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By Victor Bocharnikov

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I. Introduction

Any space-time signal can be described by a set of basic functions [1]. This allows us to obtain a signal spectrum that reflects the basis functions proportion of the content in the original signal. Such a decomposition is often useful for signal analysis. For example, the spectral representation is effective for analyzing the compressibility of signals, the synthesis of compression algorithms with minimal losses, the signals filtering problems solving, the synthesis of optimal regulators, etc.

The transition to the spectrum can be carried out using orthogonal and unitary transformations. Most often, to obtain a spectrum, the decomposition in orthogonal functions is used [2]. For example, the spectra obtained on the basis of decomposition into a Fourier series [3] (with harmonic basis), Walsh series [4] (using a non-harmonic orthogonal system of rectangular functions with values of ± 1), wavelet transform [5, 6], etc. In [7], a new approach to the time-frequency analysis of non-stationary signals was proposed. In this paper, we consider an algorithm implementing the approach developed in [7] and present the results of its application to the time-frequency analysis of non-stationary signals.

II. Research and Publications Analysis and the Problem Statement

A detailed analysis of existing approaches to the spectral-temporal analysis of signals is given in [7]. From the analysis it follows that the following elements are common to all the main approaches:

1. Arbitrary of the function $\varphi(t)$ tend to be decomposed into a set of basis functions $\{\Phi(k,t)\}$. Such functions form, as a rule, an orthogonal basis. Each function from this system conditionally plays the role of a coordinates axis.
2. To determine the projection on such an coordinates axis, the integral convolution is used $c_k = \int_0^T [\varphi(t) \cdot \Phi(k,t)]dt$, where c_k are decomposition coefficients determining the degree of coincidence of the original function $\varphi(t)$ and $\Phi(k,t)$. The values of c_k are perceived as coordinates on the respective axes of the basis orthogonal functions. Thus, the convolution integral or its discrete analogue is perceived as a measure of the similarity of the original function $\varphi(t)$ and the function $\Phi(k,t)$.

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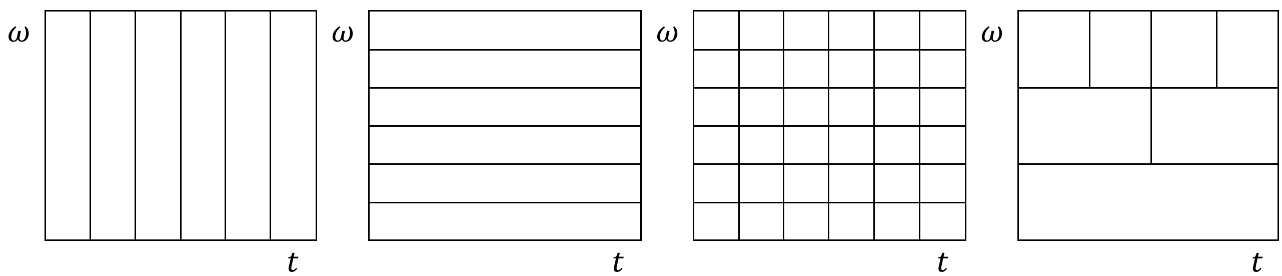
3. To study non-stationary signals and obtain the time-frequency decomposition of a signal a "window" is used, which is "cutting out" a portion of the function $\varphi(t)$ over a certain time interval. For each time interval, a measure of similarity with the basis functions $\Phi(k,t)$ is found.

4. Window accounting when analyzing the original function can be carried out explicitly or implicitly. In the first case, the window function $\varpi(t)$ is explicitly introduced into the expression for integral convolution. This function may have various forms, for example, in the form of a Gaussian function. In the second case, the window is accounted for by direct selection of the basis orthogonal functions, which are called bursts or wavelets. The use of wavelets allows to further determine the frequency localization.

5. Depending on the selected basic orthogonal functions and window functions, different time-frequency localization is obtained (Fig. 1). In any case, such localization is determined in advance before the beginning of the analysis and affects its result.



a) b) c) d)

Fig. 1: Time-frequency localization for various approaches:

a) Time Localization at Shannon signal discretization;
b) Frequency localization during Fourier transform;
c) Window transformations of the instantaneous spectrum methods [8] (D. Gabor, J. Ville etc.)
d) Wavelet transform [9] (I. Daubechies, Y. Meyer, R. Coifman, etc.)

Analysis of existing approaches to the frequency-time analysis of signals revealed a number of important points:

1. The "grid" of the time-frequency localization superimposed on the original signal may not "coincide" with the characteristics of the original function $\varphi(t)$, which leads to distortions of the time-frequency signal evaluation. That is why in the existing approaches special attention is paid to the selection of basic functions and window modeling functions.
2. In some cases, such as when using wavelet transforms, the time window has fuzzy boundaries, which leads to distortions of the time-frequency estimate due to the windows overlapping.
3. The choice of functions integral convolution instrument as a measure of their similarity affects the assessment of initial and basic functions coincidence degree. The measure of similarity can be constructed in other ways and be more effective. In addition, in practice, the use of integral convolution introduces its own errors, which are associated with numerical integration methods.
4. The rigid "grid" of time-frequency localization used in existing approaches is not adaptive. It does not take into account the behavior of the non-stationary function of the signal. At present, localization is more effective, which is used in wavelet transforms (Fig. 1d). However, it is also rigid, which leads to the need for a more careful selection of wavelet basis functions. Selection of basic functions adapted to a signal is used in the method of signal analysis proposed by N. Huang (Huang-Hilbert Transform) [10]. However, studies have shown that this approach also has several disadvantages [11].
5. It can be assumed that if the function $\varphi(t)$ behaves not stationary, then the time-frequency localization should be adaptive and adapt to the behavior of the signal under study (Fig. 2).

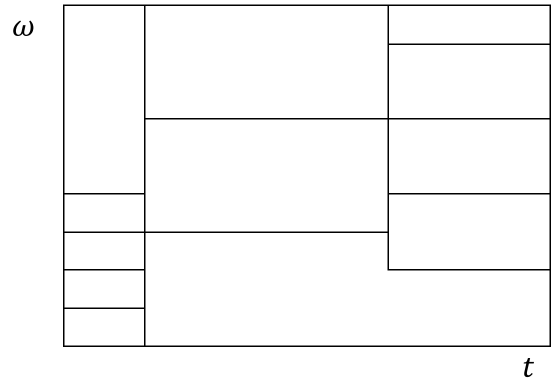


Fig. 2: Adaptive time-frequency localization

Thus, to ensure the adaptation of the time-frequency signal localization, it is necessary to determine the local areas of stationary behavior for the system that generates the signal. These areas are defined as a locally-invariant constraints on system behavior.

Earlier, theoretical studies were presented in [7], which allow us to obtain estimates of the frequency spectrum of a signal in an adaptive version. The approach is based on the fact that the behavior of the system has locally invariant sections, in which the signal spectrum is relatively stable. To identify these sections, it was proposed to use a behavior functions change in the system [12]. This behavior function is proposed to build using the mathematical basis of the p - adic calculus in the form of possibility measure distribution. The moments of the behavior function change allow us to determine the coordinates of the impulse function that models the original signal. It is noteworthy that periodic functions (for example, harmonic) are described by impulse functions, in which the coordinates of the pulses correspond to the members of arithmetic progressions In the above presented work, an approach is proposed to determine the current signal spectrum estimates based on the coincidence degree of the impulse functions of this signal and the harmonic signals of the basis functions. To ensure the effectiveness of the approach, it is necessary to develop an algorithm for solving the problem and test its workability for non-stationary signals.

III. The Purpose and Tasks of the Study

The purpose of the study, the results of which are presented in the article, was to develop an algorithm for solving the problem of frequency-time analysis of signals, which implements the approach proposed in [7], as well as testing the efficiency of the algorithm based on a model example of a non-stationary signal. To achieve the goal, three tasks were solved:

1. Development of a time-frequency analysis algorithm for signals based on behavior functions and arithmetic series.
2. Determination of the signal spectrum under study based on the algorithm, comparison of the results obtained with the ideal spectrum of the model signal and errors estimation.
3. Estimation of the error in the restoration of the original signal based on the obtained spectra.

IV. Research Methods

To solve these problems the following methods were used: the theory of fuzzy sets and fuzzy measures; system analysis based on changes in system behavior; p-adic analysis; linear programming; discrete signal modeling; spectral analysis

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V. Research Results

Based on the approach proposed in [7], an algorithm for time-frequency analysis of non-stationary signals was developed. The algorithm allows to obtain the spectrum of the signal at each point in time when observing the signal. The delay in estimating the spectrum is determined by the time for the formation of the observation window. Below is the synthesized algorithm for solving the problem:

Step. 1. Initial data set:

1. The observation time W and the sampling interval of the signal (for example, in seconds).
2. The set of basis functions of impulse sines [7] $\Phi = \{ si_k(t)\}, k = \overline{1, N}, t \in W$. Pulse sines are called impulse functions, in which the coordinates of the pulses are determined by the formula of an arithmetic progression:

$$a_m^k = a_1^k + (m-1) \cdot d_k,$$

where $a_m^k \in W$, k – index k–th sinusoid, $d_k = const$ – is a step of arithmetic progression. In fact, the functions $si_k(t)$ are given by two parameters a_1^k and d_k, which determine the phase and frequency of the harmonic signal, respectively.

3. The basis of the p - adic number ($p \geq 2$), as well as the number of L observation channel blocks [13] to represent the data that describe the signal. The number of blocks is usually from 3 to 9.
4. The set of shift parameters $\rho_k \in Z, k = \overline{1, |M|}$ is determined for the whole set of sample variables M [14] describing the signal.
5. The set of cut-off thresholds $\{\Delta_u\}, \Delta_u \in [0; 1]$ to determine the impulse function. Thresholds are chosen at regular intervals in the range 0.12 – 0.25 in the amount of 10 – 15.
6. The algorithm parameter for determining the pulses coordinates for the impulse function of the studied series: an integer $m \geq 1$.
7. Select the type of window. There are four options:
   a. Asymmetrical window. The boundaries of the window lie in the middle between the coordinates of nearby pulses of the signal that is being studied. Calculation of the window using the formula:

$$LR_n = [\tau_n - \varepsilon_L(n); \tau_n + \varepsilon_R(n)] \subseteq W.$$

   where $\varepsilon_L(n) = 0.5 \cdot (\tau_n - \tau_{n-1})$, $\varepsilon_R(n) = 0.5 \cdot (\tau_{n+1} - \tau_n)$, τ_n – pulse coordinate of the signal under study;

   b. Minimum window. Calculation of the window LR_n from the condition:

$$\varepsilon_L(n) = \varepsilon_R(n) = \varepsilon(n) = min\{\varepsilon_L(n); \varepsilon_R(n)\};$$

   c. Maximum window. Calculation of the window LR_n from the condition:

$$\varepsilon_L(n) = \varepsilon_R(n) = \varepsilon(n) = max\{\varepsilon_L(n); \varepsilon_R(n)\};$$

   d. Window with averaged deviation values. Calculation of the window using the formula:

$$LR_n = [\tau_n - 0.25 \cdot (\tau_{n+1} - \tau_{n-1}); \tau_n + 0.25 \cdot (\tau_{n+1} - \tau_{n-1})] \subseteq W.$$

8. A method is chosen for determining the influence degrees of the k-th impulse sine function from a set of basis functions Φ for building the signal spectrum: method 1 - based on optimization (linear programming); method 2 - on the basis of an approximate approach.

Step 2. The maximum normalizing integer for the time series is determined on the basis of the p - adic number formula of the form [15]:

$$b_{max} = \sum_{l=0}^{L} \alpha_l \cdot p^l,$$

where $\forall l \in \{0, \dots, L\}$ $\alpha_l = p - 1$.

Step 3. All values of the time series of the signal under study φ_n are normalized so that the minimum value of the series is 0, and the maximum value of the series b_{max}. The resulting values are rounded to integers.

Step 4. Each value of the series is decomposed into p - adic number and is represented in canonical form [16]: $b(t) = \{\alpha_0, \alpha_1, \dots, \alpha_l, \dots, \alpha_L\}$. In fact, this is a representation of a number in the p - adic number system. If $p = 2$, then these are binary numbers. The value of α_l determines the value of the l-th variable.

Step 5. For each point in time, the confidence distribution is calculated for each system variable describing the signal. Let the value of the system variable $v_i \in V_i$, be determined on the set of states $V_i = \{v_{i1}, \dots, v_{i,L+1}\}$, where $v_{ij} = \alpha_{l+1}$, $j = l + 1, l = \overline{0, L}$. Then the state of the system for this variable for $t \in W$ is determined by the distribution function of the possibility $\mu_t(v_{i,j}): W \times V_i \to [0,1]$, which is given on the basis of p - adic number $b(t)$ in the form:

$$\mu_t(v_{ij}) = \alpha_{l+1}(t) \cdot \left(\max_{l=\overline{0,L}} \alpha_l(t)\right)^{-1},$$

where $\alpha_l(t)$ – value of l-th element of canonical form p - adic number with $t \in W$.

Step 6. A set of sample variables that determine the current state of the signal is defined. Sample variables are given by the relation $s_{k,t} = v_{i,\zeta_k(t)} \in V_i \equiv S_k$,, where $s_{k,t}$ is the state of the k-th sample variable with the parameter $t \in W$, $v_{i,\zeta_k(t)} \in V_i$ - the state of the variable $v_i \in V_i$ when the value of the parameter $\zeta_k(t) = t + \rho_k$, $\rho_k \in Z$. The full set of signal states will be defined as $C = S_1 \times S_2 \times \dots \times S_{|M|}$, where $|M|$ - the power of the set of sample variables. The distribution of the possibility on the set of values of the sample variable is defined as $\mu_t(s_{k,j}) = \mu_{\zeta_k(t)}(v_{i,j}) \in [0,1]$.

Step 7. The pulses coordinate of the time series of the signal under study φ_n are determined. The algorithm is calculated for all thresholds $\{\Delta_u\}$. It is carried out iteratively on the basis of the metasystem identification subalgorithm described below:

Step. 7.1. At the first step, the initial conditions for the parameter $t = 1$ and the coefficient of the algorithm $k = 1$ are accepted.

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Step 7.2. For the data subset $[t, t+m]$, the behavior function $f_1(c)$ is defined as the distribution of the possibility by the formula:

$$f(c) = \left\{\sum_{t\in\Delta W} f_t(c)\right\} \cdot \left\{\max_{e\in C} \sum_{t\in\Delta W} f_t(e)\right\}^{-1}.$$

where

$$f_t(c) = \min_{k=\overline{1,|M|}} \{\mu_t(s_k[c])\},$$

where $c \in C$ is the system state. The specific sample variable s_k in the state $c \in C$ takes the value $s_k[c] \in S_k \equiv V_i$. $\mu_t(s_k[c])$ - the possibility to observe the state $c \in C$ with the sample variable s_k at the time $t \in W$.

Step 7.3. The index value of generating fuzziness of the system [17] $U(f_1(c))$ is calculated by the formula:

$$U(f(c)) = \sum_{j=1}^{|C|} \left(f(c_j) - f(c_{j+1})\right) \cdot \log_2(j),$$

where $f(c)$ is ordered by descending of $\forall j, f(c_j) \geq f(c_{j+1})$ behavior function with a fictitious element $f(c_{|C|+1}) = 0$, |C| - power set of states.

Step 7.4. $k = k + 1$. is given. $t + k \cdot m \notin W$, then go to step 7.7.

Step 7.5. The behavior function $f_k(c)$ is determined for the data subset $[t, t + k \cdot m] \subseteq W$ and the generating fuzziness $U(f_k(c))$ is determined.

Step 7.6. If $|U(f_k(c)) - U(f_{k-1}(c))|/\max\ \left(U(f_k(c)), U(f_{k-1}(c))\right) < \Delta_u$, then go to Step 7.4. If the condition is not satisfied, then the point $t + (k-1) \cdot m \in W$ is taken as an approximation of the replacement point of the metasystem elements. For this point, the value $k = 1$ is assumed and the transition to Step 7.2 is performed.

Step 7.7. Stop. At the moments of change in the behavior of the system, single impulses form. For a fixed threshold $\Delta_u \in [0,1], u = \overline{1, N_\Delta}$ for discrete moments $t \in W$, we obtain a two-dimensional impulse function:

$$r(u,t) = \begin{cases} 1, t = \tau_n; \\ 0, t \neq \tau_n. \end{cases}$$

Step 8. There is a generalized impulse function of the signal under study by the formula:

$$g(t) = \sum_{\Delta_u \in \{\Delta\}} \Delta_u \cdot r(u,t),$$

where $r(u,t)$ is the impulse function for the threshold Δ_u of the metasystem identification algorithm.

The time coordinates $\tau_n \in W$ of the approximating impulse function $r(t)$ for the signal under study are defined as the local maxima of the function $g(t)$. The coordinates τ_n will specify a numeric sequence with variable step $d(j)$. The total number of pulses of the function under study N. Thus, there will be N local intervals, where the original signal has relatively stable frequency characteristics.

Step.9. For each pulse τ_n, a window $LR_n \subseteq W$ is determined based on the selected window type in accordance with Step 1.

Step 10. For each pulse of the signal under investigation with the coordinate τ_n, for each basis function of the pulse sines, the coefficients of the balance equation [7] are determined by the formula:

$$\Lambda_k(\tau_n) = \{\beta_k(\tau_n) \cdot \tau_n - S_k(LR_n)\},$$

where the coefficient $\beta_k(\tau_n) = \sum_{a_m^k \in LR_n} (-1)^{m-1} \in \{-1,0,1\}$. $S_k(LR_n)$ – is the partial sum of the sign-variable number series obtained from the arithmetic progression for the pulse sine $si_k(t)$:

$$\sum_{a_m^k \in LR_n} (-1)^{m-1} \cdot a_m^k = S_k(LR_n),$$

where $LR_n \subseteq W$ is the time window near the pulse coordinate τ_n of the function under study. The partial sum $S_k(LR_n)$ takes into account the influence of all the pulses of the function $si_k(t)$ falling into the window LR_n.

Step 9. For each k-th impulse sine function, the degree of its influence on the formation of the resulting impulse $r(\tau_n)$ is determined. For each pulse $r(\tau_n)$ the determination of the coefficients of the influence degrees $x_k(\tau_n) \in [0,1]$ is found either by the optimization method (method 1) or by the approximate method (method 2).

According to method 1, the coefficients $x_k(\tau_n) \in [0,1]$ are found from the solution of an optimization linear programming problem based on the simplex method [18]:

$$\begin{cases} \sum_{k=1}^{K} x_k(\tau_n) \cdot \Lambda_k(\tau_n) \to \min\limits_{\{x_k(\tau_n)\}} \\ \sum_{k=1}^{K} x_k(\tau_n) = 1, \forall k, x_k(\tau_n) \geq 0, \end{cases}$$

According to method 2, the coefficients $x_k(\tau_n) \in [0,1]$ are found by the formula:

$$x_k(\tau_n) = \begin{cases} |\beta_k(\tau_n)| \cdot \left[1 - \dfrac{2 \cdot |\Lambda_k(\tau_n)|}{(\varepsilon_R(n) + \varepsilon_L(n))}\right], & S_k(LR_n) \neq 0, \\ x_k(\tau_n) = \lambda_k \cdot x(\tau_{n-1}), & S_k(LR_n) = 0, \lambda_k \in [0,1]. \end{cases}$$

where $\varepsilon_R(n)$ and $\varepsilon_L(n)$ are the right and left deviations from the coordinate of the pulse τ_n. Coefficient $\lambda_k \in [0,1]$ is determined from the correct consideration condition of the low-frequency components of the signal spectrum:

$$\lambda_k = \begin{cases} 0, & \left| \tau_n > \max_{\bigcup_{j\in\{1,n-1\}} LR_j} (a_m^k) + d_k \right. \\ 1, & \left| \tau_n \le \max_{\bigcup_{j\in\{1,n-1\}} LR_j} (a_m^k) + d_k \right. \\ [0{,}45; 0.55] & \left| \begin{array}{c} \tau_n \le \max_{\bigcup_{j\in\{1,n-1\}} LR_j} (a_m^k) + d_k, \\ \nexists k', \{S_{k'}(LR_n) \ne 0\} \wedge \{\Lambda_{k'}(\tau_n) = 0\}. \end{array} \right. \end{cases}$$

For all pulses of the signal under study, in the vicinity of the coordinates τ_n, a function of the conditional instantaneous spectral density $\omega_n(k): \Phi \to [0,1]$ is formed, where $\omega_n(k) = x_k(\tau_n)$.

Step 10. Spectrum estimates are determined based on fuzzy filtering. The estimated spectrum is determined by the fuzzy filter formula [19, 20]. In the simplest case, a filter is used in the form

$$\widehat{\omega}_n(k) = \widehat{\omega}_{n-1}(k) + \alpha \cdot \{\omega_n(k) - \widehat{\omega}_{n-1}(k)\},$$

where $\alpha \in [0,1]$ is the gain of a fuzzy filter [21].

Step 11. Stop.

The function $\widehat{\omega}_n(k)$ is taken as the basis for the time-frequency analysis of the signal under study. It is also used to restore the original signal. In this case, the reconstructed signal is found as the sum of sinusoidal functions, which determine the set of basis functions Φ with amplitudes multiplied by the value $\widehat{\omega}_n(k)$. The accuracy of the algorithm is checked by the error criterion, which is calculated, for example, by the degree of correlation [22] between two functions. To test the performance of the algorithm, studies were performed on a model example of a non-stationary signal.

VI. THE DISCUSSION OF THE RESULTS

To substantiate the performance of the proposed algorithm for time-frequency analysis of non-stationary signals, it is necessary to solve two classical interrelated tasks:

1. To determine the spectrum of the investigated signal. Compare the results obtained with the actual signal spectrum. Estimate the errors of the proposed algorithm.
2. Based on the obtained spectra, restore the signal and compare it with the original signal. Rate the error.

For the study of the algorithm, a non-stationary signal φ_n with a sampling interval of 0.1 s was chosen. The observation time of the signal $W = 10\,s$. The source signal is considered as a discrete representation of the addition of sinusoidal signals $f_n^i, i = \overline{1,4}$ at frequencies of 0.159 Hz, 0.334 Hz, 0.446 Hz, 0.637 Hz with different amplitudes at subintervals $W_1 = [0;5]\,s$ and $W_2 = [5;10]\,s$. The values of amplitude-frequency characteristics for signals f_n^i in the intervals W_1 and W_2 are shown in Fig. 3

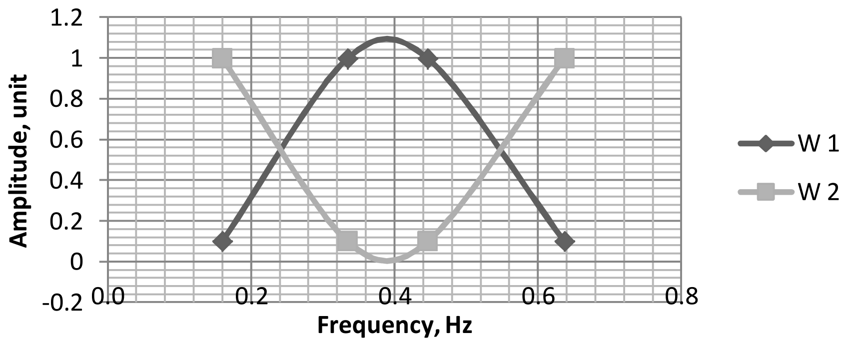

Fig. 3: Graphs of amplitude-frequency characteristics for signals f_n^i

Time series graphs of the of the signal under study and its components $f_n^i, i = \overline{1,4}$. are shown in Fig 4

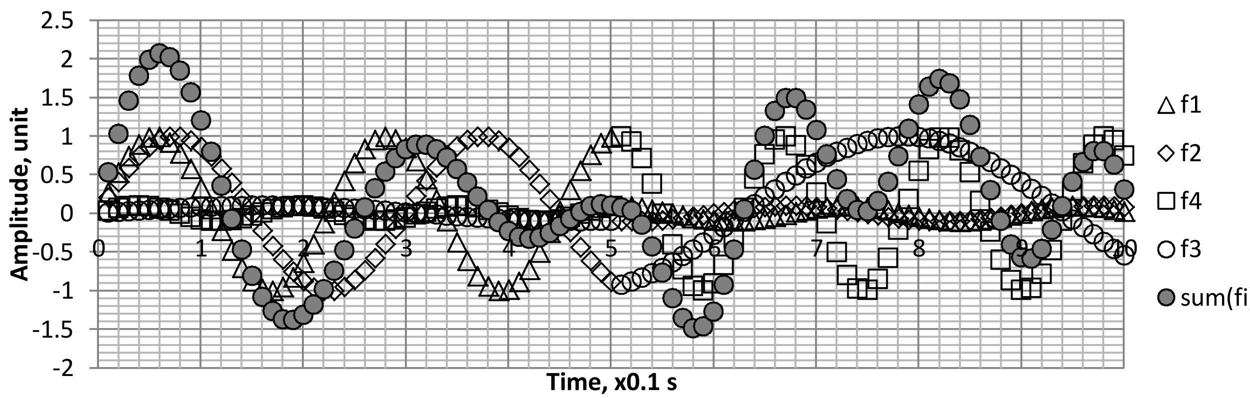

Fig. 4: Time series of the investigated signal and its components

The following parameters were chosen as initial data for the algorithm investigation:

1. 1. In the set of basic functions, sinusoidal functions are considered, represented by impulse sines of the form:

$$\Phi = \left\{ si_k\left(t \middle| a_1^k, d_k\right)\right\} = \{ si_1(t|0;15);\ \ si_2(t|0;11);\ \ si_3(t|0;20);\ \ si_4(t|0;5);\}, \ k = \overline{1,4}.$$

2. We will assume that quantization by level of the signal under study provides the power of the set of signal values $Card(Z_{ts}) = 128$. To represent the signal value, we will use p -adic numbers with $p = 2$. In this case, to represent one data in the data system, the number of blocks of the observation channel will be $L = 7$.
3. To determine the set of sample variables and build the function of the system behavior, we will use the simplest mask with the shift parameter $\rho = 0$ for all variables of the system $v_{i,\zeta_k(t)} \in V_i$. In the metasystem identification algorithm, the set of cut-off threshold values $\{\Delta_\mathrm{u}\}$ is presented in Table 1.

Table 1: The set of cut-off thresholds taken in the study

u	1	2	3	4	5	6	7	8	9	10	11
Δ_u	0.205	0.202	0.199	0.196	0.193	0.19	0.187	0.184	0.181	0.178	0.175

4. In the balance equations, we will use a symmetric window $\mathrm{LR_n}$ with the averaged deviation $\varepsilon(n) = 0.5 \cdot \left(\varepsilon_L(n) + \varepsilon_R(n)\right)$.

5. To test the algorithm performance in determining the influence degrees of the k-th impulse sine function $x_k(\tau_n)$ from the set of basis functions Φ, we will use an approximate approach.

Based on the initial data, as a result of applying the algorithm proposed above, the data system D was obtained for the signal under study. Data system D is a matrix $V \times W$ with a dimension of 7 × 100. In Table 2 shows a fragment of this matrix for a time subset up to $t \in [0; 1]\ s$.

Table 2: Fragment of the data matrix for the time series under study

t	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
v_1	1	1	1	1	1	1	1	1	1	1
v_2	0	1	1	1	1	1	1	1	1	1
v_3	1	0	0	1	1	1	1	1	1	0
v_4	0	0	1	0	1	1	1	1	0	0
v_5	0	0	1	1	1	1	1	0	0	1
v_6	0	0	0	1	0	1	0	0	0	0
v_7	1	0	1	0	1	1	1	0	0	1

As a result of using the metasystem identification algorithm, the full two-dimensional impulse function $r(u, t)$ was obtained (Fig. 5). Based on this function, a generalized impulse function $g(t)$ was obtained (Fig. 6).

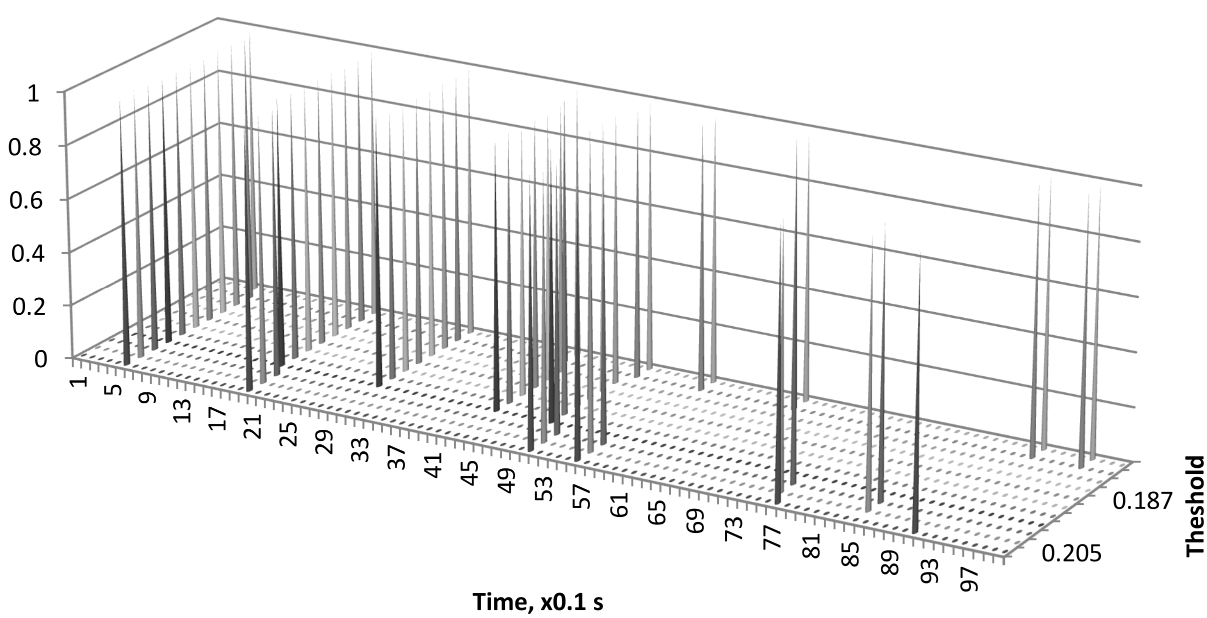

Fig. 5: Full two-dimensional pulse function $r(u, t)$ for the signal

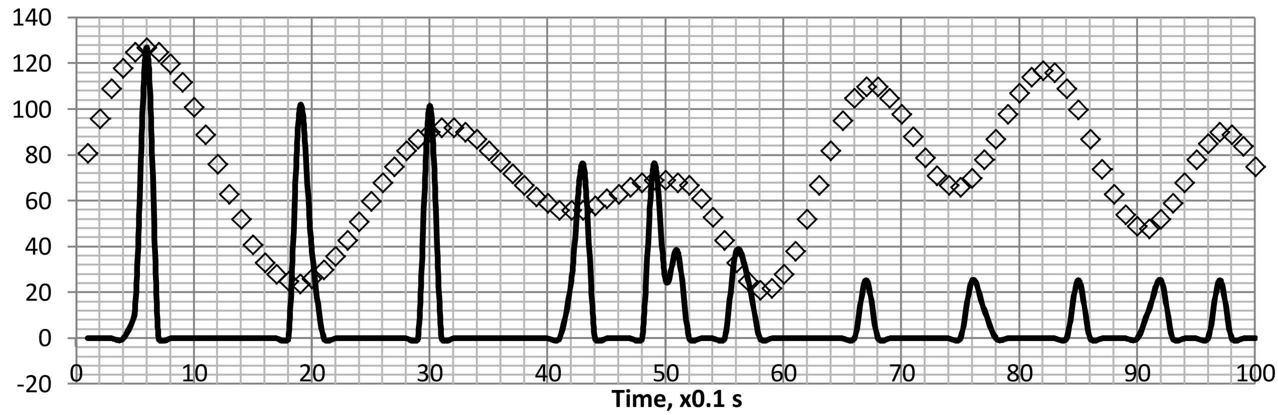

Fig. 6: The time series of the signal under study and the corresponding generalized impulse function $g(t)$ (normalized to the maximum value of the series under study)

The function $g(t)$ allows to obtain the impulse function of the signal under study $r(\tau_n)$. The coordinates of the pulses τ_n were determined as the local extrema coordinates of the function $g(t)$ (Fig. 7).

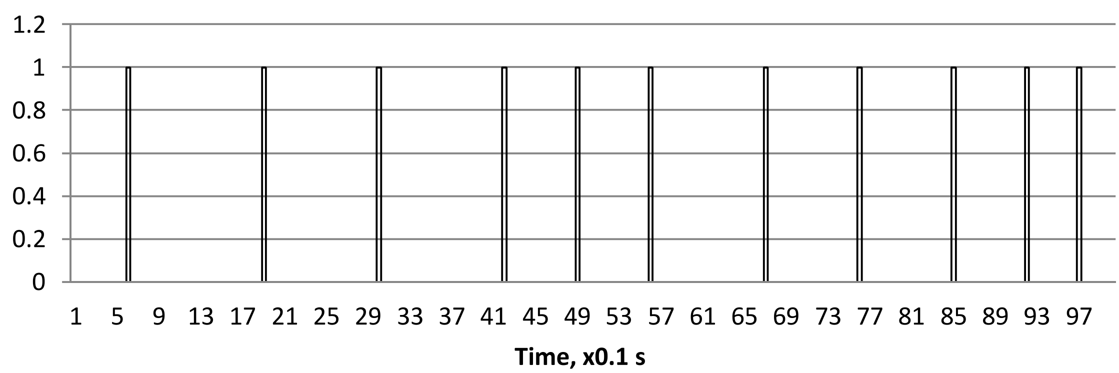

Fig. 7: Pulse function of the signal under study $r_{(\tau_n)}$

Based on the obtained impulse function $r(\tau_n)$ for a given set of impulse functions $\Phi = \{ si_k(t)\}, k = \overline{1,4}$, we obtained the coefficients of the balance equation for all coordinates of the impulses $\tau_n, n = \overline{1,N}$. For example, for the coordinate $\tau_2 = 19$ and for the average deviation, the window will be defined as:

$$LR_n = [\tau_2 - 0.25 \cdot (\tau_3 - \tau_1); \tau_2 + 0.25 \cdot (\tau_3 - \tau_1)] =$$
$$= [19 - 0.25 \cdot (30 - 6); 19 + 0.25 \cdot (30 - 6)] = [13; 25] \subseteq W.$$

In this case, the value of the partial sum for the pulse sine function $si_1(t|0;15)$ will be: $S_1(LR_2) = -21$, and the coefficient $\beta_1(\tau_2) = (-1)^{2-1} = -1$. Then the coefficient $\Lambda_1(\tau_2)$ for the balance equation will take the value:

$$\Lambda_1(\tau_2) = \{\beta_1(\tau_2) \cdot \tau_2 - S_1(LR_2)\} = -1 \cdot 19 - (-21) = 2.$$

Similarly, the coefficients of the balance equation are calculated for all basis functions and coordinates $\tau_n, n = \overline{1,N}$. (Table 3).

Table 3: The coefficients of balance equations

τ_n	Λ_1	Λ_2	Λ_3	Λ_4
19	2	-2	-	5
30	-	2	-4	5
43	-	4	3	5
49	2	-1	-	2
56	-	-	-	0
67	1	-	-1	5
76	-	-	-	0
85	-	2	-1	-1
92	-	2	-	-1

In Table 4 estimates of the coefficients $x_k(\tau_n)$ (normalized to unity) obtained using the direct approximate approach to their definition are presented. Table 4a presents the directly obtained estimates of the coefficients $x_k(\tau_n)$, and in Table 4b are estimates of the coefficients $x_k(\tau_n)$ after filtering with the gain $\alpha = 0.53$.

Table 4: Estimates of the coefficients $x_k(\tau_n)$ of the signal spectrum, obtained by the approximate method

Table 4a

τ_n	$x_4(\tau_n)$	$x_2(\tau_n)$	$x_1(\tau_n)$	$x_3(\tau_n)$
19	0.167	0.667	0.667	0.000
30	0.167	0.667	0.667	0.333
43	0.000	0.200	0.667	0.400
49	0.333	0.667	0.333	0.400
56	1.000	0.333	0.167	0.200
67	0.000	0.000	0.800	0.800
76	1.000	0.000	0.400	0.400
85	0.750	0.500	0.000	0.750
92	0.667	0.333	0.000	0.750

Table 4b

τ_n	$x_4(\tau_n)$	$x_2(\tau_n)$	$x_1(\tau_n)$	$x_3(\tau_n)$
19	0.111	0.444	0.444	0.000
30	0.100	0.402	0.402	0.096
43	0.047	0.272	0.468	0.213
49	0.124	0.332	0.322	0.222
56	0.370	0.260	0.203	0.167
67	0.174	0.122	0.360	0.343
76	0.376	0.057	0.287	0.279
85	0.376	0.159	0.135	0.330
92	0.378	0.176	0.063	0.382

The change in the $x_k(\tau_n)$ coefficients of the signal spectrum over time is shown below (Fig.8).

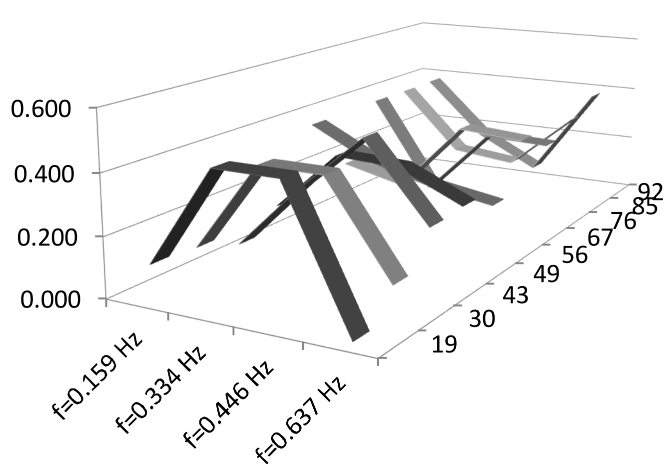

Fig. 8: Change dynamics of the $x_k(\tau_n)$ coefficients of the signal spectrum

From Fig. 8, it follows that the algorithm provides the correct spectrum estimate for a non-stationary signal. In Fig. 9 a comparison of the true and estimated signal spectrum at the initial and final measurement interval is shown.

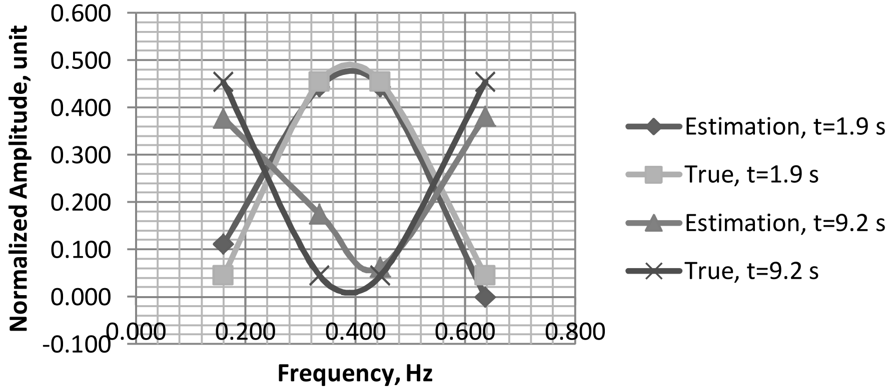

Fig. 9: Comparison of the true and estimated spectrum

At the same time, over the entire observation interval, the degree of correlation between the spectra at each frequency remains high (more than 0.7, Fig. 10).

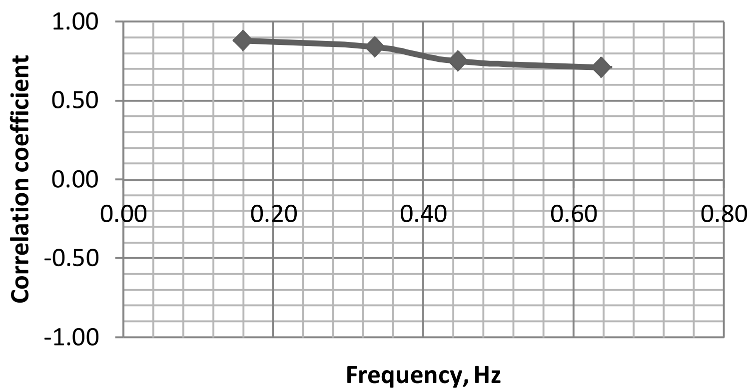

Fig. 10: The correlation coefficient between the spectra for each frequency over the entire time interval

The change in the value of the correlation coefficient between the true and estimated spectra over the entire time interval is shown in Fig. 11.

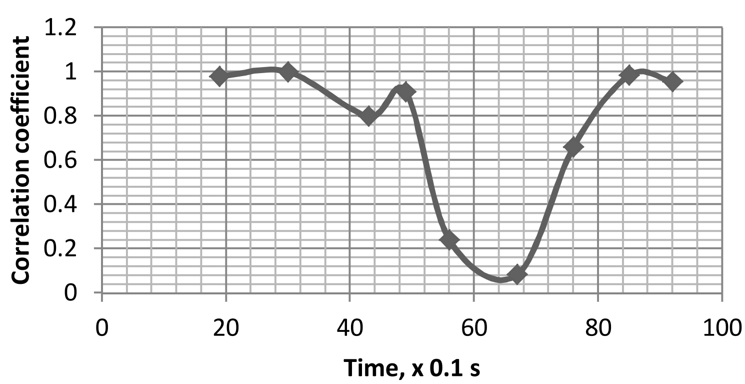

Fig. 11: Change in the correlation coefficient between the true and estimated spectra

The correlation degree of the spectra at the stationary parts of the signal refers to a high one (the correlation coefficient is more than 0.7). At the moment of changing the spectra of a true signal, the algorithm provides fast adaptation to a new signal spectrum (within 1 - 2 pulses of an impulse function simulating a true signal). Thus, with a large number of measurements, the algorithm will provide an increase in the accuracy of estimation of the current signal spectrum.

Based on the estimated spectra in each LR_n windows, the estimated signal is easily restored. In this case, the coefficients $x_k(\tau_n)$ as amplitudes of the sinusoidal signals normalized to one are used. These sinusoidal signals correspond to the impulse sines $si_k\left(t\middle|a_1^k, d_k\right)$ from the set of basis functions Φ. Fig. 12 shows the true and reconstructed signals from the estimated spectrum.

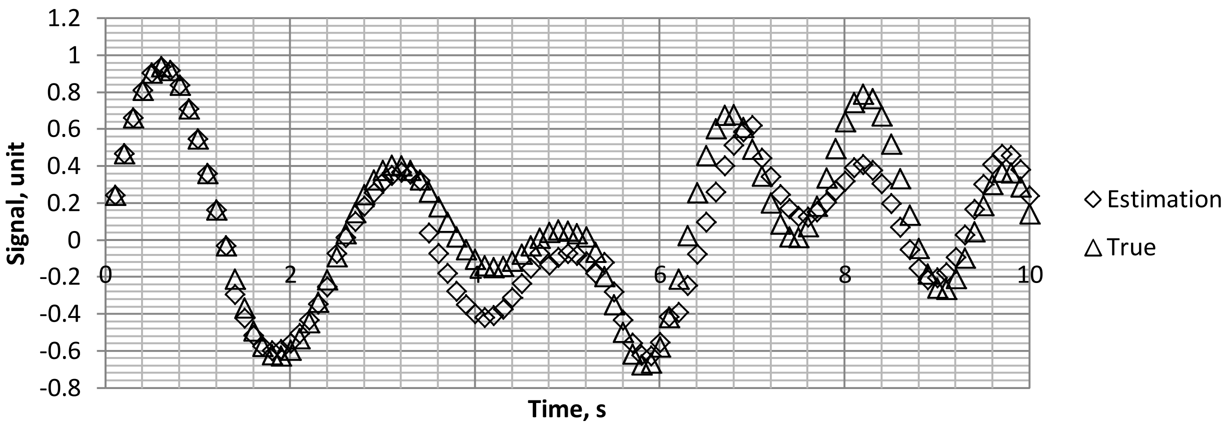

Fig.12: True and reconstructed signals from the estimated spectrum.

The correlation coefficient of the signals is 0.933, which corresponds to a high level of correlation between the true and reconstructed signals. The average error for the Hamming distance is about 0.1 for the entire observation interval of the signal, taking into account the sharp change in its spectrum. From the graph it is seen that after a sharp change of the signal spectrum, there is an error in its recovery. However, by the 9th second the signals almost coincide. Thus, a relatively high degree of adaptation of the spectrum estimation algorithm for a non-stationary signal can be made.

VII. Conclusions

The developed algorithm based on the results given in article [7] allows to reduce the problem of estimating the signal spectrum to the solution of a system of linear equations. These equations are based on the use of arithmetic progressions. In this case, the task of calculating the integral convolutions of functions that introduce additional errors in the definition of the signal spectrum is excluded. Additionally algorithm provides adaptive time-frequency localization which is linked to the measured signal. This reduces errors at the borders of the windows when determining the current spectra. These results were made possible due to the properties of the system behavior functions. These functions are the distribution of the possibility measure on a set of system states.

Studies of the algorithm have shown that the accuracy of determining the current signal spectrum will depend on the set of its parameters. In particular, the accuracy will depend on the signal discretization conditions, on parameters of the construction algorithm of behavioral functions, on algorithm for identifying the metasystem, on the choice of window, on the gain of the fuzzy filter, and other parameters. These parameters are the algorithm settings. The study of the algorithm on the example of a non-stationary signal with a sharp change in the spectrum showed the efficiency of using the algorithm and a high degree of correlation between the estimated and true signal spectra. Estimates of the signal spectra, which are obtained using the algorithm, make it possible to recover the true signal with a high degree of correlation (with a correlation coefficient higher than 0.9).

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