Thermo Dynamic Analysis on MHD Casson Nano-Fluid Flow in a Vertical Porous Space with Stretching Walls

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Abstract
This work is concerned with MHD Casson nanofluid flow in a vertical porous space with heat and mass transfer in the presence of chemical reaction. The governing non-linear partial differential equations are reduced to ordinary differential equation by employing the similarity transformations then it solved by homotopy analysis method (HAM). The results are presented with the help of graphs for different values of the involved parameters and discussed. It is found that increasing Brownian motion parameter, thermophoresis parameter and Prandtl number are lead to promote fluid temperature significantly than other parameters. Also, it is observed that increasing Lewis number lead to enhance the concentration field whereas the opposite trend can be noticed with increasing thermal parameters. Further, we have compared HAM solution with the numerical solution by using ND solver in Mathematica.

Index terms—homotopy analysis method, MHD, chemical reaction, stretching walls.

1 Introduction
The problem of mixed convective flow in vertical channels with different wall temperatures has a number of important engineering applications such as microelectronic components cooling, in the design of compact heat exchangers, industrial furnaces, power engineering and so on.

Also, convection flows with heat and mass transfer under the influence of a magnetic field, chemical reaction occurs in many branches of engineering applications and transport processes in industrial applications such as chemical industry, power and cooling industry for drying, chemical vapour deposition on surfaces, cooling of nuclear reactors and MHD power generators (See Refs. [1][2][3][4][5][6][7][8][9][10]). Moreover, MHD channel flows gained significant theoretical and practical importance owing to their applications in MHD generators, accelerators and blood flow measurements. In view of these applications, Srinivas et al. [8] have studied the effects of thermal-diffusion and diffusion-thermo effects in a two-dimensional viscous flow between slowly expanding or contracting walls with weak permeability.

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The effect of chemical reaction and thermal radiation on MHD flow over an inclined permeable stretching surface with non-uniform heat source was examined by Srinivas et al. [8]. Later, Muthuraj et al. [9] discussed the combined effects of thermal-diffusion and diffusion-thermo with space porosity on MHD mixed convective flow of micropolar fluid in a vertical channel. Immaculate et al. [10] have investigated the influence of thermophoretic particle deposition on fully developed MHD mixed convective flow in a vertical channel with thermal-diffusion and diffusion-thermo effects. More recently, effects of thermal diffusion and diffusion thermo on MHD Couette flow of Powell-Eyring fluid in an inclined porous space in the presence of chemical reaction was investigated by Muthuraj et al. [11].

In engineering applications, the flows of non-Newtonian fluid have been attracting researchers significantly during the past few decades. In particular, it occurs in the extrusion of polymer fluids, cooling of metallic plate in the bath, exotic lubricants, artificial gels, natural gels, colloidal and suspension solutions. The most important among these fluids is the Casson fluid. It can be defined as a shear thinning liquid which is assumed to have an
infinite viscosity at zero rate of shear, a yield stress below which no flow occurs and a zero viscosity at an infinite rate of shear. Human blood can also be treated as a Casson fluid due to the blood cells’ chain structure and the substances contained like protein, fibrinogen, rouleaux etc [16]. Hence the Casson fluid has its own importance in scientific as well as in engineering areas. Many researchers have used the Casson fluid model for mathematical modeling of blood flow in narrow arteries at low shear rates (See Refs. [12],[13],[14],[15],[16],[17],[18]). Nadeem et al. [15] examined MHD flow of a Casson fluid over an exponentially shrinking sheet.

Sarojamma et al. [16] have investigated MHD Casson fluid flow with heat and mass transfer in a vertical channel with stretching walls. Arthur et al. [17] have analyzed of Casson fluid flow over a vertical porous surface with chemical reaction in the presence of magnetic field. More recently, the unsteady MHD free flow of a Casson fluid past an oscillating vertical plate with constant wall temperature was analyzed by Khalid et al. [18].

Particle research is currently an area of intense scientific interest due to a wide variety of potential applications in biomedical, optical and electronic fields. It is a microscopic particle with at least one dimension less than 100 nm. Many existing studies indicate that an enormous enhancement in the emission intensity, quantity and lifetime of the molecular rectangles has been observed when the solvent medium is changed from organic to aqueous and it clearly exhibit enhanced thermal conductivity, which goes up with increasing volumetric fraction of nanoparticles [19],[20],[21],[22],[23],[24],[25],[26],[27],[28]. The model of nanofluid was first developed by Choi [19]. Later, fully developed mixed convection flow between two parallelled vertical flat plates filled by a nanofluid with the Buongiorno mathematical model using HAM was analyzed by Xu et al. [25].

Nadeem et al. [26] presented the steady stagnation point flow of a Casson nanofluid in the presence of convective boundary conditions. Khan et al. [27] analyzed the fully-developed two-layer Eyring-Powell fluid in a vertical channel divided into two equal regions. One region is filled with the clear Eyring-Powell fluid and the other with the Eyring-Powell nanofluid. The problem of MHD laminar free convection flow of nanofluid past a vertical surface was analyzed by Freidoonimehr [28]. More recently, Immaculate et al. [29] examined the MHD unsteady flow of Williamson nanofluid in a vertical channel filled with a porous material and oscillating wall temperature using HAM. To the best of our knowledge MHD Casson nanofluid in a vertical channel with stretching walls has not been studied before. In this paper, we therefore propose to analyzed the steady fully-developed mixed convection flow of MHD Casson nanofluid in a vertical porous space with stretching walls in the presence of chemical reaction. It is important to note that this type of analysis has direct applications to the study of flow and mass transfer characteristics of the flow is presented through graphs and discussed.

2 II.

3 Formulation of The Problem

We consider MHD Casson nanofluid flow in a vertical porous space bounded by two stretching walls and are maintained at different temperatures, concentrations. The channel walls are at the positions y = -L and y = L, as shown in Fig. 1. A constant magnetic field of strength B 0 is applied perpendicular to the channel walls. The fluids in the region of the parallel walls are incompressible, non-Newtonian and their transport properties are assumed to be constant.

1. If \( f(x) \) is given, then the HAM solution can be expressed as
\[
 f(x) = \lim_{n \to \infty} f_n(x)
\]
where, \( f_n(x) \) represents the nth order approximation of the HAM solution.

The corresponding boundary conditions are:

2. The HAM solution is an analytical solution that provides a series of approximations that converge to the exact solution.

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The convergence and effectiveness of the HAM solution are demonstrated by comparing it with the exact solution and other approximate solutions. The results show that the HAM solution is effective in obtaining accurate approximate solutions.
8 CONCLUSIONS

good agreement with Numerical solution which is obtained by NDSolve scheme of Mathematica (See Fig. ??). $V.\eta = 0$.

6 Results and Discussions

To study the behavior of solutions, numerical calculations for different values of magnetic parameter (M), Permeability parameter It is observe that magnitude of velocity is a decreasing function with increasing Casson fluid parameter and also we noted that increasing "M" is lead to decelerate the velocity. Physically it means that the application of transverse magnetic field produces a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity (as noted in [18]). The effect of permeability parameter a D on the velocity is displayed in Fig. ??b. It depicts that the effect of increasing the value of a D is to increase the velocity, which means that the drag force is reduced by increasing the value of the permeability parameter.

Fig. ??c illustrates the influence of thermophoresis parameter t N on velocity. It shows that increasing t N is not shown much influence on velocity distribution.

The quite similar effect can be noticed by varying Brownian motion parameter b N on the velocity (See Fig. ??d).

Fig. ??a is graphed to see the effect of Lewis number on temperature distribution. It is seen that temperature field is an increasing function in the left half of the channel whereas the behavior is reversed in the other region.

Fig. ??b describes that, increasing chemical reaction parameter gives opposite behavior that of Fig. ??a. Fig. ??c is plotted to see the influence of Brownian motion parameter on temperature distribution. It is evident that increasing b N is to increase the fluid temperature significantly. The similar effect can be noticed with increasing t N and r P , which are shown in Figs. ??d and 4e. Physically speaking, increasing thermal parameters is to increase momentum diffusivity, which leads to enhance the fluid temperature. Further, it is noted that t N , r P shows the significant influence on temperature field than other parameters. Fig. ??a shows the variation in concentration field with different values of Lewis number e L . It depicts that increasing e L lead to enhance species concentration significantly. Also, it is observed that when increasing e L from 0 to 5 there is nearly 45% increase in concentration whereas increasing e L from 5 to 10 there is only 20% (approx) decrease in the same, which means that low values of e L dominates on concentration field. The opposite trend can be seen if L while increasing is not shown much influence at the wall ?=-1. At the other wall, the opposite trend is noticed with increasing e L .

7 VI.
8 CONCLUSIONS

This article looks at flow, heat and mass transfer characteristics of a MHD Casson nanofluid in a vertical porous space with stretching walls in the presence of chemical reaction. HAM is adopted to obtain analytical solutions of the reduced set of ordinary differential equations.

The results are presented through graphs for various values of the pertinent parameters and the salient features of the solutions are discussed graphically. This type of investigations is very important for mathematical

$$L_{3}[\phi_{m}(\eta) - \chi_{m}\phi_{m-1}(\eta)] = hR_{m}^{\phi}(\eta)$$

Figure 1: D
Figure 2:

Figure 3:
CONCLUSIONS

Figure 4: $\theta'(0)$

Figure 5: $\phi'(-1)$

Figure 6: $f'$ with $N_b = 0.1, 0.2, 0.3, 0.4$
Figure 7: Global

\[ \beta = 0.2, 0.4, 0.6, 0.8 \]
\[ M = 0 \]

Figure 8: Fig. 1:

\[ \beta = 0.2, 0.4, 0.6, 0.8 \]
\[ M = 5 \]
Where the inertia coefficient, the Reynolds number, the Hartmann number, and the optimal h are considered.

Figure 9: 

\[ D_a = 0.1, 0.7, 1.5, \infty \]

Figure 10:

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3
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Figure 11:

Year 2018
4
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Figure 12: Table 1:


Figure 13:
E E and 3 E are the residual error at m-th order of HAM approximation for f , ?  and ? respectively. The average square residual error is given by:

\[ E E = 3 E \text{ are the residual error at m-th order of HAM approximation for f, } \theta, \text{ and } \theta \text{ respectively. The average square residual error is given by:} \]

\[ \frac{1}{n} \sum_{i=1}^{n} (f_i - f)^2 \]


[Mansour and El-Anssary] Effects of chemical reaction and thermal stratification on, M A Mansour , N El- Anssary , AlyA.


