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Modeling Hospital Triage Queuing System

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Modeling Hospital Triage Queuing System

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Keywords: queuing theory, waiting time, healthcare, triage, analytical technique.

I. Introduction

Queuing Theory tries to answer questions like e.g. the mean waiting time in the queue, the mean system response time (waiting time in the queue plus service times), mean utilization of the service facility, distribution of the number of customers in the queue, distribution of the number of customers in the system and so forth. Even though, queuing systems in healthcare operations are complex since patient flows through various units of a particular hospital (Gupta 2013), this paper aims to model Queuing systems which attempts to provide answers to the following questions. Why do queues form? How long customers wait to be served? How many customers wait in crowd to be served at any time t? What trade-offs must be considered by a service system architect when choosing system parameters?

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Customers go into a hospital to get service increasingly equate service quality with rapid service. However, Long waiting list or waiting time in public organization is a notorious problem in most of the countries all over the world. Particularly in healthcare delivery systems waiting in queue crowd lines are ubiquitous (Lade, et al. 2015). Waiting in a crowd queue is not usually interesting; especially waiting for non-value adding activity is undesirable. Because, delay in receiving needed services can cause prolonged discomfort and economic loss when patients are unable to work and possible worsening of their medical conditions that can increase subsequent treatment costs and poor health outcomes. In extreme cases, long queues can delay diagnosis and/or treatment to the extent that death occurs while a patient waits (S.Olorunsola, R.Adeleke and T. 47-53). By awaking this, more and more scholars and companies are focusing on queuing analysis. More recently, Health policy investigators have also sought to apply queuing analysis techniques more widely across entire healthcare systems even though models lack real-world validation (S.Olorunsola, R.Adeleke and T. 47-53). This paper aims to model phenomenon of waiting in lines using average queue length, and average waiting time in queue and analyze their implications on queue crowd management. Moreover,

All patients arriving Hospital with all case like referral, personally, emergency or scheduled appointments by OPD are received by Hospital triage. At triage patients will be welcomed (received), registered, pay registration fee, receive approval of free of charge (credit), screened receive personal cards and will be sent to the outpatient case team. Any emergency cases found here are directly sent to emergency case team without delay. Also, serves to identify priorities for patient care in emergency departments and most surge situations in which resources are rarely limited. Even though, registering and opening patient document/card and assigning them to the right physicians and keeping their documents in appropriate way and place is necessary activities in hospital, it is non-value adding activity. Hence, eliminating or at least reducing waiting time in Triage is important components of quality improvement. Furthermore, the accuracy of resulting expressions of the performance metrics at point of hospital entry or Hospital Triage is mandatory for hospital clinics queuing systems analysis. Thus, By awaking this, this paper intended to make it possible to write equations that describe how the number of

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customers in case hospital Triage queue system changes over time; using mathematical modeling Approach.

Queueing theory is the mathematical study of waiting line models. A mathematical model usually describes a system by a set of variables and a set of equations that establish relationships between the variables. Usually, the inputs of a queueing model are the distribution of an arrival process and characteristics of the system under study. The characteristics of the system include the number of servers, the service order and discipline, and the distribution of service times. The output of a queueing model is a description of the performance attributes of the system under a specific policy. The solution of a queueing model determines, for example, the fraction of time that each server is idle, the expected waiting time of customers, the expected number of customers waiting in the queue, and the number of servers necessary to ensure some level of performance for the system (Gross and Harris, 1985). In this section mathematical model modeling Technique of the queuing system will be discussed.

A mathematical model is an abstract model that uses mathematical language to describe the behavior of a system. Mathematical models are tractable when closed-form or recursive formulae can be obtained, and in such cases the resulting expressions for the performance metrics are referred to as analytical results (Gupta 2013). The purpose of mathematical models of queues is to obtain closed-form or recursive formulae that allow system designers to analyze performance metrics such as average queue length, average waiting time, and the proportion of customers turned away. Thus, this paper model patient arrival and service distribution, write equations that describe queue pattern change over time and attempts to provide substantial answers to the following questions. How long customers wait to be served? How many customers wait in queue crowd to be served at any time t? Why do queues form?

II. Development of the Model

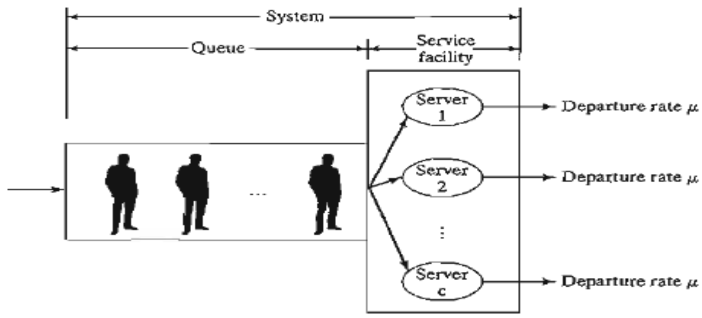
To develop a mathematical model of a hospital in the form that describes the queuing systems requires some background study on Arrival pattern and distribution, service nature and distribution, service mix, arrival and service volume. The entry of a patient into the system (patient arrival) and the release of a patient upon completion (patient departure/exit) are considered as two main events that cause an instantaneous change in the state of the system.

In reality, number of patients arrives vary from shift to shift and time to time. Even, the entire system is not a black box; customers arrive before service start. Consequently, an arrival to a queuing system starts before service start while a departure from a queuing system starts empty. Moreover, time-decisive arrival and service parameters that have been in operation, such that time t, affects the distributions of number in system after that time and the performance of a system. To model observed arrival and service pattern, this paper uses Cumulative number of arrival and exit model and drive equations of the phenomenon of waiting in lines using representative measures of performance, such as average queue length and average waiting time in queue at working time x of a system. Total number of customers arrived and served up to any time can be used to determine all the basic measures of performance. Using Cumulative arrival and service the performance of a system with time-decisive parameters that has been in operation for a sufficiently long time such that time t no longer affects the distributions of number in system, number in different queues, waiting times, and total delay. Thus, to make analytical technique possible and write equations that describe queue crowd changes over time, this paper present Cumulative Approach Modeling Technique.

Let $Na(x)$ denote total number of customers arrived up to time x and $Ns(x)$ denote total number of customers served up to time x where time x is server working time. Using these basic setup basic measures of performance can be determined in the following order.

i. The expected number of customers waiting in the queue at any time x is equal to the expected total number of customers arrived up to time x minus the expected total number of customers served up to time x. $Lq(x) = Na(x) - Ns(x)$
ii. Expected waiting time in the queue of the customer arrived at any time t is equal to the Expected time at service, x, minus arrival time, t. assuming first-in-first-out (FIFO) service protocol Expected time at service, x, of the customer arrived at any time t can be derived from $NA(t) = NS(x)$. thus, $Wq(t) = x - t$
iii. Expected number of customers in the system at any time x is equal to the expected number of customers in queue plus in service at time x. Where, the value of expected number of customers in service is equal to server utilization at time x. $Ls(x) = Lq(x) +$ Expected number of customers in service where, Expected number of customers in service at time x, $\rho(x)$ is: $\rho(x) = Na(x)/Ns(x)$
iv. Expected waiting time in the system of the customer arrived at any time t is equal to the expected waiting time in queue plus the expected service time.
v. In most case, arrival rate of customer, μ, and/or the service rate of server, λ, is uneven or varies from time to time. Using discrete Data along a continuum on Total number of customers arrived and/or served up to any time x, The rate of change in these values with respect to time x can be denoted by fitting a curve along the discrete data points. In case, when arrival rate of customers, μ, and/or the service rate

of server, λ, is uniform or constant, total number of customers arrived up to time x is arrival rate of customers times time and total number of customers served up to time x is effective service rate times time, where effective service rate is number of servers, c, times service rate. Thus, Na(x) $= \mu$*x and Ns(x) =λ*c*x



Using Discrete Data collected for values along a continuum and curve fitting Technique, trend lines equations for total number of customers arrived and total number of customers served up to time x can be easily derived and analytical result of the performance of a system of interest can be easily computed. The basic idea is to fit a curve or a series of curves that pass directly through each of the points. Using this Technique, this paper make it possible to estimate required points between these discrete values and model a function that approximately fit parameters of system of interest. This basic model, identifies the arrival and service pattern of the hospital and the variables used to determine the characteristics of queuing system. To shows how to apply this modeling technique, This paper model case Hospital Triage queuing network composed of two servers and approximate equations describing the queue system of service mechanism.

III. Case Hospital

The Case hospital, Hawassa University Referral Hospital (HURH) in Ethiopia which established in 1994 E.C, is providing Teaching and training service to health science students and medical service to 12,000,000 estimated populations with 350Beds capacity. As might be expected of any hospital, HURH have Triage worker designated to only patients with emergency cases to directly send them to emergency case team without delay, beside regular workers. Thus, this paper studied all patients entered and registered in the regular Hospital Triage.

When patient enter hospital, Hospital Triage front-desk clerk ask them to provide name and reason for visit. The clerk also clarifies if patient was pre-registered for this service or not. If the answer is yes, the clerk gets patient's documentation ready for the registration representative. Then the patient receives an assigned number and is asked to wait in waiting or triage area for registration representative to call the name and number. Registration representative determines if the patient ever receives the service at the hospital and if so, pull up patient's data and verifies patient's personal information. If the patient is visiting the hospital for the first time, CT clerk creates patient's profile in the Hospital Database card. An attendant nurses in this room identifies and determines patient's Triage (OPD) Clinics and creates new account and then orders the carter to transport the card once a patient has paid a registration fee.

In collaboration with HURH Hospital Triage workers Statistical Data were collected on number of patients enter the system within one hour time Interval (T) for consecutive four weeks. It shows that on average 286 Patients visits hospital daily with varying arrival rate. On average 164 and 122 number of patients arrive in Morning and Evening Shift respectively as shown in table below.

Table 1: Number of Patients Enter The System Within One Hour Time Interval

Patients at Morning Time AM		Patients at Evening Time PM	
Intervals	N. Arrived	Intervals	N. Arrived
Before 8:00	19	Before1:00	3
8:00-9:00	62	1:00-2:00	58
9:00-10:00	48	2:00-3:00	37
10:00-11:00	23	3:00-4:00	16
11:00-12:00	12	4:00-5:00	8

Similarly, Statistical Data on patients service time were collected for consecutive two weeks (10 working days) using 300 Sample patients selected randomly shows that CT patient service time varies from 2 to 3.8 minute per patient with 2.55 minute/patient mean server service time. This means 23 patients per hour per server service rate. Thus, HURH have effective mean service rate or service capacity of 46 patients per hour and 368 patients per day, assuming 8 working hour per day, which is much more than average number of Patients visits hospital daily (286). Moreover, Data from hospital Documentation Unit reveals insignificant correlation between days, Monday to Friday, which varies randomly. In general, The Hospital Triage system consist of one stations with two number and configuration of servers and Triage clinics with no customer classes, FIFO service protocols, and unlimited sizes of waiting room are modeled.

IV. Application of the Model

This paper model the phenomenon of waiting in lines using representative measures of performance, such as average queue length, and average waiting time in queue based on Arrival and service distribution trend lines curve fitting equations of Cumulative Approach Analytical Technique (CAAT). Based on Discrete Data collected for values along a continuum, of cumulative number of patients arrived and served up to time x are determined. Using these data representing all values along a continuum, equations of interest changes over time were derived. Estimating required points between these discrete values, equations were derived for every single curve that represents the general trend of the service data, Morning and Evening Arrival data trend lines where $0 \le x \le 4$ with 0.997 R -squared (R^2) value or Square of the correlation coefficient. Thus, functions representing Total number of patients arrived and served up to morning and evening time x are denoted by:

NAm(x) = 0.2198x3 - 10.196x2 + 73.425x + 19 at $R^2 = 0.9978$

NAe(x) = 1.0676x3 - 15.022x2 + 72.708x + 3 when $R^2 = 0.9989$

NS(x) = 46x with the r-squared value of 1.

Table and figure below shows data and trend lines equations representing Arrival and Service data.

Table 2: Morning and Evening Arrival data

Morning shift					Evening shift				
$NA_m(x) = 0.2198x^3 - 10.196x^2 + 73.425x + 19$				Total Served NS(x)= 46x	$NA_a(x) = 1.0676x^3 - 15.022x^2 + 72.708x + 3$				
X	NA_m	equation (Y1)	e		x	NAa	equation	e	NS(x)
0	19	18.642	0.358	0	0	3	2.8103	0.1897	0
1	81	82.426	-1.426	46	1	61	61.7344	-0.7344	46
2	129	126.852	2.148	92	2	98	96.8721	1.1279	92
3	152	153.42	-1.42	138	3	114	114.7232	-0.723	138
4	164	163.63	0.37	184	4	122	121.7875	0.2125	184

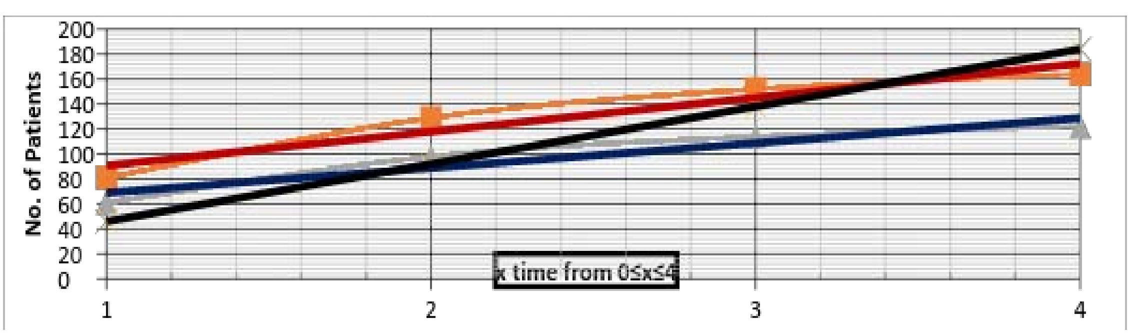


Figure 1: Arrival and Service data trend lines equations

Using this basic setup, Expected number of patients in the queue at time x Lq(x) are denoted by:

Morning Lqm(x) = NAm(x)-Ns(x)

Which is Lqm(x) = 0.2198x3 - 10.196x2 + 27.425x + 19 and

Evening Lqe(x) = NAe(x)-Ns(x)

Which is *Lqe(x) = 1.0676x3 - 15.022x2 + 26.708x + 3* Therefore, Time where an expected number of patients in the queue is zero is where Lq(x)=0 and Time t at which an expected patient in the queue is maximum is where slope of Lq(x) curve is zero which means dLqdx=0 and maximum Expected number of patients in the queue is Lq at time t (Lq(t)). Thus, Expected number of patients in the queue Lqm is zero at 11:29: 10 AM when x = 3.4863 and Lqe is zero at 04:13:02 PM when x=2.21744 morning and evening shift respectively. As a result, Maximum Expected number of patients in the queue is 38 patients at 09:24:32 AM and 16 patients at 01:59:38 PM when x= 1.409100125 and x= 0.9939966 in morning and evening shift respectively as shown in figure below.

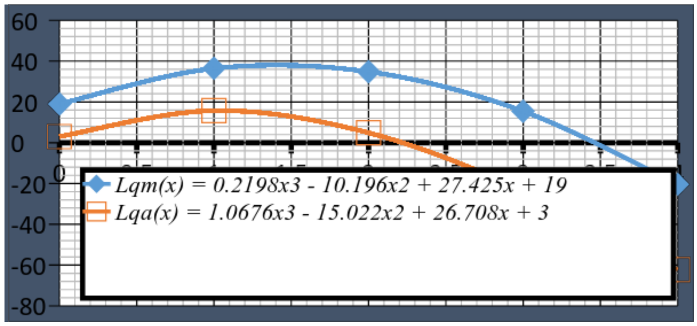


Figure 2: Average numbers of patients in the queue

Similarly, expected patient waiting time in the queue is denoted by Expected time to Service x minus customer arrival time t, which is Wq(x) = x-t. Since it has FIFO service protocol, arrival time t of patient to Service at time x where 0≤t≤x≤4 can be mode led using NA (t) =NS(x). Therefore, Expected time to Service x of patient arrival at time t is:

Morning $46x = 0.2198t3 - 10.196t2 + 73.425t + 19$ *and*
$Xm=0.004778t3 - 0.22165t2 + 1.5962t + 0.413$
Evening $x= 1.0676x^3 - 15.022x^2 + 72.708x + 3$ *and*
$Xe= 0.0232t^3 - 0.3266t^2 + 1.5806t + 0.0652$

Hence, expected waiting time in the queue of the customer arrives at morning and Evening time t, Wqm (t) and Wqe (t) respectively, are denoted by:

$Wqm (t) = 0.004778t3 - 0.22165t2 + 0.5962t + 0.413$
$Wqe (t) = 0.0232t3 - 0.3266t2 + 0.5806t + 0.0652.$

Thus, Figure below shows, expected time in the queue of the customer arrives at Morning and evening time t graphs.

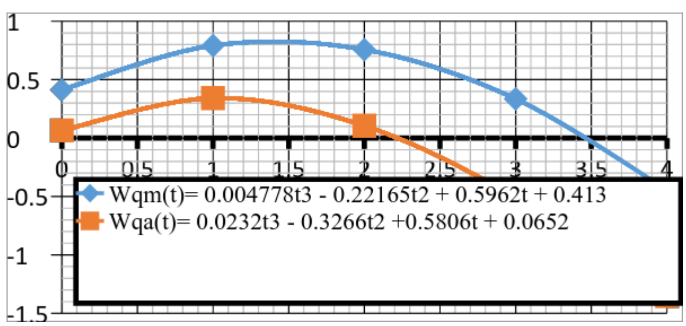


Figure 3: Average times spent in the queue

Time at which an expected patient waiting time in the queue is zero is where Wq(x)=0. Thus, customer arrives after 11:29:10 AM when time t = 3.48629 and after 04:13:02 PM when time t= 2.216859 at morning and evening shift respectively, Expect zero time in queue. The customer arrives at Time t when time in the queue is maximum is where slope of Wq (t) curve is zero which means dWq/dx=zero, Where, 0≤t≤4. Expected Arrival time of patient spend Maximum time in queue are when: is Wq when

Morning time *tm= 1.409100125*
Evening time te= 0.9939966

As a result, Maximum expected patient waiting time in the queue is 0.826hr (49.58 minutes) by patients arrived at 09:24:32 AM and 0.3425 hr (20.55 minutes) by patients arrived at 01:59:38 PM in morning and evening shift, respectively.

Results of the studied triage showed that the queue characteristics of the studied triage during the situation analysis were very undesirable in both morning and evening shifts. There were a big number of patients waiting in the queue and they waited for a long time before being registered. Thus, the average Maximum numbers of patients in the queue were 38 patients at 09:24:32 AM and 16 patients 01:59:38 PM in the morning and evening shift, respectively. The Maximum times spent in the queue by patients arrived at 09:24:32 AM were 49.58 minutes in the morning and 20.55 minutes by patients arrived at 01:59:38 PM in the evening.

As shown in figures, this analytical technique shows how time customers arrive determines the time customers wait in queue lines and analysis the relationship between patient arrival time and average times the customer spent in the queue. customer arrives at 08:30 and 09:30 spent less time in queue waiting line than customer arrives at 09:00. The result has also revealed correlation between patients' waiting times and the number of patients waiting; a positive for patients arrives before number in queue reach its maximum and negative for patients arrives after as shown in figure xx above. Note that queue crowd increase until 09:24:32 AM and 01:59:38 PM when number of arriving patients is greater than server's effective service capacity. In this instance, for each unit of time that the server is available, the average time in queue increases as number of patients in the queues increases and decrease as number of patients in the queues decreases with the same rate.

Briefly, when total number of patients arrived per unit time is greater than total number of patients served per unit time queues continue to grow over time. When total number of patients arrived up to time t is greater than total number of patients served up to time t and total number of patients served per unit time interval t is greater than arrived, queues continue to decelerate over time interval. When Total numbers of patients arrived and served are equal, expected number of customers in queue and time in queue of the customer arrives after time t is zero. In addition, when total number of patients arrived up to time is less than total number of patients served up to time, crowd in queue is zero continuously over time. The customer arrives at time t when number of patients in the queue is Maximum, Expect maximum waiting time in queue and expected waiting time in queue is zero for the customer arrives exactly after time t at which number of patients in the queue is zero.

Furthermore, the analytical results showed that the time patients in queue share 87.157% and 55.421% of system service time while time at which no patients in queue share 12.843% and 44.579% of system service time in the morning and evening shift respectively. Hence servers are capable of serving all arriving patients, queue occurrence not due to server capacity. However, Queues form when customers arrive at a service facility at time they cannot be served immediately upon arrival. Thus, increasing number of server further increase time at which no patients in queue, which means server idleness increased. By specifying reasonable limits on conflicting measures of performance such as average time in the queue and idleness percentage of the servers, anyone can determine an acceptable range of the service level through effective arrival management system. To manage arrival pattern, the arrival rate should be decreased during busy times and increased during "slow" periods. Since, This Analytical techniques show every fluctuation and pattern of queue characteristics of the system changes over time, it can be used to forecast the pattern of waiting time and pre inform customers. Using updated data, healthcare manager can recommend the best moment at which customer arrives and get service without waiting for long time in queue line.

In general, the findings show that, using Cumulative arrival and service parameters up to stationary time that has been in operation, this model limit random variables exist and time-decisive parameters that affects the distributions of number after that time t. Hence, it establish steady-state performance that has been in operation for a sufficiently long time such that time t no longer affects the distributions of number in system, number in different queues, waiting times, and total delay.

V. Conclusion

This paper develop Modeling *Technique* and model the phenomenon of waiting in line, using representative measures of performance, such as average queue length and average waiting time in queue at working time x, of tertiary teaching hospital Triage. This paper showed that, Developed Modeling *Technique, Cumulative Approach Modeling Technique, make it* possible to write equations that describe how the number of customers in each queue in the system of interest changes over time for Hospital Triage and facilities, which are open for a fixed amount of time during the day and experience time-varying customer arrival patterns. Using this model, this paper measures average queue length, and average waiting time in queue which describes the phenomenon of waiting in lines and performance of queuing systems over change of time. Thus, this model suit arrival and service pattern reality, and make it possible to write equations to analyze patients' waiting times and the number of patients waiting at any working time x of both shifts.

The first conclusion was that the Cumulative Approach Analytical Technique (CAAT) model is feasible to limits random variables exist, establish steady state system and drive equations of the phenomenon of waiting in lines using representative measures of performance, such as average queue length and average waiting time in queue at working time x, which can be used to simulate a queuing system's performance. Using this model, analytical result of the performance of a system with time-decisive parameters that has been in operation for a sufficiently long time such that time t no longer affects the distributions of number in system, number in different queues, waiting times, and total delay are possible.

The second conclusion was that the Cumulative Approach Modeling Technique is useful since, it shows how time customers arrive determines the time customers wait in queue lines crowd and analysis the relationship between patient arrival time and average times the customer spent in the queues and queue crowd. On the other hand, it helps us to identify source of queue crowd at any time and easily specify reasonable limits on conflicting measures of performance such as average time in the queue and idleness percentage of the servers. Moreover, this model is useful to indicate how and time at which improvement in system change the queue performance indicators and at what time the queue performance indicators changed very little.

A third conclusion was that the model is flexible. While simple linear models were used in this application, no difficulty is foreseen in adapting the model for nonlinearities in either patient demands or service costs. In addition, the inherent flexibility of the model would permit it to adapt easily to sub models of patient admission rates in the various medical categories.

Finally, the author concludes that, the application of appropriate analytical techniques can offer better queue performance and queue crowd analysis result. Cumulative approach is useful to analyze patients' in queue crowd and waiting times to receive services in both shifts at any working time t belter than transient queues techniques.

VI. Acknowledgements

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