Unit Commitment Scheduling with Wind power generation Uncertainty and Emission Consideration

By Mohadese Bagheri, Majid Shahabi

Abstract: Nowadays, in order to achieve environmental goals, renewable energy sources especially wind, has been seemed useful while wind generation does not directly produce air pollutants emission. So it seems necessary to consider air pollutants emission level in wind-thermal scheduling problems. This paper proposes two methodologies for wind–thermal scheduling in a power system with high penetration of wind power subject to consider air pollutants emission reduction. Also a stochastic programming market-clearing model has been applied for solving unit commitment problem to overcome stochastic nature of power. In this stochastic security model, wind generation uncertainty is modeled by scenario tree in scheduling time horizon. The usefulness of the proposed approach was demonstrated through an IEEE 30-bus test system over 6 hours.

Keywords: Emission, Stochastic Scheduling, unit commitment, wind power generation.

Classification: GJRE-J Classification: FOR Code: 660202, 850509
Abstract: Nowadays, in order to achieve environmental goals, renewable energy sources especially wind, has been seemed useful while wind generation does not directly produce air pollutants emission. So it seems necessary to consider air pollutants emission level in wind-thermal scheduling problems. This paper proposes two methodologies for wind-thermal scheduling in a power system with high penetration of wind power subject to consider air pollutants emission reduction. Also a stochastic programming market-clearing model has been applied for solving unit commitment problem to overcome stochastic nature of power. In this stochastic security model, wind generation uncertainty is modeled by scenario tree in scheduling time horizon. The usefulness of the proposed approach was demonstrated through an IEEE 30-bus test system over 6 hours.

Keywords: Emission, Stochastic Scheduling, unit commitment, wind power generation.

I. INTRODUCTION

Nowadays because of low cost of energy generation and its environmental advantages, using wind energy in electric power generation, has been seemed useful. On the other hand because of variability and uncertainty of this energy, using it has made some challenges to power-system operators. In order to adjust the unforeseeable nature of the wind power, planned productions and uses in electricity market must be improved during the real operation of the power system.

Because of the stochastic nature of the wind speed, we need to consider the probable considerations in its modeling equations. Without considering the probable issues, scheduled the system will be determined with deterministic security, and because of extra reserve allocation this method will impose extra cost to the system [1]. In this paper stochastic optimization method has been used for system operation scheduling. By using the stochastic security, a balance can be established between the advantages of using the wind power generations – because of its low cost- and increasing the operation cost – because of the necessity of increasing the needed reserve of the system, and also the required reserve level of the system can be determined optimizing. In this method, besides its normal use, system operation planning will be economical for all the probable scenarios. In stochastic method, in order to establish the power balance between generation and consumption for the probable scenarios with low demand, if the cost of involuntary load shedding is low the involuntary load shedding will be used. [2]. Nowadays decrease of production of the air pollutant gases is under consideration as a behavioral pattern in countries industries. So the level of produced gases by plants must be minimized in operation planning of them. Commitment of the wind plants in power generation increases the importance of considering the generating pollution of thermal units. Because on one hand these wits are not producers of the air pollutant gases, but on the other hand the generating pollution curve of the thermal units is in a way that by high decrease in their generating power level, their generated pollution level increases. By increasing the penetration of wind power generation and providing the load by it, power level of the thermal units decreases, (This case is more apparent in low demand or medium demand hours).

Which means the increase in air pollutant gases emission, in this paper, in order to consider the pollutant gases level in power system operation planning, two methods are offered. These methods include a multi – objective optimization method and considering the maximum permissible generating pollution for plants.

Reference [3] has used a stochastic optimization method for planning the units in a power system with the presence of wind plant, also in [1] the Monte Carlo simulation method has been used in order to estimate the required, spinning reserve of the system. Also in [4] a stochastic optimization method has been used for planning the units. In these reference the uncertainty of forecasting the wind and the demand are considered simultaneously. But the generating pollution of the thermal units in scheduling are not considered in any of these references.

II. MODEL

In the present paper, for considering the pollutant emission by thermal units in power system operation planning, two methods are offered. The first method is to use multi-objective optimization in power system planning; and the goal of this method is to
decrease the operation cost of the power system and air pollutant gases emission simultaneously. The second method considers the power system optimization scheduling by use of stochastic optimization with considering the limit of the maximum generation permissible pollution. While the air pollutant gases level is high and this environmental problem is very important, considering the generating pollution of the generating units as a limit seems logical.

a) The First Method

Multi-objective optimization the multi-objective optimization method which is used in this paper is the weighting method. This model includes two objectives.

i. The first objective

The first objective is the operation cost, and by using stochastic optimization it is the equation (1).

\[
F^{\text{cost}} = \sum_{t=1}^{N_T} EC_t = \sum_{t=1}^{N_T} \sum_{i=1}^{N_G} C_{ui}^S + \sum_{j=1}^{N_S} d_j \left( \sum_{i=1}^{N_G} \left( a_i F_g^S + b_i F_g^S + c_i \right) - \sum_{j=1}^{N_S} \lambda_{it} L_j^S \right) + \sum_{i=1}^{N_G} \left( C_{\text{R}i}^U R_i^U + C_{\text{R}i}^D R_i^D + C_{\text{R}i}^\text{NS} R_i^\text{NS} \right) + \lambda_t^{\text{WP}} P_{i WP, S}^t + \sum_{a=1}^{N_{\text{w}}} \pi_a \sum_{i=1}^{N_G} C_{a i}^A
\]

\[
= \sum_{i=1}^{N_G} \sum_{t=1}^{N_T} \left( a_i F_g^S + b_i F_g^S + c_i \right) - \sum_{j=1}^{N_S} \lambda_{it} L_j^S + \sum_{i=1}^{N_G} \left( C_{\text{R}i}^U R_i^U + C_{\text{R}i}^D R_i^D + C_{\text{R}i}^\text{NS} R_i^\text{NS} \right) + \lambda_t^{\text{WP}} P_{i WP, S}^t + \sum_{a=1}^{N_{\text{w}}} \pi_a \sum_{i=1}^{N_G} C_{a i}^A
\]

(1)

Where \( EC_t \) are the expected cost of the system in period \( t \) and \( C_{ui}^S \), \( C_{\text{R}i}^U \), \( C_{\text{R}i}^D \), and \( C_{\text{R}i}^\text{NS} \) respectively the offer costs of the up-down, and nonspinning reserves of unit \( i \) in period \( t \). Also \( \pi_a \) is the probability of occurring the scenario \( o \) and \( d_j \) is the length of each time period \( t \) in the scheduling horizon. We assume that the wind generators are not competitive factors, so they do not offer a cost in the market (\( \lambda_t^{\text{WP}} = 0 \)).

ii. The second objective

In the second objective, the generating pollution of the thermal units is considered. Are of the most important air pollutants which are generated by thermal units \( SO_x \) and \( NO_x \). Generally, the air pollutant gas level which is generated by unit \( i \) in the time horizon \( t \) is estimated by the equation (2) [5].

\[
E_{i,t}(P_g^S) = d_i \left( \alpha_i + \beta_i P_g^S + \gamma_i P_g^S + \lambda_i \exp(\lambda_i P_g^S) \right) u_{it}
\]

Where \( E_{i,t}(P_g^S) \) is the generating air pollutant level by the unit \( i \) in time horizon \( t \). Where the \( \alpha_i \), \( \beta_i \), \( \gamma_i \), \( \lambda_i \), and \( \lambda_i \) coefficients of the air pollutant objective by unit \( i \) and \( d_i \) is the length of the time horizon \( t \). In this paper regarding the stochastic nature of the case and the planning and pollution level is considered as stochastic planning and also it has considered in each scenario according to the probability of occurrence of each scenario, so the considered pollution is as equation (3):

\[
F^{E\text{mission}} = \sum_{a=1}^{N_{\text{w}}} \sum_{t=1}^{N_T} \sum_{i=1}^{N_G} E_{i,t}(P_g^S)
\]

\[
= \sum_{a=1}^{N_{\text{w}}} \sum_{t=1}^{N_T} \sum_{i=1}^{N_G} \left( \alpha_i + \beta_i P_{\text{Gito}} + \gamma_i P_{\text{Gito}}^2 + \lambda_i \exp(\lambda_i P_{\text{Gito}}) \right) u_{it}
\]

(3)

iii. The objective of the multi-objective optimization, weighting method

In this method the multi-objective optimization of the objective has been combined by some coefficients and them from the main objective of the optimization.

\[
F = \min \left\{ \eta \cdot F^{\text{cost}} + (1 - \eta) \cdot E\text{mission} \right\}
\]

Where \( E\text{mission} \) is the emission control constant, in \$/ton unit. This constant is used for the cost of operation and the pollution and in fact, if is the cost of controlling the pollution. Also \( 0 \leq \eta \leq 1 \) is a compromise factor.

b) Model limitations

The limitations of the model are categorized in 3 general categories:

i. Operation limits related to the normal mode operation.

1. Market Equilibria

\[
\sum_{i=1}^{N_G} P_{i S} + P_{i WP, S} = \sum_{j=1}^{N_L} L_j^S, \quad \forall t.
\]

(5)

2. Production limits

\[
P_{i \text{min}} u_{it} \leq P_{i S} \leq P_{i \text{max}} u_{it}, \quad \forall i, \forall t.
\]

(6)

3. Wind generation limits

\[
P_{i WP, \text{min}} \leq P_{i WP, S} \leq P_{i WP, \text{max}, \forall t}.
\]

(7)

Where, \( P_{i WP, \text{max}} \) and \( P_{i WP, \text{min}} \) are parameters offered as part of the wind producer energy offer.

4. Demand Bounds

\[
L_{i j, \text{min}}^S \leq L_{i j}^S \leq L_{i j, \text{max}}^S, \quad \forall j, \forall t.
\]

(8)
5. **Scheduled Reserve Determination Constraints**

a. **Spinning**

\[
0 \leq R_{ii}^U \leq R_{ii}^{U,\text{max}}, \forall i, \forall t.
\] (9)

\[
0 \leq R_{ii}^D \leq R_{ii}^{D,\text{max}}, \forall i, \forall t.
\] (10)

b. **Non-spinning**

\[
0 \leq R_{ii}^{NS} \leq R_{ii}^{NS,\text{max}} (1-u_{ii}), \forall i, \forall t.
\] (11)

6. **Start-Up Cost**

\[
C_{it}^{\text{SU}} \geq \lambda_{it}^{\text{SU}} (u_{ii} - u_{i,i-1}), \forall i, \forall t.
\] (12)

\[
C_{it}^{\text{SU}} \geq 0, \forall i, \forall t
\] (13)

ii. Operation limits related to planning in each scenario this part of the relationships includes actual system operation (second-stage variables)

7. **Power Balance constraints**

a. Power Balance at Every Node \( n \) (Different from node \( n' \) at which the wind power is injected.

\[
\sum_{i(i,n)\in MG} P_{it}^G - \sum_{j(j,n)\in ML} (L_{jt}^S - L_{jt}^{\text{shed}}) = \sum_{r(n,r')\in A} f_{i\rightarrow r}(n,r) = 0, \forall n \neq n', \forall t, \forall \omega.
\] (14)

b. Power balance at node \( n' \) at which the wind power generation is injected.

\[
\sum_{i(i,n')\in MG} P_{it}^G - \sum_{j(j,n')\in ML} (L_{jt}^S - L_{jt}^{\text{shed}}) + P_{it\rightarrow n'} - S_{it\rightarrow n'} = \sum_{r(n,r')\in A} f_{i\rightarrow r}(n,r) = 0, n = n', \forall t, \forall \omega.
\] (15)

c. Power flow through line from \( n \) to \( r \)

\[
f_{i\rightarrow r}(n,r) = \frac{P_{it}^{\text{loss}}(n,r)}{2} + B(n,r)(\delta_{i\rightarrow r} - \delta_{i\rightarrow r}), \forall (n,r) \in A, \forall t, \forall \omega.
\] (16)

2. **Generation Limits**

\[
P_{it}^G \geq P_{i}^{\text{min}} v_{i\rightarrow t}, \forall i, \forall t, \forall \omega.
\] (17)

\[
P_{it}^G \leq P_{i}^{\text{max}} v_{i\rightarrow t}, \forall i, \forall t, \forall \omega.
\] (18)

3. **Transmission Capacity Constraints**

\[-f_{ij}^{\text{max}}(n,r) \leq f_{i\rightarrow j}(n,r) \leq f_{ij}^{\text{max}}(n,r) \] (19)

The positive or negative power flow through the lines is related to the different directions of the power flow through a line.

4. **Involuntary Load Shedding Constraints**

\[0 \leq L_{jk}^{\text{shed}} \leq L_{jk}^S, \forall j, \forall t, \forall \omega.
\] (20)

5. **Limits of Wind Power Generation Spillage**

\[0 \leq S_{it}^W \leq P_{it}^W, \forall t, \forall \omega.
\] (21)

iii. Constraints linking the normal model and scenario scheduling

1. **Decomposition of Generator Power Outputs:**

\[P_{itw}^G = P_{it}^S + r_{itw}^U + r_{itw}^{NS} - r_{itw}^D, \forall i, \forall t, \forall \omega.
\] (22)

2. **Deployed Reserve Determination Constraints:**

a. **Spinning**

\[0 \leq r_{itw}^U \leq R_{itw}^U, \forall i, \forall t, \forall \omega
\] (23)

\[0 \leq r_{itw}^D \leq R_{itw}^D, \forall i, \forall t, \forall \omega
\] (24)

b. **Non-spinning**

\[0 \leq r_{itw}^{NS} \leq R_{itw}^{NS}, \forall i, \forall t, \forall \omega
\] (25)

c. **Second-Stage Start-Up Cost Adjustments:**

\[C_{itw}^{\text{SU}} = C_{itw}^{\text{SU}} - C_{it}^{\text{SU}} - C_{itw}^{\text{SU}} A, \forall i, \forall t, \forall \omega
\] (26)

\[C_{itw}^{\text{SU}} \geq \lambda_{it}^{\text{SU}} (v_{itw} - v_{i,t-1,\omega}), \forall i, \forall t, \forall \omega
\] (27)

\[C_{itw}^{\text{SU}} \geq 0, \forall i, \forall t, \forall \omega
\] (28)

Note that variable \( C_{itw}^{\text{SU}} \) accounts for the start-up cost incurred by generating unit \( i \) during the actual operation of the power system in period \( t \) and scenario \( \omega \). The important advantages of the planning with stochastic security are planning with the goal of minimizing the operation cost and the pollution in normal mode and in all scenarios [3].

3. **The Second Method**

The emission of the maximum permissible pollutant gases by each generating unit is considered in this method, for more about, In this method the...
stochastic planning objective includes the operation cost in normal mode and in each scenario, which is the cost objective of the equation (1). As it is mentioned, in this method each generating unit depending on the climate and environment is allowed to generate only a particular amount of pollution. This permissible pollutant emission can be modeled as the following equation.

$$\sum_{i=1}^{E} E_{i,t} (P_{it}^S) \leq E_{i,\text{target}}^t, \forall i.$$  

(29)

Where $E_{i,\text{target}}^t$ is the permissible amount of generating pollution of unit $i$ during the expected horizon. The time horizon of considering the pollution limits is weekly or monthly. Other limits will be similar to the limits of section (2.2).

### III. Case Study

The system which is being studied in this paper is the IEEE 30-bus system [6]. It is assumed that the wind plant is located in a 22-bus system. This system consists of six generators and their data have been extracted from the reference [6]. This planning is tested over a 6-h scheduling horizon. The general hourly demand in 6-h scheduling has been considered 450,420,200,150,120 and 100 MW. The prediction of the hourly wind shown in table 1. Just three wind power scenarios are considered: as forecast, high and low, with probabilities 0.6, 0.2, and 0.2, respectively. Modeling the wind prediction in 6 hours has been considered a scenario tree. Also in order the conducted planning has not been considered. And also in order to access to a better answer, by using the integer linear programming, the non linear parts of the objective has become linear by a linearing method. The expected model has been coded and performed by using the mixed integer linear programming in the powerful GAMS software [7].

#### Table 1: Wind Power Scenario

<table>
<thead>
<tr>
<th>Period</th>
<th>$P_{it}^{\text{wp}}$ (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>As forecast</td>
</tr>
<tr>
<td>1</td>
<td>60</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
</tr>
<tr>
<td>3</td>
<td>70</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
</tr>
<tr>
<td>5</td>
<td>70</td>
</tr>
<tr>
<td>6</td>
<td>80</td>
</tr>
</tbody>
</table>

In order to analyze the results, the units generating costs and the pollutant emission of each unit curves are presented in Fig (1). In order to note the importance of considering the pollutant emission in operation scheduling, planning with the goal of minimizing the system operation cost has been performable separately and these results have been compared with the results of the offered methods. The results of the system planning with the goal of minimizing the operation cost have been estimated in table (2). As it can be seen in this table, to provide the required power of the system, units with lower cost offering are in priority for power providing. Tables 3 and 4, respectively, show the results of the planning's at which the units generating pollution level is considered as a limit and the multi-objective planning is applied in a weighting method. The amount of the maximum generating pollution in the time horizon of planning for am generators is similar and is equal to 0.17 ton. Also in multi-objective optimization method is considered as $\eta = 0.6$. As it is seen in table 3, results of this kind of planning have been changed. One of these changes is the decrease of the number of unit 4 (unit with the cheapest objective of offering energy). It is obvious that, this change is because of limiting the permissible generating pollution of the units during the scheduling horizon.

![Figure 1- Out Put Power And Units Generating Pollution Curves.](image-url)
As it is seen in table (3) unit 2, which as the lowest rate of pollution production according to its pollution generating curve, has participated in the whole time planning in power providing, with considering the pollutant emission. Also it can be seen in the table that in the 6th hour, wind power has been planned at its low level; because by adjusting the wind power at its predicted level, unit 2 is being planned for a lower power production.

As it is seen the generating pollution curve of the units, by decreasing the power production to 20 MW, pollutants emission of this unit will increase. So in low demand condition, in spite the fact that wind units are not pollution producers, high level of their production may lead to increase in produced pollution by each thermal unit. This fact shows the importance of considering the pollutants emission by thermal units in planning. It can be seen in table 4 that, also in multi – objective optimization with weighting method, the priority of power production is adjusted upon the offering cost of units. In these results, at the low demand hours unit 4 (the cheapest unit fro the point of view of power production) is the provider of the required power of the system. One of advantages of the weighting method is ability of adjusting the importance of objectives that is, the power system operator, regarding the importance of environmental issues. Can choose the amount of n which is effective in planning results.

Table 3- Results of Economical Planning of The System Operation With Considering The Generating Pollution Limit of Thermal Units

<table>
<thead>
<tr>
<th>t</th>
<th>generators</th>
<th>R_1</th>
<th>R_2</th>
<th>R_3</th>
<th>R_4</th>
<th>R_5</th>
<th>R_6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 0 0 0 0 0</td>
<td>0 0 0 0 0 0</td>
<td>0 0 0 0 0 0</td>
<td>0 0 0 0 0 0</td>
<td>0 0 0 0 0 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>24 0 0 28 0 0</td>
<td>30 0 0 5 0 0</td>
<td>5 0 0 40 0 0</td>
<td>0 0 0 0 0 0</td>
<td>0 0 0 0 0 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>95 0 0 68 0 0</td>
<td>30 0 0 5 0 0</td>
<td>5 0 0 40 0 0</td>
<td>0 0 0 0 0 0</td>
<td>0 0 0 0 0 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>120 0 0 120 0 0</td>
<td>100 0 0 70 0 0</td>
<td>70 0 0 40 0 0</td>
<td>0 0 0 0 0 0</td>
<td>0 0 0 0 0 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>80 0 0 60 0 0</td>
<td>30 0 0 5 0 0</td>
<td>5 0 0 40 0 0</td>
<td>0 0 0 0 0 0</td>
<td>0 0 0 0 0 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>60 0 0 60 0 0</td>
<td>30 0 0 5 0 0</td>
<td>5 0 0 40 0 0</td>
<td>0 0 0 0 0 0</td>
<td>0 0 0 0 0 0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4- Results of System Operation Planning With the Goal of Simultaneous Decrease of Operation Cost and Generating Pollution with Weighting Method

<table>
<thead>
<tr>
<th>t</th>
<th>generators</th>
<th>R_1</th>
<th>R_2</th>
<th>R_3</th>
<th>R_4</th>
<th>R_5</th>
<th>R_6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14 0 0 32 0 0</td>
<td>0 0 0 0 0 0</td>
<td>0 0 0 0 0 0</td>
<td>0 0 0 0 0 0</td>
<td>0 0 0 0 0 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>21 0 0 47 0 0</td>
<td>30 0 0 7 0 0</td>
<td>0 0 0 0 0 0</td>
<td>0 0 0 0 0 0</td>
<td>0 0 0 0 0 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>84 0 0 7 0 0</td>
<td>30 0 0 7 0 0</td>
<td>0 0 0 0 0 0</td>
<td>0 0 0 0 0 0</td>
<td>0 0 0 0 0 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>120 0 0 120 0 0</td>
<td>100 0 0 70 0 0</td>
<td>70 0 0 40 0 0</td>
<td>0 0 0 0 0 0</td>
<td>0 0 0 0 0 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>80 0 0 80 0 0</td>
<td>30 0 0 7 0 0</td>
<td>0 0 0 0 0 0</td>
<td>0 0 0 0 0 0</td>
<td>0 0 0 0 0 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>60 0 0 60 0 0</td>
<td>30 0 0 7 0 0</td>
<td>0 0 0 0 0 0</td>
<td>0 0 0 0 0 0</td>
<td>0 0 0 0 0 0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
IV. Conclusion

In this paper, a method for the commitment of units in presence of wind power production has been offered with considering the decrease in generating pollution of units. Also the units commitment scheduling is presented with the goal of covering the wind power uncertainty with stochastic security. This paper present two effective method for decreasing the units generating pollution. The first method is a multi-objective optimization method, with the goal of decreasing the operation cost and the pollutant gases emission produced by the units, simultaneously. Also another method is presented which can be used in a condition that limiting the air pollutant gases has the most priority. The suggested method has been tested on an IEEE 30-buses system and the results have been analyzed. The results of this test are representative of the effectiveness of the presented method.

References


List of symbols

\[ C_{st} \] Cost due to the scheduled start-up of unit \( i \) in period \( t \) [\$]

\[ P_{st} \] Power output scheduled for unit \( i \) in period \( t \) [MW]

\[ L_{jt} \] Power scheduled for load \( j \) in period \( t \) [MW]

\[ R_{ui} \] Spinning reserve up scheduled for unit \( i \) in period \( t \) [MW]

\[ R_{dj} \] Spinning reserve down scheduled for load \( j \) in period \( t \) [MW]

\[ R_{ps} \] Nonspinning reserve scheduled for unit \( i \) in period \( t \) [MW]

\[ P_{st}^{WP} \] Scheduled wind power in period \( t \) [MW]

\[ P_{st}^{G} \] Power output of unit \( i \) in period \( t \) and scenario \( \omega \) [MW]

\[ r_{ui} \] Spinning reserve up deployed by unit \( i \) in period \( t \) and scenario \( \omega \) [MW].

\[ r_{di} \] Spinning reserve down deployed by unit \( i \) in period \( t \) and scenario \( \omega \) [MW].

\[ r_{ps} \] Nonspinning reserve deployed by unit \( i \) in period \( t \) and scenario \( \omega \) [MW].

\[ f_{n} \] Power loss in line \((n,r)\) in period \( t \) and scenario \( \omega \) [MW].

\[ \delta_{n} \] Voltage angle at node \( n \) in period \( t \) and scenario \( \omega \) [rad]

\[ P_{st}^{WP} \] Random variable modeling the wind power generation in period \( t \) [MW].

\[ V_{j}^{LOL} \] Value of load shed for consumer \( j \) in period \( t \) [\$/MWh]

\[ V_{s}^{s} \] Cost of wind power spillage in period \( t \) [\$/MWh]