expressed as

$$P_{un} = F_q \gamma A H \quad (5)$$

The breakout factor, F_q is given as

$$F_q = 1 + 2\left[1 + m\left(\frac{H}{D}\right)\right] K_u \left(\frac{H}{D}\right) \tan\phi \quad (6)$$

Graphs or tables are used to obtain the coefficient, m.

Vesic (1971) analysed the case of an explosive point charge for the expansion of a spherical cavity located close to the surface of a semi-infinite, homogeneous and isotropic ground. At the ground level, the failure surfaces made an angle of (45° $-\phi/2$). case of a circular anchor embedded in sand, the breakout pressure, q_u was computed as

$$q_u = \gamma H A F_q \quad (7)$$

where, A is the area of circular anchor and F_q is the breakout factor. The values of F_q are computed for ϕ varying in the range, 0° to 50° along with embedment ratios in the range, 0.5 to 8.

Clemence and Veesaert (1977) studied the results of laboratory experiments and made an approximation of the observed failure surface to an inverted truncated cone with an apex angle of $\phi/2$, going upwards from the anchor base. The breakout resistance includes the weight of soil within this cone and the shearing resistance developed along the failure surface. For shallow laid circular anchors, the net breakout resistance, P_{un} was estimated in terms of the breakout factor, F_q as given by the following expressions.

$$F_q = \frac{P_{un}}{\gamma A H} \quad (8)$$

Or

$$F_q = 4K_0(\tan\phi)\cos^2\left(\frac{\phi}{2}\right)\left(\frac{H}{D}\right)^2\left[\frac{0.5}{\left(\frac{H}{D}\right)} + \frac{\tan\frac{\phi}{2}}{3}\right] + \left(4 + 8\left(\frac{H}{D}\right)\tan\left(\frac{\phi}{2}\right) + 5.33\left(\frac{H}{D}\right)^2 \tan^2\left(\frac{\phi}{2}\right)\right) \quad (9)$$

where, K_0 is the coefficient of lateral earth pressure at rest.

Murray and Geddes (1987) have reported the solutions with both limit equilibrium and limit analyses and made a comparison of the solutions with experimental results for a circular anchor. With the limit equilibrium analysis, the ultimate breakout resistance, P_u was expressed by the following equation.

$$\frac{P_u}{\gamma A H} = 1 + 2\frac{H}{D}\left(\sin\phi + \sin\frac{\phi}{2}\right)\left(1 + \frac{2}{3}\frac{H}{D}\tan\frac{\phi}{2}(2 - \sin\phi)\right) \quad (10)$$

In the above equation, A is the area of circular anchor.

With upper bound limit solution, the breakout resistance was expressed by the following equation.

$$\frac{P_u}{\gamma A H} = 1 + 2\frac{H}{D}\tan\phi \quad (11)$$

Saeedy (1987) estimated the uplift capacity of circular plate anchors embedded in sand with the assumption of a failure surface as an arc of a logarithmic spiral. The effect of deep condition and compaction during the uplift were considered in this analysis. To account for these conditions, the uplift capacity was expressed as

$$P_u = (F_q \gamma A H)\mu \quad (12)$$

where, μ is the compaction factor which is the function of relative density of compaction.

Semi-empirical relationships are also available to estimate the breakout resistance of anchors in sand. This refers to the field and/or model testing on horizontal circular anchors or belled piles by Balla (1961), Sutherland (1965) and Baker and Konder (1966), Mors (1959), Giffels et al. (1960), Turner (1962), Ireland (1963), Mariupol'skii (1965), Kananyan (1966), Adams and Hayes (1967) and Sakai et al. (2007). A number of these studies were primarily concerned with testing foundations for transmission towers (Mors, 1959; Balla, 1961; Turner, 1962 and Ireland, 1963).

In the present study, a total of seven experimental results (Balla, 1961; Baker and Konder, 1966; Bemben and Kupferman, 1975; Ovesen, 1981; Sutherland et al., 1982; Illampurathi et al., 2002; Murray and Geddes, 1987) and two field test results (Sutherland et al., 1982; Tucker, 1987) are referred for comparison.

II. Proposed Analysis Method

Kötter's (1903) equation is used to compute the vertical soil reaction, R_v along the failure surface. This equation which is valid for the plane strain condition was employed for the analysis of a retaining wall by Dewaikar and Halkude (2002a), for the stability analysis of open cuts in soil by Dewaikar and Halkude (2002b), for the computation of bearing capacity factor, $N\gamma$ by Dewaikar and Mohapatro (2003), analysis of rectangular and square anchors in cohesionless soil by Deshmukh et al. (2010) and uplift capacity of pile anchors in cohesion less soil by Deshmukh et al. (2010). On integration along a plane or a curved failure surface, this equation gives the soil reactive pressure distribution and with further integration, it yields the resultant soil reaction on the failure surface.

b) *Soil Reaction on the Plane Failure Surface (Refer Fig.1)*

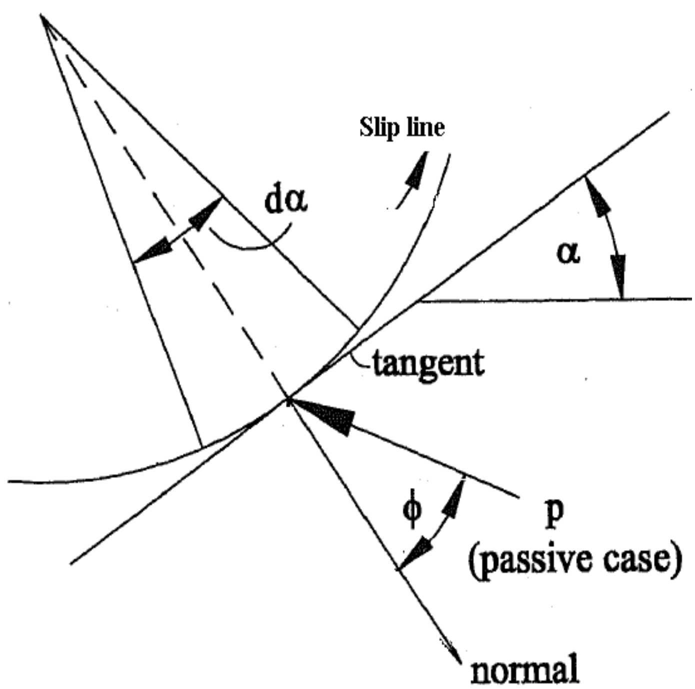

Figure 1: Kötter's (1903) equation for a curved failure surface

For a soil medium cohesionless in nature and in the passive state of equilibrium, Kötter's (1903) equation for a curved failure surface for the plane strain condition is given as

$$\frac{dp}{ds} + 2p\tan\phi\,\frac{d\alpha}{ds} = \gamma\sin(\alpha+\phi) \qquad (14)$$

where, dp is the elemental soil reaction pressure along the failure surface, ds is the elemental failure surface length, ϕ is the soil friction angle, $d\alpha$ is the elemental angle and α is the angle of failure plane made by the tangent at the point under consideration with the horizontal.

The analysis is confined to embedment ratios, $\lambda = H/D \leq 12$. The failure surface geometry corresponds to the frustum of a cone, making an angle α with the horizontal and meeting the ground level.

To compute the vertical soil reaction, R_v acting on the failure surface, Kötter's (1903) equation is integrated.

The breakout resistance is finally obtained with the summation of R_v and total weight, W of soil mass contained in the failure zone.

a) *Failure Surface Geometry*

The angle, α is a function of soil friction angle, ϕ and according to Meyerhof and Adams (1968), α varies in the range, (90° − ϕ/3) to (90° − 2ϕ/3) with an average value of (90° − ϕ/2). Based on this observation and some initial trials, the following expression for α is chosen for the analysis.

$$\alpha = 90 - 2\phi/3 \qquad (13)$$

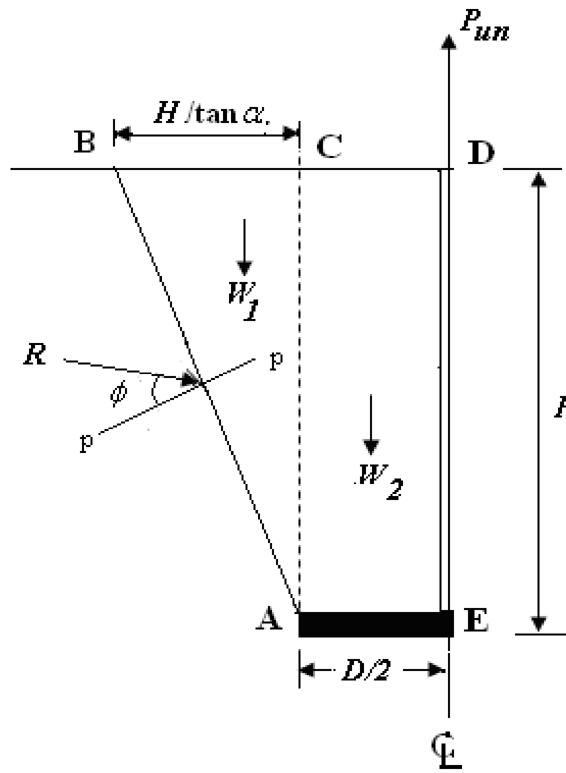

Figure 2: Forces on a failure wedge under plane strain condition

In the force diagram as shown in Fig. 2, AB is a part of the failure wedge, ABC in the case of a strip anchor under plane strain condition. The forces that come into play are the passive thrust P_p, weight ,W_1 of failure wedge ABC and soil reactive force ,R on the failure plane AB. In respect to a plane failure surface ,$d\alpha/ds$ becomes equal to zero and Eq. (14) takes the following form.

$$\frac{dp}{ds} = \gamma \sin(\alpha + \phi) \tag{15}$$

Integration of Eq. (15) gives,

$$p = \gamma \sin(\alpha + \phi)s + C_1 \tag{16}$$

Eq. (16) gives the soil reactive pressure distribution on failure plane, AB, and s is the distance measured from point B (Fig. 2). The integration constant, C_1 in Eq. (16) is obtained from the condition that, pressure p has zero value at point B, corresponding to $s = 0$. Using this condition, C_1 becomes zero and Eq.(16) finally becomes

$$p = \gamma \sin(\alpha + \phi) s \tag{17}$$

c) Soil Reaction for the Axi-symmetric Condition

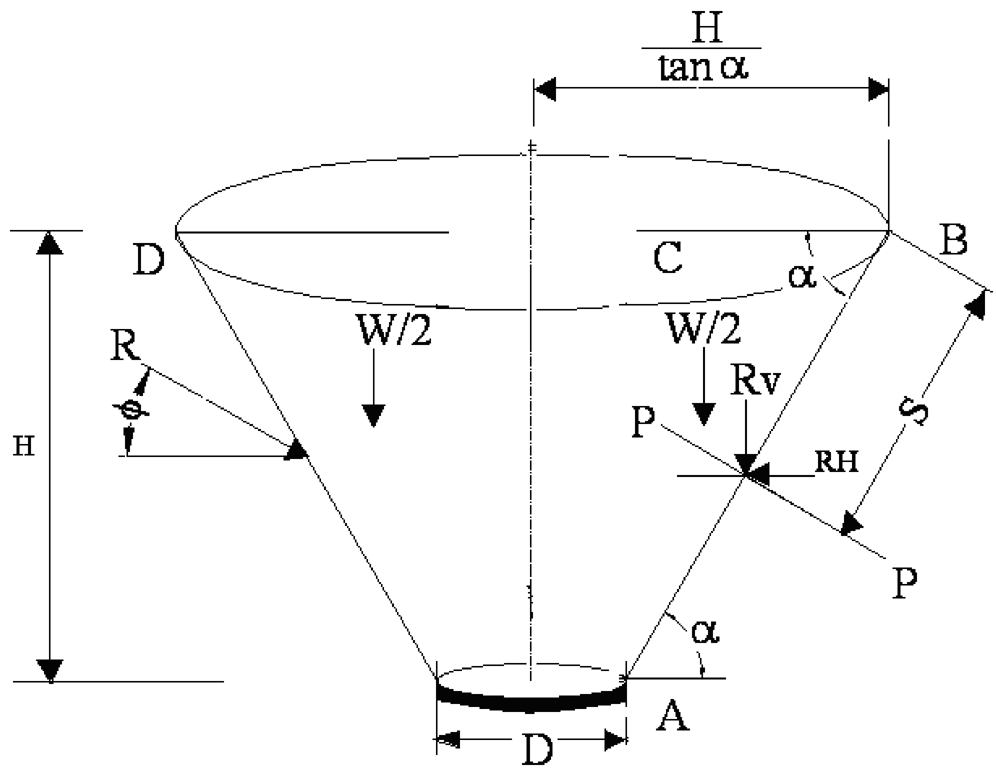

Figure 3a: Free-body diagram for the horizontal circular plate anchor in the axi-symmetric condition

At the instant of breakout of horizontal circular plate anchor in a cohesionless soil medium, failure surface in the form of a conical frustum is developed as shown in Fig. 3a. The breakout force is countered by the vertical component, R_v of the resultant soil reactive force and the weight, *W* of soil.

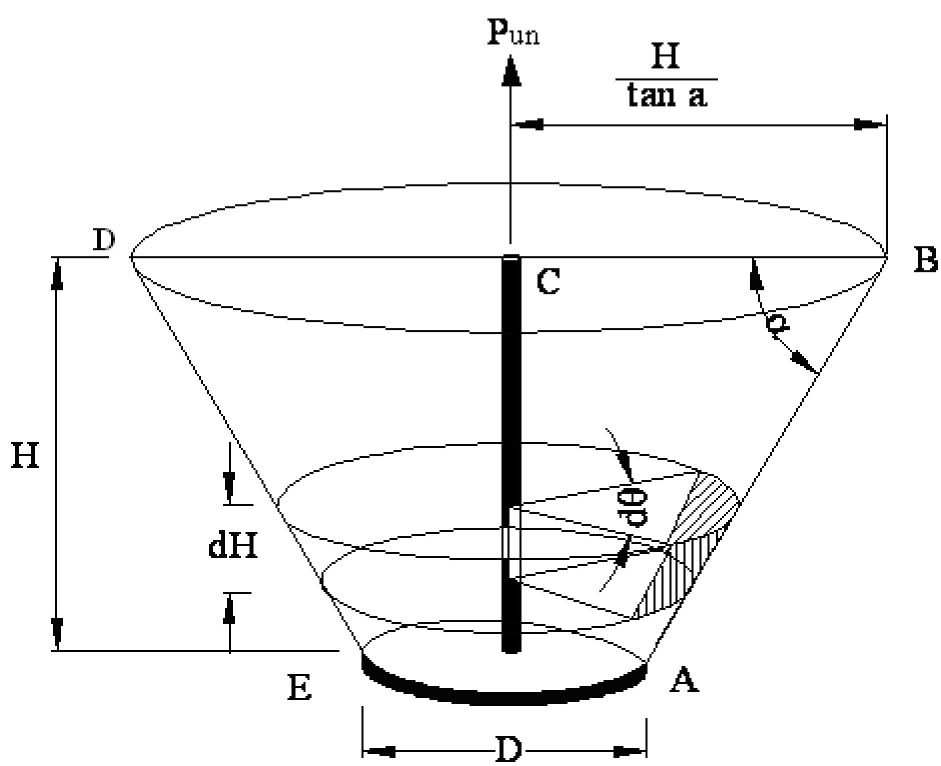

Figure 3b: Axi-symmetric solid body of revolution

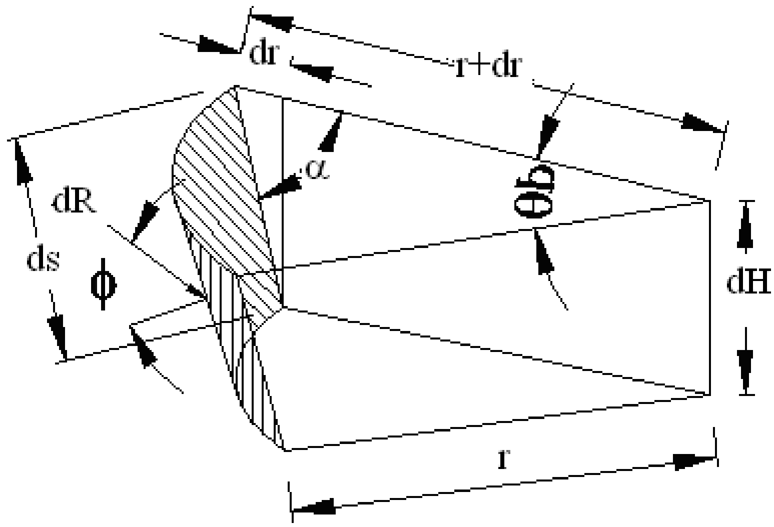

Figure 3c: Elemental forces

In the failure wedge shown in Figs. 3b and 3c, an element making an angle $d\theta$ with radius r is referred. With dp as the elemental reactive pressure, dR becomes the elemental soil reaction on the element area ($r.\ d\theta.$). ds). The height of this element is dH, with a slanted height ds and it is located at a distance, s as measured from the ground surface.

The elemental soil reaction, dR is then expressed as

$$dR = dP.dA \tag{18}$$

where, $dA = r\,d\theta\,ds$

From Fig.3c, $ds = dr/\cos\alpha$

Therefore,

$$dA = r\,d\theta \frac{dr}{\cos\alpha} \tag{19}$$

Substituting Eqs. (18) and (19) into Eq. (17), the elemental soil reaction, dR is obtained as

$$dR = rd\theta \frac{dr}{\cos\alpha} \gamma \sin(\alpha+\phi) s \tag{20}$$

From Fig. 3a, the distance, s is obtained as

$$s = \frac{\left[\dfrac{H}{\tan\alpha} + \dfrac{D}{2}\right] - \left(r + \dfrac{dr}{2}\right)}{\cos\alpha} \tag{21}$$

Substituting Eq. (21) into Eq. (20), the elemental soil reaction, dR is rewritten as

$$dR = \frac{\gamma \sin(\alpha+\phi)}{\cos^2\alpha}\left[\left(\frac{H}{\tan\alpha} + \frac{D}{2}\right) - \left(r + \frac{dr}{2}\right)\right] rdr\ d\theta \tag{22}$$

Or,

$$dR = \frac{\gamma \sin(\alpha+\phi)}{\cos^2 \alpha}\left[\left(\frac{H}{\tan\alpha}+\frac{D}{2}\right) r\,dr - \left(\frac{2r^2\,dr + r dr^2}{2}\right)\right] d\theta \tag{23}$$

With $dr^2 \cong 0$, Eq. (23) becomes

$$dR = \frac{\gamma \sin(\alpha+\phi)}{\cos^2 \alpha}\left[\left(\frac{H}{\tan\alpha}+\frac{D}{2}\right) r dr - r^2\,dr\right] d\theta \tag{24}$$

The elemental vertical component, dR_v is then obtained as

$$dR_v = \frac{\gamma \sin(\alpha+\phi)}{\cos^2 \alpha}\left[\left(\frac{H}{\tan\alpha}+\frac{D}{2}\right) r dr - r^2\,dr\right] \cos(\alpha+\phi) d\theta \tag{25}$$

After performing integration (r varying from $D/2$ to $H/\tan\,\alpha$ and θ varying from 0 to 2π), vertical soil reaction component, R_v is computed as

$$R_v = \frac{\pi\gamma \sin(\alpha+\phi)\cos(\alpha+\phi)}{6\cos^2\phi}\left[\left(\frac{H}{\tan\alpha}+\frac{D}{2}\right)+\frac{D^2}{4}\left(D-3\left(\frac{H}{\tan\alpha}+\frac{D}{2}\right)\right)\right] \tag{26}$$

d) Computation of Weight of Axi-symmetric Solid Body of Revolution

The net weight of the axis-symmetric solid body of revolution is considered into two components; W_1 corresponding to the weight of inverted circular cone and W_2 for the weight of the inverted cone below the circular anchor. Then, the net weight, W of the axis-symmetric solid body of revolution is computed as [Ref. Fig. 2]

$$W = \frac{\gamma\pi\tan\alpha}{3}\left[\left(\frac{H}{\tan\alpha}+\frac{D}{2}\right)^3 - \frac{D^3}{8}\right] \tag{27}$$

e) Net Breakout Resistance

Referring to Fig. 3a and considering vertical force equilibrium, the net breakout resistance, P_{un} is obtained as

$$P_{un} = W - 2R_v \tag{28}$$

Substituting for R_v and W from Eqs. (26) and (27) respectively into Eq. (28) and with some algebraic transformations, the following result is obtained.

$$P_{un} = \frac{\gamma}{6\sin\left(\frac{2}{3}\phi\right)}\left[2\pi\cos\left(\frac{2}{3}\phi\right)\left(C^3 - \frac{D^3}{8}\right) + C^3 + \frac{D^2}{4}(D-3C)\right] \tag{29}$$

where, $C = \left[\frac{D}{2} + H\tan\left(\frac{2}{3}\phi\right)\right]$ and D = diameter of the circular anchor plate.

The above simple expression gives the net breakout resistance of a horizontal circular plate anchor in cohesionless soil medium. It is easy for hand calculations with no need of any tables or graphs.

The breakout factor, F_q is given as

$$F_q = \frac{P_{un}}{\gamma A H} \tag{30}$$

where, A is the area of horizontal circular anchor plate.

III. Comparison with the Experimental Data

The results of theoretical predictions (Balla, 1961; Meyerhof and Adams, 1968; Vesic, 1971; Clemence and Veesaert, 1977; Murray and Geddes, 1987; Saeedy, 1987 and proposed solution) compared with the experimental data (Balla, 1961; Baker and Konder, 1966; Bemben and Kupferman, 1975; Ovesen, 1987; Sutherland et al., 1982; Illampurathi et al., 2002;

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Murray and Geddes, 1987) are presented in Table 1a and comparisons with two field results reported by Sutherland et al. (1982) and Tucker (1987) are presented in Tables 1b and 1c. The percentage deviations of the theoretical solutions with respect to the experimental results are reported in Tables 2a and 2b.

Table 1a: Comparison of breakout factor (F_q) of experimental data with the theoretical solutions

Exp. Results	H m	γ kN/m^3	ϕ(°)	λ	Exp. values	Proposed Method	Method 1	Method 2	Method 3	Method 4	Method 5	Method 6
	0.05		38	0.55	2.96	1.96	1.95	N.A.	N.A.	6.571	2.216	1.63
Balla	0.10		38	1.11	4.45	3.207	3.200	3.0	3.31	9.782	3.826	2,41
(1961)	0.15	18	38	1.68	6.11	4.74	4.768	4.78	5.157	13.517	5.773	3.3
D = 0.09 m	0.20		38	2.22	8.51	6.56	6.258	6.42	7.090	17.913	8.127	4.05
	0.24		38	2.77	11.0	8.66	8.594	7.51	9.476	22.811	10.804	6.27
	0.30		38	3.33	11.78	11.059	10.982	11.20	11.718	28.392	13.902	6.52
Bemben			46	1	5.26	3.61	4.054	N.A.	N.A.	10.608	4.024	2.5
and			46	2	11.13	7.71	9.232	N.A.	N.A.	20.131	8.658	4.68
Kupferman		-	46	3	27.66	20.36	16.534	N.A.	N.A.	32.569	14.091	7.23
(1975)			46	5	40.24	28.91	37.51	N.A.	N.A.	66.19	32.215	14.63

Table 1a: Contd

Exp. Results	H m	γ kN/m^3	ϕ (°)	λ	Exp. values	Proposed Method	Method 1	Method 2	Method 3	Method 4	Method 5	Method 6
Ovesen	0.02		45	1	4.77	3.52	3.83	4.142	3.251	10.407	3.957	4.45
(1987)	0.04		45	2	10	7.44	8.688	7.14	6.569	19.587	8.471	4.615
$D = 0.02$	0.06		45	3	19	12.44	15.415	12.413	11.081	31.540	14.542	7.11
m	0.08	-	45	4	30	19.50	24.064	19.64	16.195	46.265	22.168	10.45
	0.10		45	5	37	27.63	34.635	25.862	N.A.	63.764	31.351	14.36
Murray	0.05		44	1	3.52	3.43	3.72	4.13	3.23	10.21	3.89	2.5
and	0.08		44	1.63	5.4	5.64	6.337	6.02	5.34	15.47	6.483	4.0
Geddes	0.15		44	3	14.54	12.26	14.405	12.32	11.033	30.537	14.182	7.0
(1987)	0.23		44	4.6	27.66	23.15	27.968	23.20	N.A.	54.38	26.74	12
$D = 0.0508$	0.25		44	5	35.19	26.40	32.056	25.86	N.A.	61.406	30.49	14.08
m	0.30	-	44	6	47.25	35.46	43.496	34.81	N.A.	80.79	40.899	19.39

Table 1a: Contd

Exp. Results	H m	γ kN/m^3	ϕ (°)	λ	Exp. values	Proposed Method	Method 1	Method 2	Method 3	Method 4	Method 5	Method 6
	0.08		43	0.84	3.47	2.89	3.06	N.A.	N.A.	8.92	3.290	2.44
Ilamparuthi	0.19		43	1.9	7.13	6.52	7.22	6.97	5.892	17.572	7.604	4.48
et al.	0.28		43	2.87	12.15	11.0	12.59	11.87	10.382	27.986	12.996	6.64
(2002)	0.39	17	43	3.91	18.98	17.29	19.98	17.98	15.20	41.767	20.295	10
D = 0.1 m	0.47		43	4.75	24.74	23.27	27.19	24.50	N.A.	54.874	27.336	13.76
	0.59		43	5.97	35.64	33.54	39.64	33.18	N.A.	77.055	39.387	18.50
	0.69		43	6.91	48.36	42.73	50.83	43.18	N.A.	96.687	51.243	23.35
Sutherland			41	1	4.47	3.21	3.284	3.10	3.170	9.642	3.686	2.41
et al.			41	3	15.76	11.0	11.629	11.30	10.622	27.68	13.105	6.52
(1982)		-	41	4	20	16.73	17.688	16.85	15.849	40.07	19.836	9.66
			41	7	65.15	40.57	43.417	40.0	N.A.	90.77	48.122	22.48
			41	8	85.16	50.82	54.510	50.0	N.A.	112.97	60.247	27.82

Table 1a: Contd

Exp. Results	*H* m	γ kN/m³	ϕ (°)	λ	Exp. values	Proposed Method	Method 1	Method 2	Method 3	Method 4	Method 5	Method 6
Baker and Konder (1966) *D*=0.0756m	0.52	17.9	42	7	40.607	41.642	47.571	41.877	N A	108.951	49.641	24.162
	0.45	17.93	42	6	32.760	32.048	36.622	32.048	N A	74.450	39.635	18.133
	0.37	17.89	42	5	24.543	24.048	27.211	23.785	N A	56.893	29.616	15.088
D =0.0504m	0.45	17.92	42	9	55.140	63.846	73.602	63.043	N A	142.079	78.727	35.319
	0.37	17.92	42	7.5	45.731	46.693	53.500	46.693	N A	56.830	57.496	27.157
	0.30	17.92	42	6	32.695	32.139	36.677	32.139	N A	74.652	39.734	18.061
D = 0.0378 m	0.45	17.97	42	12	68.635	106.073	123.259	105.85	N A	230.974	39.736	N A
	0.37	17.97	42	10	61.657	75.957	88.550	75.957	N A	117.018	85.533	N A
	0.30	17.97	42	8	50.738	51.723	48.275	51.723	N A	142.032	60.261	28.078

N A: Not applicable

Note: Method 1: Meyerhof and Adams (1968)
Method 2: Saeedy (1987)
Method 3: Balla (1961)
Method 4: Clemence and Veesaert (1977)
Method 5: Murray and Geddes (1987)
Method 6: Vesic (1971)

Table 1b: Comparison of net breakout resistance (P_{un} in kN) of field tests data with the theoretical methods

Field Test Results	*H* m	γ kN/m³	ϕ (°)	λ	Field Test	Proposed Method	Method 1	Method 2	Method 3	Method 4	Method 5	Method 6
Sutherland et al. (1982) *D* = 2.39 m	4.57	10.37	42	1.91	1601	1351	1544	1445	1244	3655	1589	938.6
	5.18		42	2.17	2251	1777	2067	1660	1777	4738	2109	1079
	6.4		42	2.67	2064	2582	2553	2051	2195	5854	2553	1333.9
	7.0		42	2.94	2562	2659	4237	3702	3476	9088	4263	2201

Note:
Method 1. Meyerhof and Adams (1968)
Method 2. Saeedy (1987)
Method 3.Balla (1961)
Method 4.Clemence and Veesaert (1977)
Method 5. Murray and Geddes (1987)
Method 6.Vesic (1971)

Table 1c: Comparison of breakout factor (F_q) of field tests data with the theoretical methods

Field Test Results	H m	γ kN/m³	ϕ (°)	λ	Field Test	Proposed Method	Method 1	Method 2	Method 3	Method 4	Method 5	Method 6
	1.68		38	1.38	4.73	3.91	4.412	3.95	4.12	11.54	4.737	3.0
	1.93		42	1.59	7.95	5.14	5.80	5.18	4.957	14.38	6.036	3.41
Tucker	1.915		41.5	1.57	6.29	4.98	5.63	5.10	4.95	14.03	5.88	3.36
(1987)	1.732	10.37	41.5	1.42	6.69	4.48	5.021	4.78	4.39	12.83	5.273	3.32
D = 1.22 m	2.147		41.5	1.76	4.67	5.66	6.46	6.23	5.56	15.79	6.707	4.15
	1.952		41.5	1.6	7.09	5.09	5.761	5.18	4.94	14.28	6.0128	4.10
	2.196		41.5	1.8	7.27	5.95	6.82	7.02	5.86	16.33	7.068	4.29

Note:

Method 1. Meyerhof and Adams (1968)
Method 2.Saeedy (1987)
Method 3. Balla (1961)
Method 4: Clemence and Veesaert (1977)
Method 5. Murray and Geddes (1987)
Method 6. Vesic (1971)

Table 2a: Comparison of % deviations of the proposed and other theoretical methods with the experimental data

Exp. Results	H m	γ kN/m³	ϕ (°)	λ	Proposed Method	Method 1	Method 2	Method 3	Method 4	Method 5	Method 6
0.05			38	0.55	-33.784	-34.122	N.A.	N.A.	12.199	-25.135	-44.932
Balla	0.10		38	1.11	-27.933	-28.090	-32.584	-25.618	11.982	19.563	N.A.
(1961)	0.15		38	1.68	-22.422	-21.964	-21.768	-15.597	12.123	-5.516	-45.990
D = 0.09 m	0.20	18	38	2.22	-22.914	-26.463	-24.559	-16.686	11.049	-4.501	-52.409
	0.24		38	2.77	-21.273	-21.873	-31.727	-13.855	10.737	-1.782	-43.000
	0.30		38	3.33	-6.121	-6.774	-4.924	-0.526	14.102	18.014	-44.652

Bemben and			46	1	-31.369	-22.928	N.A.	N.A.	10.167	-23.498	-52.471
Kupferman			46	2	-30.728	-17.053	N.A.	N.A.	8.087	-22.210	-57.951
(1975)			46	3	-26.392	-40.224	N.A.	N.A.	1.775	-49.056	-73.861
			46	5	-28.156	-6.784	N.A.	N.A.	6.449	-19.943	-63.643

Table 2a: Contd.

Exp. Results	H m	γ kN/m³	ϕ (°)	λ	Proposed Method	Method 1	Method 2	Method 3	Method 4	Method 5	Method 6
Baker and	0.52	17.9	42	7	-2.174	11.789	-2.174	N.A.	127.258	20.987	-43.395
Konder	0.45	17.93	42	6	-2.013	10.872	-3.087	N.A.	131.812	20.671	-33.96
(1966) D=0.0756m	0.37	17.89	42	5	15.789	33.483	14.334	N.A.	157.671	42.777	N.A.
	0.45	17.92	42	9	2.104	16.99	2.104	N.A.	24.272	25.728	26.861
D =0.0504m	0.37	17.92	42	7.5	-1.7	12.181	-1.7	N.A.	128.329	21.53	27.762
	0.30	17.92	42	6	54.545	79.585	54.226	N.A.	236.523	-42.105	N.A.
	0.45	17.97	42	12	23.191	43.617	23.191	N.A.	89.787	38.723	N.A.
D= 0.0378m	0.37	17.97	42	10	1.942	-4.854	1.942	N.A.	179.935	18.77	122.33
	0.30	17.97	42	8	-31.369	-22.928	N.A.	N.A.	10.167	-23.498	-52.471

Table 2a: Contd.

Exp. Results	H m	γ kN/m³	ϕ (°)	λ	Proposed Method	Method 1	Method 2	Method 3	Method 4	Method 5	Method 6
	0.02		45	1	-26.205	-19.706	-13.166	-31.845	11.943	-17.044	-6.709
Ovesen	0.04		45	2	-25.600	-13.120	-28.600	-34.310	9.587	-15.290	-53.850
(1987)	0.06	-	45	3	-34.526	-18.868	-34.668	-41.679	6.600	-23.463	-62.579
D = 0.02 m	0.08		45	4	-35.000	-19.787	-34.533	-46.017	5.422	-26.107	-65.167
	0.10		45	5	-19.919	-6.392	-30.103	N.A.	7.234	-15.268	-61.189

Murray	0.05		44	1	-2.557	5.682	17.330	-8.239	19.006	10.511	-28.977
and	0.08		44	1.63	4.444	17.352	11.481	-1.111	18.648	20.056	-25.926
Geddes	0.15	-	44	3	-15.681	-0.928	-15.268	-24.120	11.002	-2.462	-51.857
(1987)	0.23		44	4.6	-16.305	1.114	-16.124	N.A.	9.660	-3.326	-56.616
D = 0.0508	0.25		44	5	-24.979	-8.906	-26.51	N.A.	7.45	-13.356	-59.99
m	0.30		44	6	-24.952	-7.945	-26.328	N.A.	7.098	-13.441	-58.963

Table 2a: Contd.

Exp. Results	H m	γ kN/m^3	ϕ (°)	λ	Proposed Method	Method 1	Method 2	Method 3	Method 4	Method 5	Method 6
	0.08		43	0.84	-16.715	-11.816	N.A.	N.A.	15.706	-5.187	-29.683
Ilamparuthi	0.19	17	43	1.9	-8.555	1.262	-2.244	-17.363	14.645	6.648	-37.167
et al.	0.28		43	2.87	-9.465	3.621	-2.305	-14.551	13.034	6.963	-45.350
(2002)	0.39		43	3.91	-8.904	5.269	-5.269	-19.916	12.006	6.928	-47.313
D = 0.1 m	0.47	17	43	4.75	-5.942	9.903	-0.970	N.A.	12.180	10.493	-44.382
	0.59		43	5.97	-5.892	11.223	-6.902	N.A.	11.620	10.513	-48.092
	0.69		43	6.91	-11.642	5.108	-10.711	N.A.	9.993	5.962	-51.716
			41	1	-28.188	-26.532	-30.649	-29.083	11.570	-17.539	-46.085
Sutherland			41	3	-30.203	-26.212	-28.299	-32.602	7.563	-16.846	-58.629
et al.		-	41	4	-16.350	-11.560	-15.750	-20.755	10.035	-0.820	-51.700
(1982)			41	7	-37.728	-33.358	-38.603	N.A.	3.932	-26.137	-65.495
			41	8	-40.324	-35.991	-41.287	N.A.	3.266	-29.254	-67.332

Table 2b: Comparison of % deviations of the proposed and other theoretical methods with the field data

Field Test Results	H m	γ kN/m^3	ϕ (°)	λ	Proposed Method	Method 1	Method 2	Method 3	Method 4	Method 5	Method 6
Sutherland et al.	4.57		42	1.91	-15.61	-3.56	-9.744	-22.30	128.29	-0.74	-41.37
	5.18		42	2.17	-21.05	-8.174	-26.25	-21.05	110.48	-6.30	-52.06
(1982)	6.4	10.37	42	2.67	25.09	23.692	-0.63	6.347	183.62	23.69	-35.37
D = 2.39m	7.0		42	2.94	3.786	65.379	44.49	35.675	254.72	66.39	-14.07
	1.68		38	1.38	-17.33	-6.72	-16.49	-12.89	143.97	0.148	-32.004
Tucker	1.93		42	1.59	-35.34	-27.04	-34.84	-37.65	80.88	-24.07	-41.207
(1987)	1.91		41.5	1.57	-20.82	-10.49	-18.92	-21.30	123.05	-6.518	-40.320
D = 1.22	1.73	10.37	41.5	1.42	-33.03	-24.95	-28.55	-34.38	91.77	-21.18	-33.878
m	2.14		41.5	1.76	21.2	38.33	33.405	19.06	238.11	43.618	-35.759
	1.95		41.5	1.6	-28.21	-18.74	-26.94	-30.32	101.41	-15.19	-28.832
	2.19		41.5	1.8	-18.15	-6.19	-3.44	-19.39	124.62	-2.7785	-37.097

Note: Method 1: Meyerhof and Adams (1968)
Method 2: Saeedy (1987)
Method 3: Balla (1961)
Method 4: Clemence and Veesaert (1977)
Method 5: Murray and Geddes (1987)
Method 6: Vesic (1971)

For a better understanding of the relative predictive capability of the proposed solution, a cumulative frequency distribution of the data corresponding to the percentage deviations is further reported in Tables 3a and 3b.

Table 3a: Cumulative frequency distribution of individual deviations

Absolute deviation (%)	Proposed Method	Method 1	Method 2	Method 3	Method 4	Method 5	Method 6
0-5	9	6	12	2	4	8	0
5-10	6	12	3	2	10	8	1
10-15	1	8	4	3	16	5	0
15-20	9	8	6	6	3	10	0
20-25	9	5	3	5	1	11	0
25-30	8	5	7	2	0	5	6
30-35	7	3	8	5	0	0	3
35-40	2	2	1	2	0	1	4
40-45	1	2	2	1	0	3	8
45-50	0	0	0	1	0	1	8
> 50	1	2	1	0	19	1	19

Note:

Method 1: Meyerhof and Adams (1968)
Method 2: Saeedy (1987)
Method 3: Balla (1961)
Method 4: Clemence and Veesaert (1977)
Method 5: Murray and Geddes (1987)
Method 6: Vesic (1971)

Table 3b: Cumulative frequency distribution of cumulative deviations

Absolute deviation (%)	Proposed Method	Method 1	Method 2	Method 3	Method 4	Method 5	Method 6
0-5	9	6	12	2	4	8	0
5-10	15	18	15	4	14	16	1
10-15	16	26	19	7	30	21	1
15-20	25	34	25	13	33	31	1
20-25	34	39	28	18	34	42	1

25-30	42	44	35	20	34	47	7
30-35	49	47	43	25	34	47	10
35-40	51	49	44	27	34	48	14
40-45	52	51	46	28	34	51	22
45-50	52	51	46	29	34	52	30
> 50	53	53	47	29	53	53	49

Note: Method 1: Meyerhof and Adams (1968)
Method 2: Saeedy (1987)
Method 3: Balla (1961)
Method 4: Clemence and Veesaert (1977)
Method 5: Murray and Geddes (1987)
Method 6: Vesic (1971)

From Tables 3a and 3b it is seen that, in 28 out of 29 cases, Balla's (1961) theoretical method shows sabsolute deviations in the range of 2% to 45%. The solution proposed by Meyerhof and Adams (1968) shows deviations in the range, 2% to 45% in 51 cases and in the remaining cases, the range is 55% to 100%.

Predictions based on the solution proposed by Vesic (1971) show deviations in the range of 2% to 45% for 22 cases and in the remaining 27 cases, the deviations are as high as 50% to 100%.

The method of Clemence and Veesaert (1977) shows deviations in the range, 2% to 45% for 34 cases and in the remaining 19 cases, the deviations are as high as 50% to 100%. The solution proposed by Murray and Geddes (1987) shows absolute deviations in the range of 2% to 45% for 51 cases and in the remaining 2 cases, the deviations are as high as 50% to 100%. Saeedy's (1987) method shows deviations in the range, 2% to 45% in 46 cases and in the remaining case, the range is 55% to 100%.

The proposed solution shows deviations in the range, 2% to 45% in 52 cases and in the remaining case, the range is 55% to 100%. Proposed solution and Saeedy's (1987) method show errors in the range, 0% to 5% in 9 and 12 cases respectively, whereas, in respect to the other methods, only 0 to 8 cases show deviations in this range.

From the above discussion it is seen that, Balla's (1961) method makes better predictions in 96% of the cases when compared to the experimental data.

In general, Balla's (1961) method shows a good agreement for dense sand up-to the embedment ratio of 5. It requires a chart for using the required functions. Vesic's (1971) method shows a good performance in 45% of the cases. However, it also requires a chart or table for using a proper value of the breakout coefficient.

The method of Meyerhof and Adams (1968) makes good predictions in 96% of the cases; but two charts are needed to select the proper values of the net breakout factor and the shape coefficient. The method of Clemence and Veesaert (1977) makes good predictions in only 64% cases. It involves an assumption in respect to the coefficient of earth pressure at rest.

The proposed analysis method considers failure surface in the form of frustum of a cone. It makes predictions that are very close to the experimental values in 98% cases. Thus, the performance appears to be superior to the other methods. Although the proposed analysis makes an approximation while using Kötter's (1903) equation, it is improved with a proper selection of the angle, α as per Eq. (12). The integration is fairly simple, yielding a closed form expression for the net uplift resistance (Eq. 29), which is easy for calculations, with no need for graphs or tables. Kötter's (1903) equation plays a significant role in the analysis.

IV. Conclusions

The proposed analysis method is simple giving a closed form solution. It is also easy for hand calculations. Kötter's (1903) equation is successfully employed for axi-symmetric conditions with a proper choice of angle at which the failure surface intersects the ground level. No assumptions are necessary for the coefficient of earth pressure and the results show a very close agreement with the experimental data.

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List of symbols

The following symbols are used in this paper.

A = area of circular anchor plate
C_1 = integration constant
dp = elemental soil reactive pressure
dR = elemental soil reaction
dR_v = elemental vertical component
ds = elemental failure surface length
$d\alpha$ = elemental angle
D = diameter of circular anchor plate
F_q = breakout factor
H = height of circular anchor plate
p = soil reactive pressure distribution
P_p = passive thrust
P_u = ultimate breakout resistance
P_{un} = net breakout resistance
R = soil reactive force on the failure plane
R_v = vertical soil reaction component
W_1 = weight of inverted circular cone
W_2 = weight of the inverted cone below the circular anchor
W = net weight of the axis-symmetric solid body of revolution
α = inclination of failure plane with the horizontal
ϕ = soil friction angle
γ = unit weight of soil
λ = embedment ratio = H/D