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New Types of Transitive Maps and Minimal Mappings

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Abstract- In this paper, we have introduced the relationship between two different concepts of maps, namely topological $\alpha-$ transitive and $\delta-$transitive maps and investigate some of their properties in two topological spaces (X,τ^{α}) and (X,τ^{δ}), τ^{α} denotes the $\alpha-$topology and τ^{δ} denotes the $\delta-$topology of a given topological space (X,τ). The two concepts are defined by using the concepts of $\alpha-$irresolute and $\delta-$irresolute maps respectively Also, we studied the relationship between two types of minimal systems, namely, α- minimal and $\delta-$minimal systems, The main results are the following propositions.

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NEWTYPESOFTRANSITIVEMAPSANDMINIMALMAPPINGS

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New Types of Transitive Maps and Minimal Mappings

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Abstract- In this paper, we have introduced the relationship between two different concepts of maps, namely topological $\alpha -$ transitive and $\delta -$ transitive maps and investigate some of their properties in two topological spaces (X, τ^{α}) and (X, τ^{δ}), τ^{α} denotes the $\alpha -$topology and τ^{δ} denotes the $\delta -$topology of a given topological space (X, τ). The two concepts are defined by using the concepts of $\alpha -$ irresolute and $\delta -$irresolute maps respectively Also, we studied the relationship between two types of minimal systems, namely, α - minimal and $\delta -$minimal systems, The main results are the following propositions:

1. Every topologically $\alpha -$transitive map implies topologically $\delta -$transitive map, but the converse not necessarily true.
2. Every $\alpha -$minimal system implies $\delta -$minimal system, but the converse not necessarily true.

Keywords: *topologically $\delta -$ transitive, α -irres olute, δ - transitive, $\delta -$ dense.*

I. Introduction

Let (X, τ) be a topological space, $f : X \to X$ be α -irresolute map, then the set $A \subseteq X$ is called topologically α -mixing set[1] if, given any nonempty α -open subsets $U, V \subseteq X$ with $A \cap U \neq \phi$ and $A \cap V \neq \phi$ then $\exists\ N > 0$ such that $f^{n}(U) \cap V \neq \phi$ for all $n > N$, weakly α - mixing set[4] of (X, f) if for any choice of nonempty α -open subsets V_1, V_2 of A and nonempty α –opensubsets U_1, U_2 of X with $A \cap U_1 \neq \phi$ *and* $A \cap U_2 \neq \phi$ there exists n ∈N such that $f^{n}(V_1) \cap U_1 \neq \phi$ *and* $f^{n}(V_1) \cap U_2 \neq \phi$, *strongly α -mixing* if for any pair of open sets U and V with $U \cap A \neq \phi$ and $V \cap A \neq \phi$, there exist some $n \in \mathbf{N}$ such that $f^{k}(U) \cap V \neq \phi$ for any $k \geq n$. *A* point x *which has* α -dense orbit $O_d(x)$ in *X*. is called $\alpha - type$ hyper-cyclic point.A system is α - mixing[1] if, given α -open sets U and V in X, there exists an integer *N*, such that, for all $n > N$, one has $f^{n}(U) \cap V \neq \phi$, *topologically α -mixing if* for any *non-empty* α -open set U, there exists $N \in \mathbf{N}$ such that $\bigcup_{n \geq N} f^{n}(U)$ is α -dense in X. With the above concepts, some new theorems have been introduced and studied. Furthermore, we have the following results:

- Every topologically $\alpha -$transitive map implies topologically $\delta -$transitive map, but the converse not necessarily true.
- Every $\alpha -$minimal system implies $\delta -$minimal system, but the converse not necessarily true.
- $(E_{\alpha}) \Rightarrow (ET_{\alpha})$;
- $(TM_{\alpha}) \Rightarrow (WM_{\alpha}) \Rightarrow (TT_{\alpha})$;

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II. Preliminaries and Theorems

Definition 3.1 [2] A map $f : X \to Y$ is called α-irresolute if for every α-open set H of Y, $f^{-1}(H)$ is α-open in X.

Proposition 2.2 The product of two topologically α -mixing systems must be topologically α -mixing.

Proof: Suppose that (X, f) *and* (Y, g) are two α -mixing systems, and consider any α -open sets W, W' in $X \times Y$. By definition of the product topology, there exist α -open sets $U, U' \subset X'$ and $V, V' \subset Y$ so that $U \times V \subset W$ and $U' \times V' \subset W'$.By definition of topological α -mixing of (X, f), there exists N such that for any $n > N$, $f^{n}(U) \cap V \neq \phi$. By definition of topological α -mixing[3] of (Y, g), there exists N′ such that for any $n > N'$, $g^{n}(U') \cap V' \neq \phi$. Then, for any $n > \max(N, N')$, both $f^{n}(U) \cap V$ and $g^{n}(U') \cap V''$ are nonempty, and therefore $(f \times g)^{n}(U \times U') \cap (V \times V')$ is nonempty as well. But this implies that $(f \times g)^{n}(W) \cap W' \neq \phi$, since W and W' were arbitrary, this implies that $(X \times Y, f \times g)$ is topologically α -mixing.

Theorem 2.3 The product of two α-transitive maps is not necessarily α-transitive map [4].

Corollary 2.4 The product of two topologically α-transitive systems is not necessarily topologically α-transitive.

III. New Types of Chaos of Topological Spaces

In this section, I introduced and defined α-type transitive maps[3] and α –type minimal maps[3], and study some of their properties and prove some results associated with these new definitions. I investigate some properties and characterizations of such maps.

Definition 3.1 Let X is a separable and second category space with no isolated point, if for $x \in X$ the set $\{f^n(x): n \in \mathbf{N}\}$ is dense in X thenx is called hyper-cyclic point. If there exists such an $x \in X$, then f is called hyper-cyclic function or f is said to have a hyper-cyclic point. Here, we have an important theorem that is: f is a hyper-cyclic function if and only if f is transitive.

Definition 3.2 A function $f: X \to X$ is called αr-homeomorphism if f is α-irresolute bijective and $f^{-1}: X \to X$ is α-irresolute.

Definition 3.3 Two topological systems $f: X \to X$, $x_{n+1} = f(x_n)$ and $g: Y \to Y$, $y_{n+1} = g(y_n)$ are topologically αr-conjugate if there is αr-homeomorphism $h: X \to Y$ such that $h \circ f = g \circ h$ (i.e. $h(f(x)) = g(h(x))$. We call h a topological αr-Conjugacy. Then I have proved some of the following statements:

1. The maps f and g have the same kind of dynamics.
2. If x is a periodic point of the map f with stable set $W_f(x)$, then the stable set of h(x) is $h(W_f(x))$.
3. The map f is α-exact if and only if g is α-exact
4. The map f is α-mixing if and only if g is α-mixing
5. The map f is α-type chaotic if and only if g is α-type chaotic
6. *The map f is weakly α-mixing if and only if g isweakl yα - mixing.*

Remark 3.4

If $\{x_0, x_1, x_2, \ldots\}$ denotes an orbit of $x_{n+1} = f(x_n)$ then $\{y_0 = h(x_0),\ y_1 = h(x_1),\ y_2 = h(x_2), \ldots\}$ yields an. In particular, h maps periodic orbits of f onto periodic orbits of g. orbit of g since $y_{n+1} = h(x_{n+1}) = h(f(x_n)) = g(h(x_n)) = g(y_n)$, i.e. f and g have the same kind of dynamics.

I introduced and defined the new type of transitive in such a way that it is preserved under topologically α r- conjugation.

Proposition 3.5 Let X and Y are α-separable and α-second category spaces. If $f: X \to X$ *and* $g: Y \to Y$ are αr-conjugated by the αr-homeomorphism $h: Y \to X$ then, *for each α-hyper-cyclic point y in* Y *if and only if* h(y) *is α-hyper-cyclic point in* X

Proof: Suppose that $f: X \to X$ *and* $g: Y \to Y$ are maps αr – conjugate via $h: Y \to X$ such that $h \circ g = f \circ h$, then if y∈Y is α-hyper-cyclic in Y i.e. *the orbit* $O_g(y) = \{y, g(y), g^2(y), \ldots\ldots.\}$ *is* α-dense in Y, let $V \subset X$ be a nonempty α-open set. Then since *h* is a α r-homeomorphism, $h^{-1}(V)$ is α-open in *Y*, so there exists $n \in \mathbf{N}$ with $g^n(y) \in h^{-1}(V)$. From $h \circ g^n = f^n \circ h$ it follows that $h(g^n(y)) = f^n(h(y)) \in V$,

So that $O_f(h(y)) = \{h(y), f(h(y)), f^2(h(y)), \ldots\ldots.\}$ is α-dense in *X so h(y) is hyper-cyclic in X*. Similarly, if $h(y)$ is α-hyper-cyclic in *X*, then y is α *-hyper-cyclic in* Y.

Proposition 3.6 if $f: X \to X$ *and* $g: Y \to Y$ are αr-conjugate via $h: X \to Y$. Then

(1) T is α-type transitive subset of X $\Leftrightarrow$ $h(T)$ is α-type transitive subset of Y;

(2) $T \subset X$ is α-mixing set $\Leftrightarrow$ $h(T)$ is α-mixing subset of Y.

Proof (1) Assume that $f: X \to X$ *and* $g: Y \to Y$ are topological systemswhich are topologically αr-conjugated by $h: X \to Y$. Thus, h is αr-homeomorphism (that is, h is bijective and thus invertible and both h and h^{-1} are α-irresolute) and $h \circ f = g \circ h$ Suppose T is α-type transitive subset of X. Let A, B be α-open subsets of Y with $B \cap h(T) \neq \phi$ and $A \cap h(T) \neq \phi$

$(to\, show\;\; g^n(A) \cap B \neq \varphi \;\; for \;\; some \;\; n > 0).$ $U = h^{-1}(A) \;\; and \;\; V = h^{-1}(B)$ are α –open subsets of X since h is an α -irresolute. Then there exists some n>0 such *that* $f^n(U) \cap V \neq \varphi$ since the set T is α –type transitive subset of X, with $U \cap T \neq \phi$ and $V \cap T \neq \phi$.Thus

So h(T) is α -type transitive subset of Y.

$$(as \quad f \circ h^{-1} = h^{-1} \circ g \;\; implies \;\; f^n \circ h^{-1} = h^{-1} \circ g^n).$$

$$\phi \neq f^n(h^{-1}(A)) \cap h^{-1}(B) = h^{-1}(g^n(A)) \cap h^{-1}(B).$$

Therefore,

$$h^{-1}(g^n(A) \cap B) \neq \phi \;\; implies \;\; g^n(A) \cap B \neq \phi \;\; \text{since } h^{-1} \;\; is\, invertible.$$

Proof (2) We only prove that if T is topologically α -mixing subset of Y then $h^{-1}(T)$ is also topologically α -mixing subset of X. Let U,V be two α -open subsets of X with $U \cap h^{-1}(T) \neq \phi$ and $V \cap h^{-1}(T) \neq \phi$. We have to show that there is N>0 such that for any n>N, $f^n(U) \cap V \neq \phi$. $h^{-1}(U)$ and $h^{-1}(V)$ are two α -open sets since h is α -irresolute with $h^{-1}(V) \cap T \neq \phi$ and $h^{-1}(U) \cap T \neq \phi$. If the set Tis topologically α -mixing then there is N >0 such that for any n>M, $g^n(h^{-1}(U)) \cap h^{-1}(V) \neq \phi$. S $\exists$ $x \in g^n(h^{-1}(U)) \cap h^{-1}(V)$. That is $x \in g^n(h^{-1}(U)) \;\; and \;\; x \in h^{-1}(V) \Leftrightarrow x = g^n(y) \;\; for \;\; y \in h^{-1}(U)$ .h(x) ε V. Thus, since $h \circ g^n = f^n \circ h$, so that $h(x) = h(g^n(y)) = f^n(h(y)) \in f^n(U)$ and we have $h(x) \in V$ that is $f^n(U) \cap V \neq \phi$. So, $h^{-1}(T)$ is α -mixing set.

Proposition 3.7 Let (X, f) be a topological system and A be a nonempty α -closed set of X. Then the following conditions are equivalent.

1. A is a α -transitive set of (X, f).
2. Let V be a nonempty α -open subset of A and U be a nonempty α -open subset of X with $U \cap A \neq \phi$. Then ther exists $n \in \mathbf{N}$ such that $V \cap f^{-n}(U) \neq \phi$.
3. Let U be a nonempty α -open set of X with $U \cap A \neq \phi$. Then $\bigcup_{n \in \mathbf{N}} f^{-n}(U)$ is α -dense in A.

Theorem 3.8 Let (X, f) be topological dynamical system and A be a nonempty α -closed invariant set of X. Then A is a α - transitive set of (X, f) if and only if (A, f) is α -type transitive system.

Proof: $\Rightarrow$) Let $V_1 \, and \, U_1$ be two nonempty α -open subsets of A. For a nonempty α -open subset U_1 of A, there exists a α - open set U of X such that $U_1 = U \cap A$ Since A is a α -type transitive set of (X, f), there exists n ∈N such that $f(V_1) \cap U \neq \phi$. Moreover, A is invariant, i.e., $f(A) \subset A$, which implies that $f(A) \subset A$ Therefore, $f(V_1) \cap A \cap U \neq \phi$, i.e. $f(V_1) \cap U_1 \neq \phi$. These shows that (A, f) is α - type transitive.

$\Leftarrow$) Let V_1 be a nonempty α -open set of A and U be a nonempty α -open set of X with $A \cap U \neq \phi$, Since U is an α -open set of X and $A \cap U \neq \phi$, , it follows that U ∩ A is a nonempty α -open set of A. Since (A, f) is topologically α -type transitive, there existsn ∈N such that $f(V_1) \cap (A \cap U) \neq \phi$, which implies that $f(V_1) \cap U \neq \phi$.. This shows that *A* is a α -type transitive set of (X, f).

IV. New Types of Chaos in Product Spaces

We will give a new definition of chaos for δ -irresolute self map $f : X \to X$ of a compact Hausdorff topological space X, so called δ -type chaos. This new definition induces from John Tylar definition which coincides with Devaney's definition for chaos when the topological space happens to be a metric space.

Definition 4.1 [4] Let (X, f) be a topological dynamical system; the dynamics is obtained by iterating the map. Then, *f* is said to be δ -type chaotic on X provided that for any nonempty δ -open sets U and V in X, there is a periodic point $p \in X$ such that $U \cap O_f(p) \neq \phi$ and $V \cap O_f(p) \neq \phi$.

Proposition 4.2 Let (X, f) be a topological dynamical system. The map *f* is δ -type chaotic on X if and only if *f* is δ -type transitive and the periodic points of the map are δ -dense in X .

Proof: $\Rightarrow$) If *f* is δ -type chaotic on X, then for every pair of nonempty δ -open sets U and V, there is a

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periodic orbit intersects them; in particular, the periodic points are δ-dense in X. Then there is a periodic point p and $x, y \in O_f(p)$ with x ∈U and y ∈V and some positive integer n such that $f^n(x) = y$, so that $y = f^n(x) \in f^n(U)$ therefore $f^n(U) \cap V \neq \phi$.

⇐:) The δ-type transitivity [5]of f on X implies, for any nonempty δ-open subsets U, V ⊂X, there is n such that for some x ∈U, $f^n(x) \in V$. Now,define $W = f^{-n}(V) \cap U$. Then W is δ-open and nonempty with the property that $f^n(W) \subset V$.

But since the periodic points of *f* are δ-dense in X, there is a p ∈W such that $f^n(p) \in V$. Therefore, $U \cap O_f(p) \neq \phi$ and $V \cap O_f(p) \neq \phi$. So, the map f is δ-type chaotic.

We will define some concepts as follows:

1. (TT_δ) if for every non-empty δ-open set $D \subset X$, $\bigcup_{n=1}^{\infty} f^n(D)$ is δ-dense,
2. Weak δ-Mixing (WM_δ) if $f \times f$ is topologically δ-transitive .
3. Exact δ-Transitive (ET_δ) if for every pair of non-empty δ-open set $D, W \subset X$, $\bigcup_{n=1}^{\infty} (f^n(D) \cap f^n(W)$ $is\ \delta - dense\ in\ X$,
4. Topologically δ- Mixing (TM_δ) if for every pair of non-empty δ-open set $D, W \subset X$, there exits an $N \in \mathbf{N}$ such that $f^n(D) \cap W \neq \phi$ for all $n \geq N$.
5. δ- Exact (E_δ) if for every non-empty δ-open set $D \subset X$, there exists $N \in \mathbf{N}$ such that $f^N(D) = X$
6. Then the following implications hold:

- $(E_\alpha) \Rightarrow (ET_\alpha)$;
- $(TM_\alpha) \Rightarrow (WM_\alpha) \Rightarrow (TT_\alpha)$;

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