Global Journal of Researches in Engineering: G
Industrial Engineering
Volume 17 Issue 2 Version1.0 Year 2017
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4596 & Print ISSN: 0975-5861

Predicting Waiting Time under Deferent FCFS Queue Schemes

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GJRE-G Classification: FOR Code: 290502p

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Strictly as per the compliance and regulations of:

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I. Introduction

Today, businesses compete not only on quality of products but on service level as well. Recently, the time waiting for service is acknowledged as one of the most critical attributes of service level. . M. K. Hui and D. K. Tse , K. Katz, B. Larson, and R. Larson and other research point out that Customer surveys in service systems demonstrate that waiting time is a key factor when evaluating quality of service (Nakibly, 2002). In fact, waiting time is one of the main considerations when determining staffing levels (Davis., (1991) and more). A common method is to plan for the least number of agents that suffice to satisfy a required service level based on analytical modelling or simulation result. However, Very often, the service process involves delays. Waiting for some services takes place while the customer is waiting on- a face- to-face service line (laboratory diagnosis or a telephone service) or when customers continue their regular activities (waiting for an e-mail or laboratory diagnosis result reply). often, Different factors contribute to the waiting experience result in feelings of anger and in a low customer satisfaction: waiting conditions; the interest level while waiting (filled time vs. empty time); the feeling of justice (or of injustice) in the service discipline and the amount of time that a nation's populace wastes by waiting in queues, which is a major factor in both the quality of life there and the efficiency of the nation's economy. Thus, Proper queuing system's modeling and performance analysis is important components of Customers waiting time reduction and quality improvement.

Over the last decades, customers and customer satisfaction have become the major concern of almost all companies. Surveys demonstrate that customer satisfaction can be improved without changing the waiting time itself, but by managing customer expectations or by improving the waiting experience (Maister, 1985). In addition to the waiting duration itself, customer satisfaction is also affected by the perceived waiting time and by the waiting experience that may be improved by providing information or other services while waiting; making sure the physical waiting environment is comfortable (in face-to-face service); explaining the reasons for waiting; and providing information regarding the anticipated waiting time(Taylor., 1994). However, we have become accustomed to considerable amounts of waiting in service and manufacturing systems, but still get annoyed by unusually long Waiting in a crowd queue without information regarding the anticipated waiting time, which is, Usually, not interesting and undesirable. By awaking this, this paper, like more and more scholars and companies, is focusing on queuing analysis and estimations of waiting times.

Information about anticipated waiting times has important role in service systems and also objectively improves the service level; particularly, it has an important role in service systems with invisible queues (Whitt., 1999). Cleveland and Maybe describe the difference in the waiting experience between visible and invisible queues; they suggest that when the queue is visible, customers experience dissatisfaction upon arrival, as they see that there is a queue; then, as they are advancing in queue, in a satisfactory rate, the

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feelings of dissatisfaction decrease until they receive service and happily leave the system. Where as in queues that are invisible, customers do not experience dissatisfaction upon arrival, but as they are kept on hold, feelings of anger and dissatisfaction emerge; these feelings intensify until they eventually possibly abandon.

Providing waiting information in these cases may eliminate the gap between reality and customer expectations (Cleveland & Mayben, 1999). The For any of queuing systems and FCFS in particular, waiting times estimation method should be either based on the system state at a given moment which are usually tracked in real-time and needed on-line system state or system state distribution (steady state) used to predict the general behavior of the system and is performed off-line, usually, for purposes of planning and for evaluating the performance of a service system, as opposed to the experience of a specific customer. Since individual customers are usually interested on information at a given moment, the goal of this paper is to provide information which is relevant to a specific customer at a specific time. Thus, this work focuses on estimating the waiting time given the system state at the time of estimation rather than estimating the overall performance of the system, such as the average waiting time of all customers, which is usually done assuming a steady-state.

This paper aims to model and predict Expected waiting time in queue and analyze their implications on queue crowd management. In this work, estimations of waiting times is done for the purpose of informing individuals about their anticipated delays, therefore focus on estimating times given the system state at the time of estimation (arrival or any point of time during the waiting).,. The calculations involved in this method, are usually easier, but operational effort is high and the accuracy of the estimation varies accordingly. For example, when service discipline is FCFS, if we could infer the exact service requirement of each customer upon arrival, we would have been able to anticipate the accurate delay (the system would have become deterministic). Since we are dealing with stochastic systems, there is no possible way to predict the exact waiting time. The best one can do is estimate the waiting time distribution. Using model this paper *predicts mean* Expected queue length, and waiting time in queue of kth customer based on the system state at the time of estimation and pre inform customers. Hence estimations of waiting times depend on the information provided, system states, usually, the inputs[1] of a queueing model and characteristics of the system[2] under study, should first be defined. Motivated by the complexity of exact calculations, The goal of this work is to propose methods for estimating waiting times in FCFS queuing systems in general based on trend lines curve fitting equations derived from Cumulative Arrival and service data distribution.

Thus, this paper, First, focus on queueing system and develop basic model based on the arrival and service pattern of the First-Come-First-Served service discipline and the variables used to determine the characteristics of queuing system and propose general model that estimate Expected waiting time of kth customer arrive at any time x in different FCFS system characteristics queue line. Then, apply model and estimate waiting times for classic queueing models, that maintain a simple First-Come-First-Served service discipline and demonstrates the use of different estimation methods and demonstrates the use of estimation methods for FCFS systems with Identical servers and service types, independent servers and Identical service types, Identical servers and multiple service types, and independent servers and multiple service types. Finally, concludes based on result. Thus, this paper model Customers arrival and service distribution, write equations that describe queue pattern change over time and attempts to provide substantial answers to the following questions. How long kth customer arrive at any time x wait to be served? How many customers wait in queue crowd to be served at kth customer arrival time x?

II. Development Of The Model

This work focus on estimating times given the system state at the arrival or any point of time during the waiting time of estimation and study estimations of waiting times for the purpose of informing individuals about their anticipated delays based on trend lines curve fitting equations derived from Cumulative Arrival and service data distribution. To develop a mathematical model in the form that describes the queuing systems, requires some background study on Arrival pattern and distribution, service nature and distribution, service mix, arrival and service volume. The entry of Customers into the system (Customers arrival) and the release of a Customer upon completion (Customers departure/exit) are considered as two main events that cause an instantaneous change in the state of the system. Hence, the types and number of servers, the service order and discipline, and the distribution of service times are variables used to determine the mean server service rate and total number of customers served up to time x, this paper predict Expected queue length and waiting time in queue of jth customer arrived at time x, using a Cumulative Approach phenomenon of waiting in lines modeling and Analytical Technique.

[1] Usually, the inputs of a queuing model are the distribution of an arrival process

[2] The characteristics of the system include the number of servers, the service order and discipline, and the distribution of service times.

Thus, using Cumulative Approach Modeling Technique estimations of waiting times for the purpose of informing individuals about their anticipated delays and basic measures of performance are modeled as follows, assuming exponential service time distributions. Let:

- Na(x) denote total number of customers arrived up to time x
- Ns(x) denote total number of customers served up to time x where time x is server working time
- Nq(x)The expected number of customers waiting in the queue at any arrival time x of kth customer
- Wt.(x)=Expected waiting time in the queue of kth customer arrive at any time x
- Aj(x) denote total number of type j customers arrived up to time x and j (1, 2, …. n)
- T- Expected time to service of kth customer arrive at any time x
- S- number of servers and n- number of service type
- μe(x)- mean effective service rate at time x and μ(x)I- mean server I service rate.

Assuming infinite queue, an arriving customer is immediately entering service if there is an available agent and joins the queue if all agents are busy. Since it is first-in-first-out (FIFO) service protocol, the expected number of customers waiting in the queue at any time x is equal to the expected total number of customers arrived up to time x minus the expected total number of customers served up to time x and Expected waiting time in the queue of the customer arrived at any time x is the difference between the Expected time to service T and arrival time, x, where Expected time to service, T, of the customer can be derived from NA(x) =NS(T).

Since the characteristics of the system include the number of servers, service types and the distribution of service times are different for different FCFS systems, estimation methods for S- number of servers and n-number of service type can be denoted by mean effective service rate assuming exponential service time distributions.

where tj is service time of service type j at server i. i (1,2, …, c) and j (1,2, …, n), mean effective service rate can be:

$$\mu\mathrm{e}(\mathrm{x}) = \sum_{i-1}^{s} \mu(\mathrm{x})i;\ but\ \mu(\mathrm{x})i = [\frac{NA(x)}{\sum_{j=1}^{n} A(x)j * tj}]i$$

$$\mu\boldsymbol{e}(\boldsymbol{x}) = \sum_{\boldsymbol{i-1}}^{s} \left[\frac{\boldsymbol{NA(x)}}{\sum_{j=1}^{n} \boldsymbol{A(x)j * tj}}\right]\boldsymbol{i} \ldots\ldots \qquad \textit{Equation 1 a}$$

Thus, total number of customers served up to time x, Ns(x) is area under μeve (x) curve

$$\boldsymbol{Ns(x)} = \int_{0}^{x} \boldsymbol{e(x)} \ldots\ldots \qquad \textit{Equation 1b}$$

$$\boldsymbol{Ns(x)} = \int_{0}^{x} \sum_{i-1}^{s} \left[\frac{\boldsymbol{NA(x)}}{\sum_{j=1}^{n} \boldsymbol{A(x)j * tj}}\right]\boldsymbol{i} \ldots\ldots \qquad \textit{Equation 1}$$

Note that: where mean effective service rate at time x is constant or μe(x)= μe, total number of customers served up to time x, NS(x)= $\mu e * x$,.

similarly, based on service types and the distribution of service times, total number of customers arrived up to time x of n- number of service type can be denoted by:

$$\mathrm{Na(x)} = \sum_{j=1}^{n} Aj(x),\ \ldots\ldots \qquad \textit{Equation 2}$$

Thus: expected number of customers waiting in the queue at any time x Nq(x) = Na(x) - Ns(x) is:

$$Nq(x) = \sum_{j=1}^{n} Aj(x) - \int_{0}^{x} \sum_{i-1}^{s} \left[\frac{\boldsymbol{NA(x)}}{\sum_{j=1}^{n} \boldsymbol{A(x)j * tj}}\right]\boldsymbol{i} \qquad \textit{Equation 3}$$

Expected time to service and are:

$$\mathrm{T} = \frac{\sum_{j=1}^{n} Aj(x),}{\sum_{i-1}^{s}\left(\sum_{j=1}^{n} \mu\mathrm{j}\right)i},\ \ldots\ldots \qquad \textit{Equation 4}$$

Expected waiting time in the queue of kth customer arrive at any time x of queuing system under study

$$\mathrm{Wt.(x)} = \frac{\sum_{j=1}^{n} Aj(x),}{\sum_{i-1}^{s}\left(\sum_{j=1}^{n} \mu\mathrm{j}\right)i} - \mathrm{x}\ \ldots\ldots \qquad \textit{Equation 5}$$

by Using Cumulative Approach Analytical Technique (CAAT) and,

As illustration, assuming exponential service time distributions, this paper drive difference equations and predict expected waiting time in the queue of kth customer arrive at any time x based on formulated trend lines equations for: the number of servers, service types and the distribution of service times are different for different FCFS systems, model developed to estimate waiting times in classic queuing systems.

Furthermore, based on Cumulative Approach Modeling Technique, using Microsoft excel scatter diagram curve fitting technique, equations estimating required points between the discrete values for every single curve that represents the general trend of the cumulative arrival values along a continuum and served up to time x are determined; and hence, estimations of waiting times for the purpose of informing individuals about their anticipated delays and basic measures of performance are determined and illustrated as follows, assuming exponential service time distributions.

III. Application Of The Model

This section, apply Cumulative Approach phenomenon of waiting in lines modeling and Analytical Technique and predict Expected queue length and waiting time in queue of jth customer arrived at time x and demonstrates the use of estimation methods for FCFS systems with Identical servers and service types, independent servers and Identical service types, Identical servers and multiple service types, and independent servers and multiple service types. As illustration, the characteristics of the Queuing system consist of one station with no customer classes, FIFO service protocols, two servers and two service types with unlimited sizes of waiting room were used to demonstrate the use of different estimation methods using customers' arrival Data collected within one-hour time Intervals shown in table below

Table 1: Arrival data

Arrival Data Collected			
Intervals	A1	A2	A
Before 8:00	9	3	12
8:00-9:00	34	28	74
9:00-10:00	18	27	119
10:00-11:00	13	16	148
11:00-12:00	4	8	160

Based on data shown in table, total number of Customers arrived up to time x and cumulative number of arrival data trend lines below were derived, Where $0 \le x \le 4$, Na(x) is Total number of Customers arrived up to time x, A1(x) and A2(x) are Total number of service type 1 and type 2 Customers arrived up to time x, respectively. Using formulated Trend lines equations representing Arrival data and equations with 0.999 R-squared (R2) value or Square of the correlation coefficient shown in Table and figure below basic measures of performance under three cases can be determined as follow in order.

Table 2: Customers arrived up to time x

Arrival Time		Customers arrived up to time x		
Intervals	x	A1(x)	A2(x)	NA(x)
Before 8:00	0	9	3	12
8:00-9:00	1	43	31	74
9:00-10:00	2	61	58	119
10:00-11:00	3	74	74	148
11:00-12:00	4	78	82	160

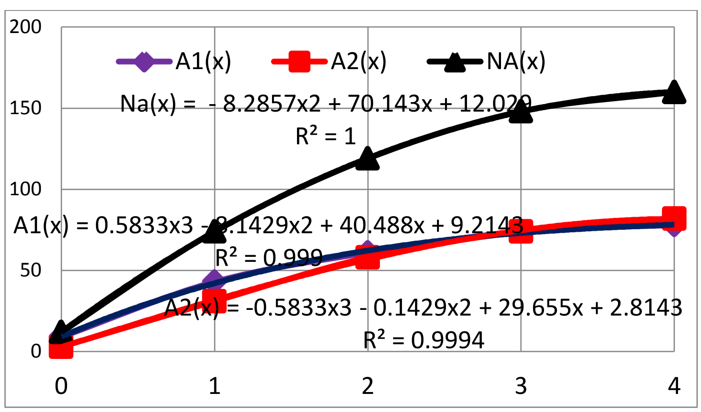


Figure 1: Arrival data trend lines and curve fitting equations

Case I: two identical servers with μ server service rates

The characteristics of the system include two number of identical servers with mean service rate of 23 customers per hour and assuming identical service types with identical distribution of service times,

a. *Arrival:* Since service types service time distribution is identical arrival pattern is over all arrival to system which is Total number of Customers arrived up to time x.

$$\mathbf{Na(x) = -8.2857x^2 + 70.143x + 12.029}$$

b. *Service:* Since the system characterized with two number of identical servers and 23 customers per hour mean server service rate, which is, μ= 23 and S=2. mean effective service rate is μ*S

$$\mathbf{Ns(x)=46x;}\ \text{Where, } \mu*S= \mathbf{\mu eve = 46}$$

c. *Expected waiting time in the queue of kth customer arrive at any time x*

$$\mathbf{Wt(x)} = \frac{\boldsymbol{Na(x)}}{\boldsymbol{\mu ef}} - \boldsymbol{x.}\ \ \boldsymbol{thus};\ \mathbf{Wt(x)}$$
$$= -\mathbf{0.18x2} + \mathbf{0.525x} + \mathbf{0.2615}$$

d. *The expected number of customers waiting in the queue at any arrival time x of kth customer*

$$\mathbf{Nq(x)} = \mu \boldsymbol{ef} * \boldsymbol{Wt(x).thus;}$$
$$\mathbf{Nq(x)} = -8.2857x^2 + 24.15x + 12.029$$

Case II: Two independent servers with μ1 and μ2 service rates

The characteristics of the Queuing system consist of one station with no customer classes, FIFO service protocols, two number of independent servers with mean server service rate of 23 and 22 customers per hour, respectively and unlimited sizes of waiting room are modeled

a. *Arrival:* Since service types service time distribution is identical arrival pattern is over all arrival to system which is Total number of Customers arrived up to time x.

$$\mathbf{Na(x) = -8.2857x^2 + 70.143x + 12.029}$$

b. *Service:* Since the system characterized with two number of independent servers with mean server service rate of 23 and 22 customers per hour, which is, μ1= 23, μ2= 22 and S=2. mean effective service rate is μeve is:

$$\mathbf{\mu eve} = \sum_{i-1}^{s} \mu i = \mathbf{23 + 22 = 45. \ Thus,\ Ns(x)=45x}$$

c. *Expected waiting time in the queue of kth customer arrive at any time x*

$$\mathbf{Wt(x)} = \frac{\boldsymbol{Na(x)}}{\boldsymbol{\mu ef}} - \boldsymbol{x}. \ \ \boldsymbol{thus}; \mathbf{Wt(x) = -0.184x^2 + 0.5587x + 0.2673}$$

d. *The expected number of customers waiting in the queue at any arrival time x of kth customer*

$$\mathbf{Nq(x)} = \mu \boldsymbol{ef} * \boldsymbol{Wt(x).thus;}\ \boldsymbol{Nq(x) = -8.2857x^2 + 25.143x + 12.029.}$$

Case III: Two Identical servers and Two service types

The characteristics of the Queuing system consist of one stations with no customer classes, FIFO service protocols, two number of identical servers, two service type with mean service time of 2.5 and 2.857 minute per customer, and unlimited sizes of waiting room

a. *Arrival:* Since service types service time distribution is not identical arrival pattern is each service types arrival to system which are Total number of service type I and II Customers arrived up to time x.

$$\mathbf{A1(x) = 0.5833x3 - 8.1429x2 + 40.488x + 9.2143}$$
$$\mathbf{A2(x) = -0.5833x3 - 0.1429x2 + 29.655x + 2.8143}$$
$$\mathbf{Na(x) = -8.2857x^2 + 70.143x + 12.029}$$

b. *Service:* Since the system characterized with two number of identical servers and two service type with mean service time of 2.5 and 2.857 minute per customer, mean server service rate, is 1/mean service time

mean SERVER service rate μ is: μ(x)

$$\mu(x) = \frac{\boldsymbol{Na(x)}}{\boldsymbol{A1(x) * T1 + A2(x) * T2}}$$

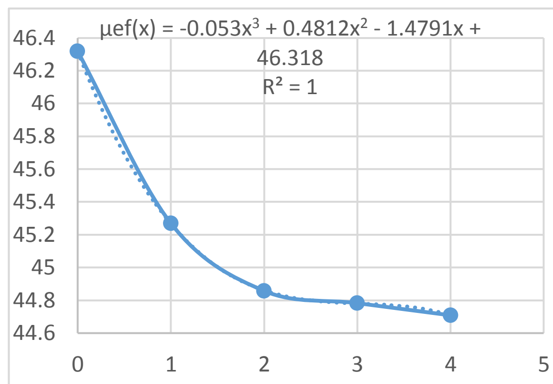
where T1=2.5 (0.0417 hr./cuts.) and T2=2.857 (0.047617 hr./customer). Thus;

$$\mu(x) = -0.0265x3 + 0.2406x2 - 0.7396x + 23.159$$

thus, mean effective service rate is μeve is: **μeve(x)** = μ(x)

$$\text{*S=}\ \ 2(\boldsymbol{-0.0265x3 + 0.2406x2 - 0.7396x + 23.159)}$$
$$\boldsymbol{\mu eve\ (x) = -0.053x3 + 0.4812x2 - 1.4792x + 46.318}$$

since area under $\boldsymbol{\mu eve\ (x)}$ ***curve is*** **Ns(x)**



$$\boldsymbol{Ns(x)} = \int_{0}^{x} \boldsymbol{-0.053x3 + 0.4812x2 - 1.4792x + 46.318}$$

$$\textbf{Thus, Ns(x)} = \boldsymbol{-0.01325x4 + 0.1604x3 - 0.7396x2 + 46.318x}$$

c. *The expected number of customers waiting in the queue at any arrival time x of kth customer*

$$\mathbf{Nq(x) = Na(x) - Ns(x)}$$
$$\mathbf{Nq(x)} = \boldsymbol{0.01325x4 - 0.1604x3 - 7.5461x2 + 23.825x + 12.029}$$

d. *The expected number of customers waiting in the queue at any arrival time x of kth customer*

$$\mathbf{Wt(x)} = \frac{\boldsymbol{Nq(x)}}{\boldsymbol{\mu ef}}.$$

$$\mathrm{Wt(x)} = \frac{\boldsymbol{0.01325x4 - 0.1604x3 - 7.5461x2 + 23.825x + 12.029}}{\boldsymbol{-0.053x3 + 0.4812x2 - 1.4792x + 46.318}}$$

$$\boldsymbol{thus;}\ \mathbf{W(x)III = -0.0017x^3 - 0.1724x^2 + 0.5377x + 0.2594} = 0.0008x4 - 0.0077x3 - 0.1576x2 + 0.5269x + 0.2597$$

In general, The basic idea is to fit a curve or a series of curves that pass directly through each points of discrete Total number of customers arrived and/or served up to any time x Data. Using Microsoft excel sheet, a function that approximately fit parameters of system of interest with more than 0.997 Square of the correlation coefficient and The rate of change in these values with respect to time x can be denoted by fitting a curve along the discrete data points. Thus, based on discrete Data along a continuum on Total number of customers arrived and/or served up to any time x, Estimation of required points between these discrete values is possible for every single curve that represents the general trend of the data from trend lines equations derived. Using these basic setup, this paper makes it possible to model a function that approximately fit parameters of system of interest, estimate Expected waiting time in queue of kth customer arrive at any time x and simulate the performance of a system on which analytical result of interest can be easily computed.

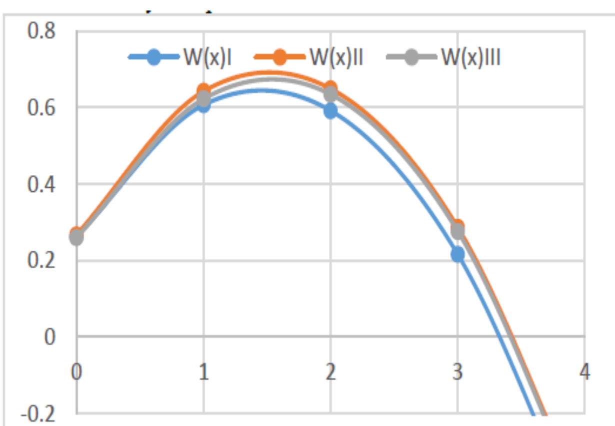


Figure 2: Expected waiting time in the queue of kth customer arrive at any time x

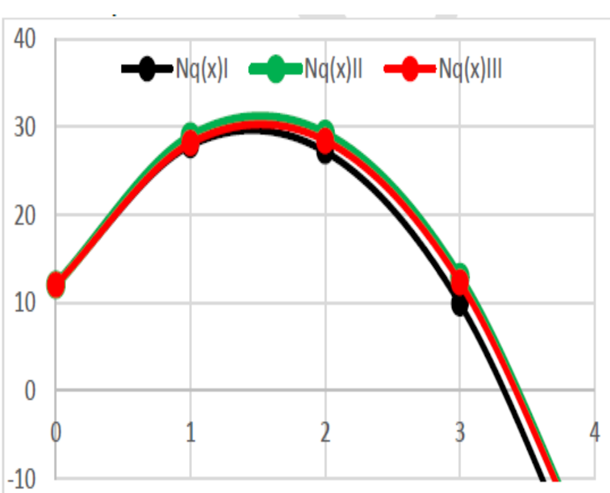


Figure 3: The expected number of customers waiting in the queue at any arrival time x of kth customer

In general, based on developed Modeling Technique, this paper shows how this model can be used to predicts mean Expected queue length, and waiting time in queue of kth customer based on the system state at the time of estimation and manage crowd in queue by integrating the movement of the Service into the actual operation of the resource performing the work.

As shown in application, This model suit to drive difference equations for The characteristics of the Queuing system consist of one stations with no customer classes, FIFO service protocols given with Identical servers and service types, S number of independent servers and Identical service types, S number of Identical servers and multiple service types, and S number of independent servers and multiple service types and predict expected kth customer waiting time in FCFS case queue systems at the time of estimation. The findings show that, using Cumulative arrival and service parameters up to stationary time that has been in operation, the expected queue length and waiting time in queue of kth customer estimation methods for different characteristics of the FCFS system with S- number of servers, n- number of service type and different distribution of service times at the time of estimation can be denoted. Thus, where, estimation methods of expected waiting time in the queue of kth customer arrives at any time x, Wt(x) and the expected number of customers waiting in the queue at any arrival time x of kth customer, (x); The General estimation methods for the system characterized with s number of servers and n service types under different schemes can be denoted as shown below. For:

a. *The system characterized with s number of identical servers is:*

$$\mathrm{Wt(x)} = \frac{Na(x)}{\mu * S} - x$$
$$\mathrm{Nq(x)} = \mu * S * Wt(x)$$

b. *The system characterized with s number of independent servers is:*

$$\mathrm{Nq(x)} = \sum_{i-1}^{s} \mu \boldsymbol{i} * Wt(x)$$
$$\mathrm{Wt(x)} = \frac{Na(x)}{\sum_{i-1}^{s} \mu \boldsymbol{i}} - x$$

c. *The system characterized with s number of identical servers and n service types is:*

$$\mathrm{Nq(x)} = \sum_{i-1}^{s} \mu \boldsymbol{i}(\boldsymbol{x}) * Wt(x)$$
$$\mathrm{Wt(x)} = \frac{Na(x)}{\sum_{i-1}^{s} \mu \boldsymbol{i}\,(\boldsymbol{x})} - x$$

$$where, \mu i(x) = \frac{\boldsymbol{Na(x)}}{\sum_{j=1}^{n} \boldsymbol{Aj(x) * Tj}}$$

d. *S no. of Independent servers and n service types*

$$Wt.(x) = \frac{\sum_{j=1}^{n} Aj(x),}{\sum_{i-1}^{s}\left(\sum_{j=1}^{n} \mu j\right)i} - x$$

$$Nq(x) = \sum_{i-1}^{s}\left(\sum_{j=1}^{n} \mu j\right)i * \mathrm{Wt(x)}$$

IV. Result and Discussion

Using ***Cumulative Approach Modeling Technique developed,*** this paper ***make it*** possible to write equations that describe how the number of customers in each queue in the system of interest changes over time for a First-Come-First-Served service discipline and facilities, which experience time-varying customer arrival patterns and ***predicts mean*** Expected queue length, and waiting time in queue for the purpose of informing individuals about their anticipated delays based on estimating times given the system state at the time of estimation. Thus, based on Information about anticipated waiting time, organization can shorten the perceived waiting time reduces the uncertainty and increases customer satisfaction.

Moreover, This Analytical technique show every fluctuation and pattern of queue characteristics of the system changes over time and forecast the pattern of waiting time. It shows how time customers arrive determines the time customers wait in queue lines and analysis the relationship between Customer arrival time and average times the customer spent in the queue. The result has also revealed correlation between Customers' waiting times and the number of Customers waiting; a positive for Customers arrives before number in queue reach its maximum and negative for Customers arrives after as shown in figure xx above. In this instance, for each unit of time that the server is available, the average time in queue increases as number of Customers in the queues increases and decrease as number of Customers in the queues decreases with the same rate. Briefly, when total number of Customers arrived per unit time is greater than total number of Customers served per unit time queues continue to grow over time. When total number of Customers arrived up to time t is greater than total number of Customers served up to time t and total number of Customers served per unit time interval t is greater than arrived, queues continue to decelerate over time interval. When Total numbers of Customers arrived and served are equal, expected number of customers in queue and time in queue of the customer arrives after time t is zero. In addition, when total number of Customers arrived up to time is less than total number of Customers served up to time, crowd in queue is zero continuously over time. The customer arrives at time t when number of Customers in the queue is Maximum, expect maximum waiting time in queue and expected waiting time in queue is zero for the customer arrives exactly after time t at which number of Customers in the queue is zero. Based on waiting information provided, manager can recommend the best moment at which customer arrives and get service without waiting for long time in queue line.

Furthermore, result showed that, this model suit to obtain closed-form or recursive formulae that measures performance of queuing systems over change of time which, allow system designers to calculate performance metrics that describes the phenomenon of waiting in lines such as average queue length, average waiting time, and the proportion of customers turned away. this paper looks at arrival and service distribution and pattern change over time write equations that calculate operational attributes of the service level: service times, waiting times, number of people in the system, percentage of abandoning customers and more and describe queue and queue crowd changes over time. developed analytical technique queueing models can be used to obtain the analytical result of performance of system such as: the time Customers in queue service time and time at which no Customers in queue.

As shown in figures, hence servers are capable of serving all arriving Customers, queue occurrence not due to server capacity, Queues form when customers arrive at a service facility at time they cannot be served immediately upon arrival. Thus, increasing number of server further increase time at which no Customers in queue, which means server idleness increased. By specifying reasonable limits on conflicting measures of performance such as average time in the queue and idleness percentage of the servers, anyone can determine an acceptable range of the service level through effective arrival management system. To manage arrival pattern, the arrival rate should be decreased during busy times and increased during "slow" periods by providing Different types of waiting information to customers. The decision of what quantile of the waiting time distribution queue-size, waiting time of the longest-waiting customers or the anticipated waiting time of an individual customer to inform, depends on the desired outcome. The service system manager should then decide what is the exact information that will be provided to customers. informing individuals about their anticipated delays based on estimating times given the system state at the time of estimation right upon arrival, Customers can decide if or whether they are willing to wait. As less customers decide to abandon after already waiting for a while, the steady-state number of customers in queue decreases and so does the percentage of customers who find the system full.

V. Conclusion

This paper developed Cumulative Approach phenomenon of waiting in line Modeling Technique and predict mean expected queue length and waiting time in queue of kth customer of a First-Come-First-Served service discipline queueing systems at time for the purpose of informing individuals about their anticipated delays based on estimating times given the system state at the time of estimation. Cumulative Approach Analytical Technique (CAAT) is feasible to model the phenomenon of waiting in lines using representative measures of performance and predict mean expected queue length and waiting time in queue of kth customer arrive at any working time x. Using this model, analytical result of the performance of a system with time-decisive parameters that has been in operation for a sufficiently long time such that time t no longer affects the distributions of number in system, number in different queues, waiting times, and total delay are possible. the Cumulative Approach Modeling Technique is useful to simulate a queuing system's performance, shows how time customers arrive determines the time customers wait in queue lines crowd and analysis the relationship between Customer arrival time and average times the customer spent in the queues and queue crowd. On the other hand, it helps us to identify source of queue crowd at any time and easily specify reasonable limits on conflicting measures of performance such as average time in the queue and idleness percentage of the servers and indicate how and time at which improvement in system change the queue performance indicators and at what time the queue performance indicators changed very little. Moreover, this model is flexible. While simple linear models were used in this application, no difficulty is foreseen in adapting the model for nonlinearities in either Customer demands or service costs. In addition, the inherent flexibility of the model would permit it to adapt easily to sub models of Customer admission rates in the various medical categories. Finally, the author concludes that, the application of Cumulative Approach Modeling Technique can easily predict mean expected queue length and waiting time in queue of kth customer arrive at any working time x and offer better queue performance analysis result.

Acknowledgements

The author would like to thank Kamil Kadir, Kadir Mamo, Muhammad Kawo, Lenco Samuel and Lalise Woliyi for their helpful comments and suggestions on the draft version of this paper. Responsibility for the contents of the paper, however, rests entirely with the author.

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