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Thermo Dynamic Analysis on MHD Casson Nano-Fluid Flow in a Vertical Porous Space with Stretching Walls

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6 Abstract

3

 $_{7}$ $\,$ This work is concerned with MHD Casson nanofluid flow in a vertical porous space with heat

 $_{\circ}$ and mass transfer in the presence of chemical reaction. The governing non-linear partial

9 differential equations are reduced to ordinary differential equation by employing the similarity

¹⁰ transformations then it solved by homotopy analysis method (HAM). The results are

¹¹ presented with the help of graphs for different values of the involved parameters and discussed.

¹² It is found that increasing Brownian motion parameter, thermophoresis parameter and

¹³ Prandtl number are lead to promote fluid temperature significantly than other parameters.

¹⁴ Also, it is observed that increasing Lewis number lead to enhance the concentration field

¹⁵ whereas the opposite trend can be noticed with increasing thermal parameters. Further, we

¹⁶ have compared HAM solution with the numerical solution by using ND solver in Mathematica.

17

18 Index terms— homotopy analysis method, MHD, chemical reaction, stretching walls.

¹⁹ 1 Introduction

he problem of mixed convective flow in vertical channels with different wall temperatures has a number of
 important engineering applications such as microelectronic components cooling, in the design of compact heat
 exchangers, industrial furnaces, power engineering and so on.

Also, convection flows with heat and mass transfer under the influence of a magnetic field, chemical reaction 23 occurs in many branches of engineering applications and transport processes in industrial applications such 24 25 as chemical industry, power and cooling industry for drying, chemical vapour deposition on surfaces, cooling of 26 nuclear reactors and MHD power generators (See Refs. [1][2][3][4][5] ??6] ??7][8][9][10]). Moreover, MHD channel flows gained significant theoretical and practical importance owing to their applications in MHD generators, 27 accelerators and blood flow measurements. In view of these applications, Srinivas et al. ??7] have studied 28 the effects of thermal-diffusion and diffusion-thermo effects in a two-dimensional viscous flow between slowly 29 expanding or contracting walls with weak permeability. 30

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The effect of chemical reaction and thermal radiation on MHD flow over an inclined permeable stretching surface with non-uniform heat source was examined by Srinivas et al. [8]. Later, Muthuraj et al. [9] discussed the combined effects of thermal-diffusion and diffusion-thermo with space porosity on MHD mixed convective flow of micropolar fluid in a vertical channel. Immaculate et al. [10] have investigated the influence of thermophoretic particle deposition on fully developed MHD mixed convective flow in a vertical channel with thermal-diffusion and diffusion-thermo effects. More recently, effects of thermal diffusion and diffusion thermo on MHD Couette flow of Powell-Eyring fluid in an inclined porous space in the presence of chemical reaction was investigated by

flow of Powell-Eyring fluid in an inclined porous space in the presence of chemical reaction was investigated by
Muthuraj et al. [11].
In engineering applications, the flows of non-Newtonian fluid have been attracting researchers significantly

during the past few decades. In particular, it occurs in the extrusion of polymer fluids, cooling of metallic plate
 in the bath, exotic lubricants, artificial gels, natural gels, colloidal and suspension solutions. The most important

44 among these fluids is the Casson fluid. It can be defined as a shear thinning liquid which is assumed to have an

infinite viscosity at zero rate of shear, a yield stress below which no flow occurs and a zero viscosity at an infinite
rate of shear. Human blood can also be treated as a Casson fluid due to the blood cells' chain structure and the
substances contained like protein, fibrinogen, rouleaux etc [16]. Hence the Casson fluid has its own importance
in scientific as well as in engineering areas. Many researchers have used the Casson fluid model for mathematical
modeling of blood flow in narrow arteries at low shear rates (See Refs. [12][13][14][15][16][17][18]). Nadeem et al.
[15] examined MHD flow of a Casson fluid over an exponentially shrinking sheet.

Sarojamma et al. [16] have investigated MHD Casson fluid flow with heat and mass transfer in a vertical channel with stretching walls. Arthur et al. [17] have analyzed of Casson fluid flow over a vertical porous surface with chemical reaction in the presence of magnetic field. More recently, the unsteady MHD free flow of a Casson fluid past an oscillating vertical plate with constant wall temperature was analyzed by Khalid et al. [18].

Nanoparticle research is currently an area of intense scientific interest due to a wide variety of potential 55 applications in biomedical, optical and electronic fields. It is a microscopic particle with at least one dimension 56 less than 100 nm. Many existing studies indicate that an enormous enhancement in the emission intensity, 57 quantum yield, and lifetime of the molecular rectangles has been observed when the solvent medium is changed 58 from organic to aqueous and it clearly exhibit enhanced thermal conductivity, which goes up with increasing 59 volumetric fraction of nanoparticles [19][20][21][22][23] ??24][25] ??26][27] ??28]. The model of nanofluid was 60 61 first developed by Choi [19]. Later, fully developed mixed convection flow between two paralleled vertical flat 62 plates filled by a nanofluid with the Buongiorno mathematical model using HAM was analyzed by Xu et al. [25]. ??adeem et al. [26] presented the steady stagnation point flow of a Casson nanofluid in the presence of convective 63 boundary conditions. Khan et al. [27] analyzed the fully-developed two-layer Eyring-Powell fluid in a vertical 64 channel divided into two equal regions. One region is filled with the clear Eyring-Powell fluid and the other 65 with the Eyring-Powell nanofluid. The problem of MHD laminar free convection flow of nanofluid past a vertical 66 surface was analyzed by Freidoonimehr ??28]. More recently, Immaculate et al. [29] examined the MHD unsteady 67 flow of Williamson nanofluid in a vertical channel filled with a porous material and oscillating wall temperature 68 using HAM. To the best of our knowledge MHD Casson nanofluid in a vertical channel with stretching walls 69 has not been studied before. In this paper, we therefore propose to analyzed the steady fully-developed mixed 70 convection flow of MHD Casson nanofluid in a vertical porous space with stretching walls in the presence of 71 chemical reaction. It is important to note that this type of analysis has direct applications to the study of 72 blood flow in the cardiovascular system subject to external magnetic field. The reduced non-dimensional, highly 73 74 non-linear, coupled system of equations is solved by HAM ??30] ??31] ??32] ??33] ??34] ??35]. The influence 75 of significant parameters on heat and mass transfer characteristics of the flow is presented through graphs and discussed. 76

77 **2 II.**

78 **3** Formulation of The Problem

⁷⁹ We consider MHD Casson nanofluid flow in a vertical porous space bounded by two stretching walls and are ⁸⁰ maintained at different temperatures, concentrations. The channel walls are at the positions y = -L and y =⁸¹ L, as shown in Fig. 1. A constant magnetic field of strength B 0 is applied perpendicular to the channel walls. ⁸² The fluids in the region of the parallel walls are incompressible, non-Newtonian and their transport properties ⁸³ are assumed to be constant.

The constitutive equation for the Casson fluid can be written as [16] y B ij c ij y B ij c c 2 e , 2 2 e , 2 ? ? ? $\mu + ? > ? ? ? ? ? ? ? ? ? ? ? = ? ? ? ? \mu + ? < ? ? ? ? ? ? ? ? ? ? ? (1)$

where B μ is the plastic dynamic viscosity of the non-Newtonian fluid, y ? is the yield stress of the fluid, ? is the product of the component of deformation rate with itself, namely, ij ij e e ? =

95 ? ? 2 2 T T B 1 D k C C u v D C T k C x y T ? ? + = ? + ? ? ? (6)

The boundary conditions of the problem areu bx = , v = 0, 1 T T = , 1 C C = at y L = ? (7) u bx = , v = 0, 2 T T = , 2 C C = at y L = (8)

- where u and v are the velocity components in x and y directions,1
- 99 T and 2 T are the wall temperatures 2 1 (T T) > , 1
- 100 C and 2 C are the wall concentrations, T

is the mean value of 1 x yT and 2 T , F C is the inertial coefficient, p C is the specific heat,?? ? = +??.

We introduce the similarity variables' u bxf () = ?; v Lbf () = ??; y L ? = ; 1 2 1 T T T T?? = ?; 1 2 103 1 C C C C ?? = ?(9)

104 Invoking the above similarity variables to equations (??)-(??) and eliminating pressure gradient, we get ()1

- 105 1 f R f f ff Hf I f f G G 0 e r c ? ? + ? ? ? ? + ? + ? = ? ? ? ? (10) ' ' ' ' 2 ' r b t e P [N N () R f] 0 ? + 106 ? ? + ? + ? = (11) ' ' ' * t e e 1 b N L (R f k) 0 N ? + ? + ? ? ?? + =(**12**)
- Its non-dimensional form is given by 'f 1 1 C 1 f () ????????????= \pm ??= \pm ??; '1 Nu ()?= \pm =? 113 ??; '1 Sh ()?= \pm =???(17)
- III. SOLUTION BY HOMOTOPY ANALYSIS METHOD (HAM) For HAM solutions, we can choose the 114 initial guesses and auxiliary linear operators in the following form: [0,1]?? be an embedding parameter and h 115 be the auxiliary non-zero parameter. We construct the following zero-order deformation equations.3 0 0 0 1 1 f 116 (); (); () 2 2 2 + ? + ? ? ?? ? = ? ? = ? ? = (18) iv '3 ' " 1 2 L (f) f L () L () = ? =? ? =? (19) 1 0 1 117 118 119 = (**21**)3 3 0 ??(1)L [(,)()] h [f(,),(,),(,)] ?? ??????????????????????????? N,?(1,)0,(1,)1?? 120 ? = ? ? = (22)121
- 123) (,) (f Hf ????, I f f G G r) (,) (,)(,), c (

125 **4 J**

- The skin friction coefficient, local heat rate transfer and the local mass diffusion rate at the walls are defined as 127 t 0 2 1 r f g (1 C) (T T) L G bx ? ? ? = ? local temperature
- Grashof number ,2 c p f 2 1 c f g ()(C C)L G bx ???? = μ is the local nano-particle Grashof number, f r * P ?= ? is the Prandtl number, f e B L D ?= is the Lewis number, * B 2 1 b f D (C C) N ?? = ? is the Brownian motion parameter * T 2 1 t f D (T T) N T ?? = ? is the thermophoresis parameter, 2 1 f k L ?= ? is the chemical reaction parameter.
- when ? increases from 0 to 1, then f (,),? ? ?(,), ? ? ? ?(,)
- In which 'h' is chosen in such a way that these series are convergent at $1 \ge 1$, therefore we have $0 \le 1 \le 1$ () 144 f() f(), ? = ? = ? + ? ? $0 \le 1$ () () (), ? = ? ? =? ? +? ? ? $0 \le 1$ () () () ? = ? ? =? ? +? ? ? 145 (29)

¹⁵² 5 IV. Convergence and the Residual Error

The convergence and rate of approximation for the HAM solution depends on auxiliary parameter 'h' (See Refs. [29] ??30] ??31] ??32] ??33] ??34]), for this purpose, we have plotted h-curves in Fig. ?? with fixing the values of involved parameters rG = 5, c G = 5, e R 1 =, I = 1, t N = 0.45, b N = 0.45, e L = 10, M = 2, r P = 2.5, a D 0.5 =, 1 K 1 = , 0.5 ? = , 0.6 ? = .

As a result, we can choose proper value of 'h' and also we obtain the optimal values of the auxiliary parameter 'h' by minimizing the average square residual error for the equations (??0) to (12). We define the residual error for above mentioned equations as:

(36) Further, we have tabulated the minimum average square residual errors for 10 th , 15 th , 20 th , 25 th order of HAM approximation for different values of parameters with optimal 'h' in Table 1. It is noted that the number of HAM approximation increases the corresponding minimum square residual error decreases significantly and hence it leads to more accurate solutions. Further, it is important to note that our present HAM solution is good agreement with Numerical solution which is obtained by NDSolve scheme of Mathematica (See Fig. ??). V.? = 0.

¹⁶⁶ 6 Results and Discussions

To study the behavior of solutions, numerical calculations for different values of magnetic parameter (M), 167 Permeability parameter It is observe that magnitude of velocity is a decreasing function with increasing Casson 168 fluid parameter and also we noted that increasing 'M' is lead to decelerate the velocity. Physically it means that 169 the application of transverse magnetic field produces a resistive type force (Lorentz force) similar to drag force 170 which tends to resist the fluid flow and thus reducing its velocity (as noted in [18]). The effect of permeability 171 parameter a D on the velocity is displayed in Fig. ??b. It depicts that the effect of increasing the value of a D is 172 to increase the velocity, which means that the drag force is reduced by increasing the value of the permeability 173 parameter. 174 Fig. ??c illustrates the influence of thermophoresis parameter t N on velocity. It shows that increasing t N is 175

not shown much influence on velocity distribution.
The quite similar effect can be noticed by varying Brownian motion parameter b N on the velocity (See Fig. ??d).

178 Fig. ??a is graphed to see the effect of Lewis number on temperature distribution. It is seen that temperature 179 field is an increasing function in the left half of the channel whereas the behavior is reversed in the other region. 180 Fig. ??b describes that, increasing chemical reaction parameter gives opposite behavior that of Fig. ??a. Fig. 181 ??c is plotted to see the influence of Brownian motion parameter on temperature distribution. It is evident that 182 increasing b N is to increase the fluid temperature significantly. The similar effect can be noticed with increasing 183 t N and r P, which are shown in Figs. ??d and 4e. Physically speaking, increasing thermal parameters is to 184 increase momentum diffusivity, which leads to enhance the fluid temperature. Further, it is noted that t N, r 185 P shows the significant influence on temperature field than other parameters. Fig. ??a shows the variation in 186 concentration field with different values of Lewis number e L. It depicts that increasing e L lead to enhance 187 species concentration significantly. Also, it is observed that when increasing e L from 0 to 5 there is nearly 45%188 increase in concentration whereas increasing e L from 5 to 10 there is only 20% (approx) decrease in the same, 189 which means that low values of e L dominates on concentration field. The opposite trend can be seen if L while 190 increasing ? is not shown much influence at the wall ?=-1. At the other wall, the opposite trend is noticed with 191 increasing e L . 192

¹⁹³ 7 VI.

194 8 CONCLUSIONS

This article looks at flow, heat and mass transfer characteristics of a MHD Casson nanofluid in a vertical porous space with stretching walls in the presence of chemical reaction. HAM is adopted to obtain analytical solutions of the reduced set of ordinary differential equations.

¹⁹⁸ The results are presented through graphs for various values of the pertinent parameters and the salient features of the solutions are discussed graphically. This type of investigations is very important for mathematical

$$L_3[\phi_m(\eta) - \chi_m \phi_{m-1}(\eta)] = hR_m^{\phi}(\eta)$$

Figure 1: D

199 200 ² ³

^{1, , , , , , , , , , , , , , ,}

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Figure 2:



Figure 3:



Figure 4: (20)



Figure 5:



Figure 6: eLG



Figure 7: Global



Figure 8: Fig. 1 :



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Figure 9: Fig. 2 : 8 Fig. 3 : Fig. 4 : 2018 JFig. 5 : 8 : Fig. 9 : G? 2 N

Year 2018 3 () Volume XVIII Issue I Version I J Global Journal of Researches in Engineering Where the inertia coefficient, is the Reynolds mbisr, the Hartmann number, © 2018 Global Journals

Figure 10:

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Figure 11:

1

Optimal h

Figure 12: Table 1 :

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Figure 13:

8 CONCLUSIONS

- E E and 3 E are the residual error at m-th order of HAM approximation for f , ? and ? respectively. The average square residual error is given by:
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