

An Approach to Determine the Initial Shear Modulus of Clean Sandy Soils

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Abstract

This study presents new empirical equations to estimate the initial shear modulus of clean sands under low strains by using soft computing methods. A series of resonant column tests were conducted on clean sand specimens. The effect of various factors, such as effective stress, saturation degree, void ratio and shear strain levels, were considered by using fuzzy expert systems, neural networks and regression analysis. A new empirical equation was developed to determine the initial shear modulus of clean sand samples and compared with the existing empirical relationships in the literature. Success of the new equation was increased by considering boundary conditions at the shear strain ranges given by the ASTM standards for the resonant column test. It was found that the new formulation has a high level accuracy for determining the initial shear modulus of Toyoura sand samples. And also it can be used for clean sandy soils having similar properties. It was suggested that new approaches can be developed by using soft computing techniques to identify the dynamic shear modulus of sandy soils.

Index terms— toyoura sand; resonant column test; shear modulus; dynamic loading, empirical equation.

1 Introduction

he maximum shear modulus, G_{max} ($< 5 \times 10^{-4}$ %), provides information about the soil strength and rigidity in cyclic loading. Shear modulus of various soils at very small levels of strains can be called maximum shear modulus, initial shear modulus, or low amplitude shear modulus, and denoted by G_{max} or G_0 (Ishihara, 2003). Shear modulus is generally determined by field tests or by laboratory tests, such as resonant column (RC), dynamic triaxial and bender element methods (Youn et al. 2008). The shear modulus can be measured at strain levels between 10^{-4} and 10^{-1} by performing resonant column test.

There are many equation and various experimental relationships proposed for the determination of the initial shear modulus of sandy soils in the literature. The most popular empirical equation based on laboratory experiments is presented by Seed and Drnevich (1972a-b). Although these tests provide adequate results, preparing high quality undisturbed samples and simulating field conditions are the main problems in the laboratory testing. Lun and Goktepe (2006) emphasized the deficiencies of the laboratory test on the cyclic response of soils, such as reconstitution of non-cohesive samples and the unidentified geological stress history affect. Cyclic testing to simulate the real cyclic response is also expensive and requires much time. Alternatively, the dynamic properties of soils can be determined by using soft computing techniques, such as artificial intelligence based on field or laboratory data (Hasal and Iyisan, 2014). However, artificial intelligence (AI) cannot fully simulate the complex response of the systems; the use of AI technologies, such as fuzzy logic, neural networks, evolutionary computations, and expert systems, provides partial simulations. In addition, using these methods provides effective feasibility analysis, early decisions and so on (Akbulut et. al. 2004).

Engineers generally prefer AI applications due to the creation of non-linear mappings between the input and output variables in optimum time and cost. As a result, many researchers began to use AI applications to evaluate

44 dynamic soil parameters. ??abalar ??010) generalized formulations to simulate the strain-stress curves and the
45 modeled dynamic stress-strain behavior of sands by using AI-based genetic programming. It can be concluded
46 from many published studies that AI models well describe the dynamic characteristics of soils.

47 Many previous study showed that the dynamic behavior of sandy soils are affected by various factors, such as
48 water content, void ratio (relative density), confining stress, particle shape and soil fabric (Salgado et al. 2000).
49 In this study, to determine the dynamic behavior using AI-based genetic applications, a series of RC tests were
50 conducted on reconstituted sand samples. Clean sand samples were prepared under fully saturated, partially
51 saturated and dry conditions. The tests were conducted under undrained and stresscontrolled conditions. The
52 input variable data (void ratio, effective stress, shear strain, and saturation degree) were obtained from the
53 tests and used by soft computing techniques to evaluate the output parameters (resonant frequency and shear
54 modulus). A new empirical equation was developed and compared with the existing equations in the literature
55 according to its accuracy level.

56 2 II.

57 3 Experimental Study

58 A resonant column (RC) test device was used to determine the cyclic response of the reconstituted sand samples.
59 The resonant column method is used to determine the dynamic properties of soils, concrete, and rocks with respect
60 to the theory of wave propagation. The details of the resonant column tests are explained by Drnevich(1985)
61 and in the Standard of ASTM D 4015-87 ??2000).

62 The RC test configuration used in this study is a fixed-free system, in which the sample is fixed at the bottom
63 and is free to rotate at the top. The wave velocity and the degree of material damping can be determined by
64 measuring the motion of the free end. Then, the shear modulus is derived from the velocity and the density of
65 the sample. The bottom of the specimen is fixed to the base of the apparatus. Sinusoidal torsional excitation
66 is applied to the top of the specimen by an electric motor system. A torsional harmonic load with a constant
67 amplitude is applied over a range of frequencies, and the response curve (strain amplitude) is calculated. The
68 output angular acceleration at the top of the sample is recorded by an accelerometer. The frequency of the
69 cyclic torque is gradually changed until the first resonance of torsional vibration is obtained. The shear wave
70 velocity is obtained from the first-mode resonant frequency. The initial shear modulus for shear strain ranging
71 from 0.001-0.009% to 0.01-0.023% is then calculated using the shear wave velocity and the density of the sample.

72 Standard uniform Toyoura sand was chosen for the study to allow for easy comparison with the literature.
73 Index and shear strength parameters of Toyoura sand is taken from different studies in the literature. Some
74 basic characteristics are given in Table 1. The test specimens were solid cylindrical samples with an approximate
75 diameter of 50 mm and a height of 130-135 mm. The initial relative density and saturation degree are the most
76 important factors regarding the cyclic behavior of sandy soils. Therefore, the test samples were prepared at
77 different initial void ratios and saturation degrees. Two methods of sample preparation, dry deposition (for dry
78 samples) and moist placement (for partially saturated and fully saturated samples), were preferred because of
79 time consuming by these methods. The experimental details of the study are shown in Table 2.

80 4 Model Study

81 Many real-world problems cannot be solved by using conventional approaches because of an inadequate amount
82 of time. Therefore, various soft computing techniques using predictive modeling are preferred for such problems.
83 In particular, fuzzy logic and neural networks are the most popular and widely used techniques because of their
84 benefits in modeling.

85 5 a) Generation of the fuzzy expert system (FES)

86 Fuzzy modeling offers control mechanisms for problems that have uncertainties and building solution steps. A
87 multi-level decision-making mechanism provides an expert system with a knowledge base, an inference mechanism,
88 and a user interface, although with a varying number of elements (Zadeh, 1994, Jang et. al. 1997). In this study,
89 Math Works MATLAB software was used for fuzzy membership functions and rules. The structure of the fuzzy
90 expert system was created by using the MATLAB Fuzzy Logic Toolbox for all test results. In this study, the input
91 variables were void ratio, effective stress, shear strain amplitude, saturation degree, and the output variables were
92 resonant frequency and maximum shear modulus. The next step was definition of the fuzzy rules to perform fuzzy
93 reasoning. The fuzzy rules were constructed on an "ifthen" structure, in which they provided the conditional
94 statements that comprised the fuzzy logic. The "ifthen"statements are defined as follows: R: IF value $x=A_i$ and
95 $y=B_i$ and $z=C_i$ THEN $n = D_i$ ($i = 1,2,\dots,k$)

96 where x , y , and z are the input variables, and n is the output parameter described by fuzzy subsets. Two
97 hundred and ten rules were written for the shear modulus and the resonant frequency. The abbreviations of
98 the membership functions are given in Table 3. The output parameters related with each input variable were
99 evaluated by using the test results. The percentages of the weighted output parameters were calculated and
100 defined in the form of "if-then" statements. A total of 516 rules for the maximum shear modulus, 492 rules for
101 the resonant frequency were defined with this way.

The rules were defined according to the MATLAB Fuzzy Logic Toolbox to construct the FES variables. Two built-in AND methods, min (minimum) and prod (product), and two built-in OR methods, max (maximum) and probor (probabilistic OR method), were used to evaluate the values of the resonant frequency and the maximum shear modulus. The best recall performances of the FES indicate that the system has an acceptable performance. The determination value (R²) for the resonance frequency and the maximum shear modulus are shown in Fig. 2(a) and Fig. 2(b), respectively.

6 b) Generation of the Artificial Neural Networks (ANNs)

An artificial neural network is a soft computing technique that provides information processing by using the simulation of nerve cells and networks (Fahman, 1988). In this study, a supervised learning network using feed forward back propagation was performed. Alyuda NeuroIntelligence (ANI) software was used to design the structure of the neural network, and the network was compared with the other structures created by MATLAB. Various results were observed by selecting different layers and neurons. The dataset was formed by randomly separation method into training and validation. The Levenberg-Marquardt (LM) algorithm was used to train all the networks. LM was used because the algorithm is known as the optimum training algorithm and gives a virtual standard in nonlinear optimization. LM is a pseudo-second-order method, i.e., it does not only work with function evaluations and gradient information but also it estimates the Hessian matrix using the sum of the outer products of the gradients. LM quickly minimizes the error function and uses the Jacobian matrix instead of the Hessian matrix. The parameter, λ , is the Marquardt parameter, used in the calculation of the Hessian matrix (Hagan and Menhaj 1994). $H(n) = J^T(n)J(n) + \lambda I(n)$

The weights and bias values are updated as follows: $w(n+1) = w(n) - \lambda^{-1} J^T(n)e(n)$

The data for the input and output parameters are given in Table 4. An exhaustive search option in the ANI was used to choose the input variables. The optimal NN architecture was found as 3-15-1 NN architecture for resonant frequency and 3-18-1 NN architecture for shear modulus. The NN architectures created by using Matlab NN are shown in Fig. 2. The same input variables were used and the linearity of the relationship between the parameters for the single-layer structure was found to be acceptable. The coefficients of determination value (R²) (Y=T) are 0.9785 and 0.9787 for the ANI predictions of resonance frequency and maximum shear modulus, respectively, which implies a significant value for R² and hence a good performance for the whole model. The results are presented in Fig. ??.

7 Resonant Frequency ANI Results

8 c) Generation of the Empirical Equation

The shear strain level is an important factor on the shear modulus. It is already known that the resonant column experiments are capable by achieving shear strain within an amplitude range of 10⁻⁶ - 10⁻³ (Ishihara 2003). In any type of laboratory test, the shear modulus of cohesionless soils at low strain is measured under different effective confining stresses (σ'_v) for various conditions presented by different void ratios (e). In the early works by Terzaghi and Richart (1972a-b), the effect of void ratio is found to be expressed by a function F(e): Thus, it is appropriate to divide the measured shear modulus (G_{max}) by the function F(e) and plot this ratio against the effective confining stress applied in the test (Kokusho 1987). The amplitude of the shear strain is obtained by converting the axial strain in the triaxial test through the following relationship: $F(e) = a(1+e)^{-n}$

A number of similar formulas are proposed for various sand types, as shown in Table 5; however, most of these formulas can be expressed in the general form of Eq. (5). (Kokusho 1987) For a sufficient small shear strain of $\gamma = 10^{-5}$, a typical formula is specified as Eq. (6): $G_0 = AF(e) \sigma'_v$

$$G_0 = 8400$$

$$(2,17e)^{2+e} (1+e)^{-0.5} \quad (7)$$

In this study, the experimental results were compared with the existing empirical relationships for the initial shear modulus obtained by performing various test devices for several types of sands at the shear strain of 10⁻⁵. All the results (0.001 < R² < 0.04) of the empirical relationships were recomputed using the void ratio and the effective stress from the test results. The empirical relationship for the shear modulus given by Kokusho (1987) was found more appropriate for the values obtained from the test results in the range of the shear strain amplitude. The comparison between experimental results and literature is shown in Fig. ?. The experimental test results were found to be in excellent agreement with the literature at a small shear strain amplitude. R² = 0.9111 (x = y) 0.001 < R² < 0.04 R² = 0.9742 (x = y) 0.001 < R² < 0.005 R² = 0.7452 (x = y) 0.005 < R² < 0.04 Fig. ?: Suitability of the experimental results and the Kokusho equation A new empirical relationship for the initial shear modulus was derived from the experimental studies at different shear strain levels by considering test data, the fuzzy expert system and the neural network results. Dynamic responses of soils can be determined at strain levels between 10⁻⁶ and 10⁻² by different test methods. But, decrease in shear modulus is observed from 0.005 strain levels to 0.0001 strain levels in the previous studies. Accuracy of the new equation was increased by considering boundary conditions at the shear strain ranges given by the ASTM standards for the resonant column test. Therefore, a new relationship to determine the initial shear modulus was divided into two parts given by Eq. (8) and Eq. (9).

$$\text{In the range of } 0.001 < R^2 < 0.005, \text{ the equation is given below, } G_0 = 8254 (2,17e)^{2+e} (1+e)^{-0.5} \quad (8)$$

$$\text{In the range of } 0.005 < R^2 < 0.04, \text{ the equation is given below, } G_0 = 7294 (2,17e)^{2+e} (1+e)^{-0.5} \quad (9)$$

9 CONCLUSIONS

162 The coefficients of determination value (R^2) ($Y=T$) are 0.9767 and 0.9362, respectively. Using these two
163 equations for the prediction of the initial shear modulus, which implies a significant value for R^2 and indicates
164 good performance for the whole model. The suitability of the derived empirical relationship according to the
165 variation of the shear strain amplitude is shown in Fig. ??.

9 Conclusions

167 A series of RC tests was conducted on clean sand samples to create a new approach by using AI techniques for
168 the prediction of the dynamic characteristics of soils. MATLAB software was used to perform data analysis and
169 modeling. Test results were analyzed by considering saturation degree, effective stress and cyclic strain. First,
170 two inference systems were performed to predict the maximum shear modulus and the database was created
171 by using the experimental study. Compared with the test results, both inference system models were found to
172 be quite suitable. Subsequently, new empirical relationships in the prediction of the dynamic characteristics of
173 Toyoura sands were derived. New equation is divided in two groups by considering boundary conditions at the
174 shear strain ranges given by the ASTM standards. Therefore, new formulation has a high level accuracy for
175 determining the initial shear modulus of Toyoura sand samples. The equations showed acceptable results that
176 were the analogous to those of the soft computing techniques. These results revealed that both methods can be
177 used for practical purposes to solve complex real life problems. This study encourages further work to explore
178 other inference systems for the estimation and generation of data from experimental studies.

V. ¹

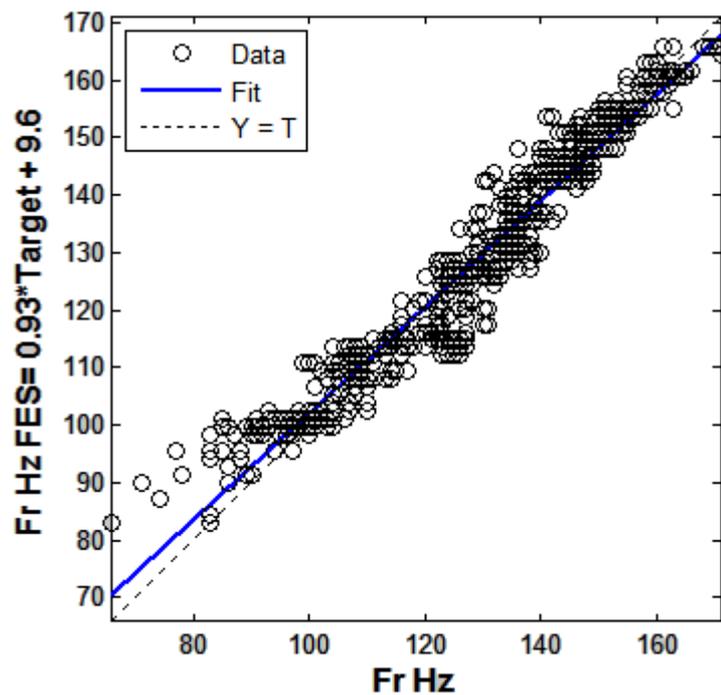
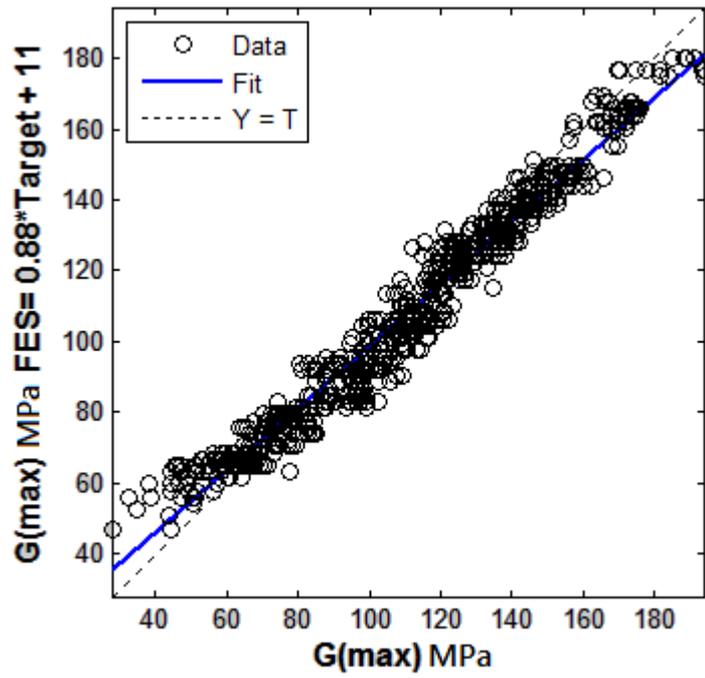


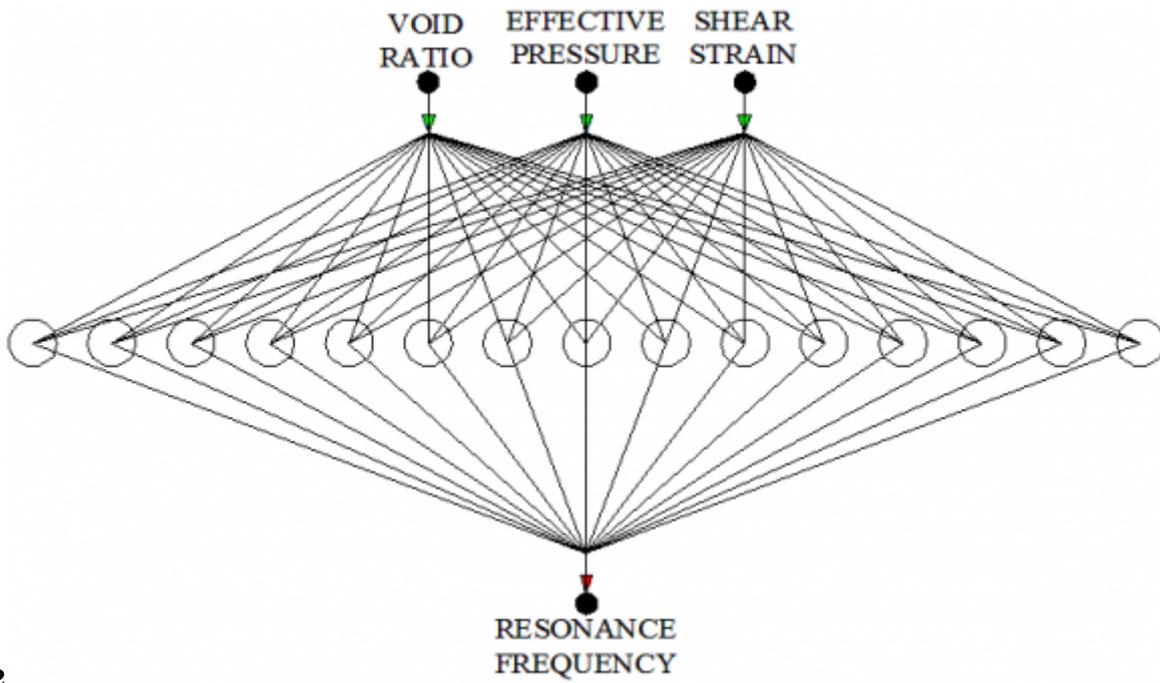
Figure 1:

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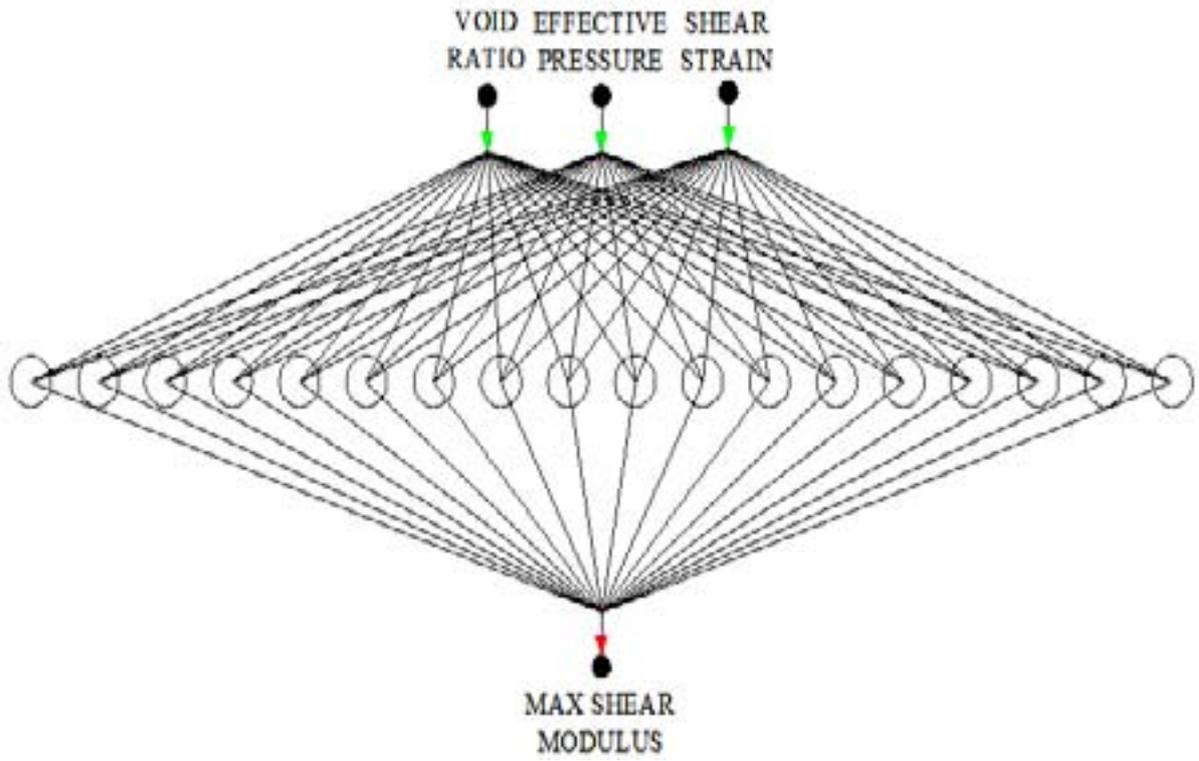
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Figure 2: Fig. 1 :



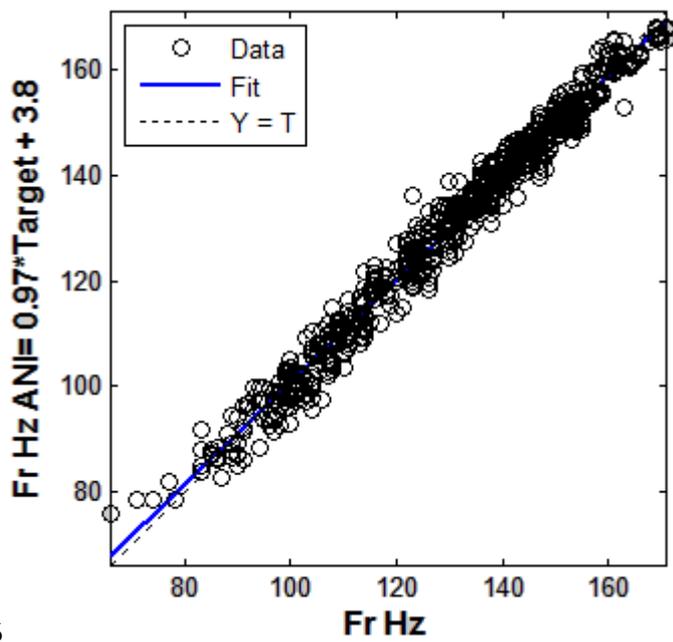
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Figure 3: Fig. 2 :



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Figure 4: R 2 = Fig. 3 :



25

Figure 5: R 2 = Fig. 5 :

1

Material	Toyoura Sand
D 50 (mm)	0.26
D 10 (mm)	0.21
C _u	1.33
C _c	0.98
G _s	2.653
e maks (Mg/m ³)	1.34
e min (Mg/m ³)	1.64
e maks	0.97
e min	0.597
c (kPa)	4
?	39?
D 10 =	

Figure 6: Table 1 :

2

Figure 7: Table 2 :

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Max. (e int)	?'(kPa)	%Dr	H	R	%S	%?
0.7959	348.8	47.66	sample (mm) 135	sample (mm) 50	99.98	0.04
Min. 06166	25.8	94.85	130	50	0	0.001

[Note: © 2017 Global Journals Inc. (US)]

Figure 8:

3

Linguistic Rule	Abbreviations
Extremely High	VR7, EP6, G8, V6, FR7
Very High	VR6, EP5, SS5, G7, V5, FR6
High	VR5, EP4, SS4, G6, V4, FR5
Medium High	G5
Medium	VR4, SS3, FR4
Medium Low	G4
Low	VR3, EP3, SS2, G3, V3, FR3
Very Low	VR2, EP2, SS1, G2, V2, FR2
Extremely Low	VR1, EP1, G1, V1, FR1

Figure 9: Table 3 :

4

Total Sample:711	Sample Input	Input	σ' (kPa)	σ (%)	Str (%)	Fr	Gmax (MPa)	Vs (m/sn)
Column Type	Input	Input	Input	Input	Input	Output	Output	Output
Format	Numerical	Numerical	Numerical	Numerical	Numerical	Numerical	Numerical	Numerical
Scaling Range	[-1?1]	[-1?1]	[-1?1]	[-1?1]	[-1?1]	[0?1]	[0?1]	[0?1]
Min	0.6166	25.8	0.001	0	66	27.97	120.34	
Max	0.7959	348.8	0.04	99.98	171	193.92	343.83	
Mean	0.733718	150.83884	0.005447	60.76166	130.734177	112.976414	248.98495	
Std. Deviation	0.046822	75.95824	0.006556	46.54837	21.815257	35.981497	45.01604	

[Note: Architecture Fitness R 2 = 0.99[3-18-1]]

Figure 10: Table 4 :

5

References	A	F(e)	n	Soil Material	Test Method
Hardin-Richart (1963)	7000 3300	(2,17 ? e) 2 (1 + e) (2,97 ? e) 2 (1 + e) ?	0,5 0,5	Round Grained Crashed Quartz Ottawa Sand Angular Grained	Resonant Column
Shibata-Soelarno (1975)	42000	(0,67 ? e) (1 + e) ?	0,5	Three types of clean sand	Ultrasonic Pulse
Iwasaki et.al. (1978)	9000	(2,17 ? e) 2 (1 + e) ?	0,38	Eleven types of clean sand	Resonant Column
Kokusho (1980)	8400	(2,17 ? e) 2 (1 + e) ?	0,5	Toyoura Sand	Cyclic axial
Yu-Richart (1984)	7000	(2,17 ? e) 2 (1 + e) ?	0,5	Three types of clean sand	Resonant Column

σ :kPa; δ :kPa; e: void ratio

Figure 11: Table 5 :

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