New Formulas for the Mutual Inductance and the Magnetic Force of the System: Thin Disk Coil (Pancake) with Inverse Radial Current Density and Thin Wall Solenoid with Constant Azimuthal Current Density

Cevdet Akyel

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Abstract

This paper deals with two coaxial circular coils (thin disk coil and thin wall solenoid) for which we calculated the electromagnetic quantities such as the mutual inductance and the magnetic force. The disk coil (pancake) is with the nonlinear inverse radial current and the wall solenoid with the constant current in the azimuthal direction. The circular coils with the nonlinear inverse radial current are well known as the Bitter coils, and the circular coils with the azimuthal current are well known as the ordinary coils. Also, the coils with the azimuthal current can serve as the superconducting coils. These calculations give the semi-analytical and the analytical expressions respectively for these electromagnetic quantities. Also, we presented the improved filament method as the comparative method.

Index terms—

1 I. Introduction

he computation of the electromagnetic quantities (magnetic field, self-inductance, mutual inductance, magnetic force, etc.) for the conventional circular coaxial coils with the constant azimuthal current density has been presented in many papers, books, monographs and studies [1][2][3][4][5][6][7][8][9][10][11][12][13][14][15][16][17][18][19]. The analytical, the semi-analytical and the numerical methods have been used to calculate these electromagnetic quantities. These calculations are used in many electromagnetic applications (tubular linear motors, magnetically controllable devices and sensors, current reactors, cochlear implants, defibrillators, instrumented orthopedic implants, in magnetic resonance imaging (MRI) systems, superconducting coils, and tokamaks, etc.).

Also, there are the nonconventional circular coils with the nonlinear inverse radial density current which are used in many technical applications such as the superconducting coils, the electromagnets for the the superconducting coils, the electromagnets for the production of the extremely powerful magnetic fields (Bitter coils) and the homopolar motors [20][21][22][23][24][25] **??**26][27][28][29][30][31][32][33][34][35][36]. The calculation of the magnetic force and the mutual inductance for these coils is essential for the design of electromagnetic inductors. In this paper, we calculated these electromagnetic quantities for the coil's combination, the disk coil (pancake) with the nonlinear inverse radial current density (Bitter disk coil) and the wall solenoid with the constant azimuthal current density (superconducting wall solenoid). All expressions are obtained in the semi-analytical form (mutual inductance) and the closed form (magnetic force). Also, all singular case has been solved and given in the closed form. The results of these calculations are expressed over the elliptic integrals of the first kind and the Heuman's Lambda function and one simple friendly integral whose kernel function is the continuous function in all interval of the integration. We used the Gaussian numerical integration, [37][38]. The improved modified filament method for the presented configuration is given as the comparative method. We use the Matlab implementation to calculate the mutual inductance and the magnetic force by two independent methods.

2 II. Basic Expressions

The Bitter disk coil and the wall solenoid in the air are with the inverse radial current density and the uniform current density respectively [29][30], (See Fig. 1) as follow:T 1 1 1 2 1 1 ln (1) I r N I J R R = 2 2 2 2 1 (z z)

Figure 1: Bitter disk coil and thin solenoid the mutual inductance and magnetic force between these coils, are respectively **??**29 -30],2 2 0 1 2 2 0 1 1 2 1 1 cos = (3) ( ) ln R z I R z R R N N R dr dzd M r z z ? µ ? ? ? ? ? ? 2 2 0 1 2 1 2 3 2 0 1 1 2 1 1 ( )cos = (4) ( ) ln R z Q I I R z R R N N I I R z z dr dzd F r z z ? µ ? ? ? ? ? ? ? ? where 2 2 2 ( ) 2 cos Q I I r z z r R r R ? = ? + + ?

Both configurations are in the air or a nonmagnetic and non-conducting environment. We obtain the integral form to calculate these two physical quantities.

3 III. Calculation Method

After four analytical integration M and Fare respectively:

where, , , 1 2 2 3 1 1 2 1 3 4 2 Q Q R R t t z z t t z z ? ? ? ? = = = = == ? == ? 2 2 0 0 2 2 2 2 2 2 2 2 ( ) 1 (,)] sign( ) 8 4 4(1) 1) 8 8 1 1 sign sign (1)(3)[(1) (1)(3) 3 [(1)] ( ) [**2 4**

] (1n n n n n n n n n n n n n n n n n n n n n n n n n n n n n I b k b b b b T l l b V l l k b k b l b E k b l K k l l l ? ? ? + ? ? ? ? + + + = + ? ? ? + ? ? + + + ? ? ? + 2 2 2 2 2 2 1 [1] ( ) [**(1)**

] ( )n n n n n n n n n n n n n n b b V k k S l b K k l b E k l l ? + = ? ? ? + + + ? z y x z 1 z 2 z Q N 1 N 2 R z y R 2 R 1 (5) 2 4 1 0 1 2 2 1 2 1 1 ( )ln n n n n N N R R z z R µ = ? = = ? ? ? M T (1) 4 1 0 1 2 1 2 2 1 2 1 1 2( )ln n n n n N N I I R F S R z z R µ = ? = = ? ? ? (1)**(6)**

, , 1, 2 ,3, 4 n n n t n R R l b n ? = = = 2 2 0 1 0 2 1 (,) sgn( )[1 (,)] n n n n n n n V k R b k ? ? ? = ? ? + + ? ? ? © 2018

)n n n n n n n l l k h l b l = = + + + 2 2 1 1 1 n n m b = ? + + 1 2 , arcsin 1 1 n n n b b ? = + + 2 2 2 , 1 arcsin 1 n n n n n k m m k ? ? = ? ? , 2 2 , 1 arcsin 1 n n n n n k h h k ? ? = ? ? /2 1 0 2 0 cos2 sinh d 1 2 n n n n b I l l p b b - = ò + +

From general cases (**??**) and (6) it is possible to obtain the special and singular cases.

The expression T n is in a semi-analytical form where we need to solve the simple integral I 0n numerically by using the Gaussian integration for example. The expression S n is in the closed form.

4 Singular Cases

Singular cases are in the analytical form (**??**) and (6) respectively: If b n = 0 and k n 2 ? 1 or b n = 0 and k n 2 =0.

If b n = 0 and kn 2 ? 1.2 2 [ ] ( ) (1) ( ) 1 n n n n n n n n n k k S l K k l E k l l = ? + + 2 2

5 4

(1)n n n n l k h l = = + If b n = 0 and k n 2 = 1.

All expressions in (**??**), (6), (7), (**??**) and (**??**) are the complete elliptical integrals K, Eand? 0 , Heuman's Lambda function [37][38]. ©[2 ( ) (,) (,)= (1) ((,)) ((,))] 2 (,) II Rr l k g l M g l K k g l E k g l k g l µ ? ? 2 0 1 2 2 [2 (,) ( ) (,) (,) ((,)) 2 ((,))] 4 ( ) 1 (,) II k g l I I z g k g l F g l E k g l K k g l Rr l k g l µ ? = ? ? ? ( ) (, ..., 0, ...,) 2 1 II II II h r l R l l n n n = + = ? + 3 4 4 3 , 2 II II R R R h R R + = = = - 11 I R R R = = , 4 3 II h R R = ? ( ) , ,..., 0,..., 2 1 a z g c g g K K K = ? = ? + 2 2 2 4 ( ) (,) (( )) ( ) II II Rr l k g l R r l z g = + +**(7) 2 0**

IV. Modified Filament Method

In this paper, we give the modified formulas for the mutual inductance and the magnetic force between two Bitter thick coils (See Fig. 2) using the filament method. Applying some modification in the mutual inductance calculation [30], we deduced the mutual inductance and the magnetic force between the Bitter disk and the wall solenoid as follows:1 2 2 1 2 1) (,) (( ) (8) (2 1)(2 1) ln g K l n g K l n II M g l N N R R r l M R K n R = = = ? = ? ? = + + ? ? 1 2 1 2 2 1 2 1) (,)**(( )**

(2 1)(2 1) ln g K l n II g K l n F g l N N I I R R r l F R K n R = = = ? = ? ? = + + ? ?

6 Examples

To validate the new approach we present some examples, which cover either the regular or the singular cases. In these examples, all coils are with the unit currents. Also, we define the coil dimensions. For the comparative filament method, the number of subdivisions for each coil is also given. Our goal is to verify the accuracy of this method, so that we will fix the number of subdivisions (K= n = 3000) in the following examples without taking into consideration the computational time in the calculations. The number of turn in each coil is 100. **??**) and (6) we obtain: M = 17.661179mH F = 7.4710846mN From (10) and (11) we obtain: **??**) and (6) we obtain: M = 26.158014mH F = 0 N From (10) and (11) we obtain:a) Example1. Wall solenoid: R= 2 m, z 1 = 0 m, z 2 = 1 m. Disk coil: R 2 = 3 m, R 4 = 4m, z Q =2 m. From (M = 17.661179mH F = 7.4710845mN b) Example2. Wall solenoid: R= 2 m, m, z 1 = 0 m, z 2 = 1 m. Disk coil: R 2 = 3 m, R 4 = 4m, z Q = 0.5 m. From (M = 26.158014mH F = 0N c) Example3. Wall solenoid: R= 3 m, z 1 = 0 m, z 2 = 1 m. Disk coil: R 2 = 3 m, R 4 = 4m, z Q =2 m.

From (??) and (6) we obtain: M = 36.827754mH F = 20.338671mN From (??0) and (??1 This case is the singular case. From (??) and (6) we obtain: M = 67.6203121mH F = 45.309445mN From (??0) and (??1 This case is the singular case. From (??) and (6) we obtain: M = 83.323296mH F = 49.855888mN From (10) and (11) we obtain:M =83.323370mH F = 49.855568mN g) Example7.

Wall solenoid: R= 4 m, m, z 1 = 0 m, z 2 = 1 m. Disk coil: R 2 = 3 m, R 4 = 4m, z Q = 0 m. This case is the singular case. From (??) and (6) we obtain: M = 83.323296mH F = -49.855888 NmN From (10) and (11) This case is the singular case. From (??) and (6) we obtain: M =91.598922mH F = 54.254023mN From (10) and (11) we obtain:M = F = 54.258225mN i) Example9. Wall solenoid: R= 3 m, m, z 1 = 0 m, z 2 = 1 m, N 1 =100. Disk coil: R 2 = 3 m, R 4 = 5m, z Q = 0.6 m, N 2 =100.

This case is the singular case. From (??) and (6) we obtain: M = 65.436644mH F = 5.7050033mN From (??0) and (??1) we obtain: M = 65.436923mH F = 5.7050600 mN

By previous examples, we confirmed that all calculated results by two different methods are in an excellent agreement. The bold digits are significant with the same accuracy in both calculations.

7 V. Conclusion

The new accurate expressions for calculating two electromagnetic quantities such as the mutual inductance and the magnetic force are presented in this work. All expressions are in the semi-analytical and the closed form. We give the improved filament method as the comparative method. Results obtained by two different methods agree at least in five significant figures. Nomenclature I 1 : Current imposed in the disk (pancake) in (m) I 2 : Current imposed in the superconducting solenoid in (m) N 1 : number of turns of the pancake N 2 : number of turns of the solenoid R 1 and R 2 : Inner and outer radius of the pancake in (m) R: The radius of the solenoid in (m) z Q : Axial position to the pancake in (m)

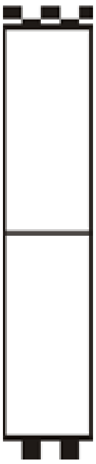

Figure 1: F

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Figure 2: Figure 2 :

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Figure 3:

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Figure 4:

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