

Low Probability of Intercept Triangular Modulated Frequency Modulated Continuous Wave Signal Characterization Comparison using the Spectrogram and the Scalogram

Daniel L. Stevens¹ and Stephanie A. Schuckers²

¹ Air Force Research Laboratory

Received: 7 December 2016 Accepted: 1 January 2017 Published: 15 January 2017

Abstract

Digital intercept receivers are currently moving away from Fourier-based analysis and towards classical timefrequency analysis techniques for the purpose of analyzing low probability of intercept radar signals. This paper presents the novel approach of characterizing low probability of intercept frequency modulated continuous wave radar signals through utilization and direct comparison of the Spectrogram versus the Scalogram. Two different triangular modulated frequency modulated continuous wave signals were analyzed. The following metrics were used for evaluation: percent error of: carrier frequency, modulation bandwidth, modulation period, chirp rate, and time-frequency localization (x and y direction). Also used were: percent detection, lowest signal-to-noise ratio for signal detection, and plot (processing) time. Experimental results demonstrate that overall, the Spectrogram produced more accurate characterization metrics than the Scalogram. An improvement in performance may well translate into saved equipment and lives.

Index terms—

1 Introduction

Frequency Modulated Continuous Wave (FMCW) signals are frequently encountered in modern radar systems [WAN10], [WON09], [WAJ08]. The frequency modulation spreads the transmitted energy over a large modulation bandwidth B , providing good range resolution that is critical for discriminating targets from clutter. The power spectrum of the FMCW signal is nearly rectangular over the modulation bandwidth, so non-cooperative interception is difficult.

Since the transmit waveform is deterministic, the form of the return signals can be predicted. This gives it the added advantage of being resistant to interference (such as jamming), since any signal not matching this form can be suppressed [WIL06]. Consequently, it is difficult for an intercept receiver to detect the FMCW waveform and measure the parameters accurately enough to match the jammer waveform to the radar waveform $s(t) = A \cos(2\pi f_c t + \pi k t^2)$ (1)

Where $h(t)$ is a short time analysis window localized around $t=0$ and $f=0$. Because multiplication by the relatively short window $h(u-t)$ effectively suppresses the signal outside a neighborhood around the analysis point $u=t$, the STFT is a 'local' spectrum of the signal $x(u)$ around t . Think of the window $h(t)$ as sliding along the signal $x(u)$ and for each shift $h(u-t)$ we compute the usual Fourier transform of the product function $x(u)h(u-t)$. The observation window allows The STFT was the first tool devised for analyzing a signal in both time and frequency simultaneously. For analysis of human speech, the main method was, and still is, the STFT. In general, the STFT is still the most widely used method for studying non-stationary signals [COH95].

The Spectrogram (the squared modulus of the STFT) is given by equation 2 as: $S(f, t) = |STFT(x(u)h(u-t))|^2$ (2)

The Spectrogram is a real-valued and nonnegative distribution. Since the window h of the STFT is assumed of unit energy, the Spectrogram satisfies the global energy distribution property. Thus we can interpret the Spectrogram as a measure of the energy of the signal contained in the time-frequency domain centered on the point (t, f) and whose shape is independent of this localization.

Here are some properties of the Spectrogram: 1) The idea of the wavelet transform (equation (3)) is to project a signal x on a family of zero-mean functions (the wavelets) deduced from an elementary function (the mother wavelet) by translations and dilations: $\psi(t, f) = \int_{-\infty}^{\infty} \psi(t - \tau, f) x(\tau) d\tau$ (3)

The wavelet transform is of interest for the analysis of non-stationary signals, because it provides still another alternative to the STFT and to many of the quadratic time-frequency distributions.

The basic difference between the STFT and the wavelet transform is that the STFT uses a fixed signal analysis window, whereas the wavelet transform uses short windows at high frequencies and long windows at low frequencies. This helps to diffuse the effect of the uncertainty principle by providing good time resolution at high frequencies and good frequency resolution at low frequencies. This approach makes sense especially when the signal at hand has high frequency components for short durations and low frequency components for long durations.

The signals encountered in practical applications are often of this type.

The wavelet transform allows localization in both the time domain via translations of the mother wavelet, and in the scale (frequency) domain via dilations. The wavelet is irregular in shape and compactly supported, thus making it an ideal tool for analyzing signals of a transient nature; the irregularity of the wavelet basis lends itself to analysis of signals with discontinuities or sharp changes, while the compactly supported nature of wavelets enables temporal localization of a signal's features [BOA03]. Unlike many of the quadratic functions such as the Wigner-Ville Distribution (WVD) and Choi-Williams Distribution (CWD), the wavelet transform is a linear transformation, therefore cross-term interference is not generated. There is another major difference between the STFT and the wavelet transform; the STFT uses sines and cosines as an orthogonal basis set to which the signal of interest is effectively correlated against, whereas the wavelet transform uses special 'wavelets' which usually comprise an orthogonal basis set. The wavelet transform then computes coefficients, which represents a measure of the similarities, or correlation, of the signal with respect to the set of wavelets. In other words, the wavelet transform of a signal corresponds to its decomposition with respect to a family of functions obtained by dilations (or The variable a corresponds to a scale factor, in the sense that taking $|a| > 1$ dilates the wavelet ψ and taking $|a| < 1$ compresses ψ . By definition, the wavelet transform is more a time-scale than a time-frequency representation. However, for wavelets which are well localized around a non-zero frequency ω_0 at a scale $a = 1$, a time-frequency interpretation is possible thanks to the formal identification $\omega = \omega_0 / a$. localization of the spectrum in time, but also smears the contractions) and translations (moving window) of an analyzing wavelet.

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A filter bank concept is often used to describe the wavelet transform. The wavelet transform can be interpreted as the result of filtering the signal with a set of bandpass filters, each with a different center frequency [GRI08], [FAR96], [SAR98], [SAT98].

Like the design of conventional digital filters, the design of a wavelet filter can be accomplished by using a number of methods including weighted least squares [ALN00], [GOH00], orthogonal matrix methods [ZAH99], nonlinear optimization, optimization of a single parameter (e.g. the passband edge) [ZHA00], and a method that minimizes an objective function that bounds the out-of-tile energy [FAR99].

Here are some properties of the wavelet transform: 1) The wavelet transform is covariant by translation in time and scaling. The corresponding group of transforms is called the Affine group; 2) The signal x can be recovered from its wavelet transform via the synthesis wavelet; 3) Time and frequency resolutions, like in the STFT case, are related via the Heisenberg-Gabor inequality. However in the wavelet transform case, these two resolutions depend on the frequency: the frequency resolution becomes poorer and the time resolution becomes better as the analysis frequency grows; 4) Because the wavelet transform is a linear transform, it does not contain cross-term interferences [GRI07], [LAR92].

A similar distribution to the Spectrogram can be defined in the wavelet case. Since the wavelet transform behaves like an orthonormal basis decomposition, it can be shown that it preserves energy: $\int \int |\psi(t, f)|^2 dt df = \int |x(t)|^2 dt$ (4)

As is the case for the wavelet transform, the time and frequency resolutions of the Scalogram are related via the Heisenberg-Gabor principle.

The interference terms of the Scalogram, as for the spectrogram, are also restricted to those regions of the time-frequency plane where the corresponding signals overlap. Therefore, if two signal components are sufficiently far apart in the time-frequency plane, their cross-Scalogram will be essentially zero [ISI96], [HLA92].

For this paper, the Morlet Scalogram will be used. The Morlet wavelet is obtained by taking a complex sine wave and by localizing it with a Gaussian envelope. The Mexican hat wavelet isolates a single bump of the Morlet wavelet. The Morlet wavelet has good focusing in both time and frequency [CHE09].

3 II.

4 Methodology

The methodologies detailed in this section describe the processes involved in obtaining and comparing metrics between the classical time-frequency analysis techniques of the Spectrogram and the Scalogram for the detection and characterization of low probability of intercept triangular modulated FMCW radar signals.

The tools used for this testing were: MATLAB (version 7.12), Signal Processing Toolbox (version 6.15), Wavelet Toolbox (version 4.7), Image Processing Toolbox (version 7.2), Time-Frequency Toolbox (version 1.0) (<http://tftb.nongnu.org/>).

All testing was accomplished on a desktop computer (HP Compaq, 2.5GHz processor, AMD Athlon 64X2 Dual Core Processor 4800+, 2.00GB Memory (RAM), 32 Bit Operating System).

Testing was performed for 2 different triangular modulated FMCW waveforms. For each waveform, parameters were chosen for academic validation of signal processing techniques.

Due to computer processing resources they were not meant to represent real-world values. The number of samples for each test was chosen to be either 256 or 512, which seemed to be the optimum size for the desktop computer. Testing was performed at three different SNR levels: 10dB, 0dB, and the lowest SNR at which the signal could be detected. The noise added was white Gaussian noise, which best reflects the thermal noise present in the IF section of an intercept receiver [PAC09]. Kaiser windowing was used, when windowing was applicable. 50 runs were performed for each test, for statistical purposes. The plots included in this paper were done at a threshold of 5% of the maximum intensity and were linear scale (not dB) of analytic (complex) signals; the color bar represented intensity. The signal processing tools used for each task were the Spectrogram and the Scalogram.

Task 1 consisted of analyzing a triangular modulated FMCW signal (most prevalent LPI radar waveform [LIA09]) whose parameters were: sampling frequency=4KHz; carrier frequency=1KHz; modulation bandwidth=500Hz; modulation period=.02sec.

Task 2 was similar to Task 1, but with different parameters: sampling frequency=6KHz; carrier frequency=1.5KHz; modulation bandwidth=2400Hz; modulation period=.15sec. The different parameters were chosen to see how the different shapes/heights of the triangles of the triangular modulated FMCW would affect the metrics.

After each particular run of each test, metrics were extracted from the time-frequency representation.

The different metrics extracted were as follows: 1) Plot (processing) time: Time required for plot to be displayed.

where E is the energy of x . This leads us to define the Scalogram (equation (4)) of x as the squared modulus of the wavelet transform. It is an energy distribution of the signal in the time-scale plane, associated with the measure dx .

2) Percent detection: Percent of time signal was detected -signal was declared a detection if any portion of each of the signal components (4 chirp components for triangular modulated FMCW) exceeded a set threshold (a certain percentage of the maximum intensity of the time-frequency representation).

Threshold percentages were determined based on visual detections of low SNR signals (lowest SNR at which the signal could be visually detected in the timefrequency representation) (see Figure ??).

Figure ??: Threshold percentage determination. This plot is an amplitude vs. time (x-z view) of the Spectrogram of a triangular modulated FMCW signal (256 samples, with SNR= -3dB). For visually detected low SNR plots (like this one), the percent of max intensity for the peak z-value of each of the signal components (the 2 legs for each of the 2 triangles of the triangular modulated FMCW) was noted (here 98%, 60%, 95%, 63%), and the lowest of these 4 values was recorded (60%). Ten test runs were performed for both time-frequency analysis tools (Spectrogram and Scalogram) for this waveform. The average of these recorded low values was determined and then assigned as the threshold for that particular time-frequency analysis tool. Note -the threshold for the Spectrogram is 60%.

Thresholds were assigned as follows: Spectrogram (60%); Scalogram (50%).

For percent detection determination, these threshold values were included in the time-frequency plot algorithms so that the thresholds could be applied automatically during the plotting process. From the threshold plot, the signal was declared a detection if any portion of each of the signal components was visible (see Figure 2).). From the frequency-intensity (y-z) view, the maximum intensity value is manually determined. The frequency corresponding to the max intensity value is the carrier frequency (here $f_c=984.4$ Hz).

5 4) Modulation bandwidth:

Distance from highest frequency value of signal (at a threshold of 20% maximum intensity) to lowest frequency value of signal (at same threshold) in Y-direction (frequency).

The threshold percentage was determined based on manual measurement of the modulation bandwidth of the signal in the time-frequency representation. This was accomplished for ten test runs of each time-frequency analysis tool (Spectrogram and Scalogram), for each of the 2 waveforms. During each manual measurement, the max intensity of the high and low measuring points was recorded. The average of the max intensity values for these test runs was 20%. This was adopted as the threshold value, and is representative of what is obtained when

performing manual measurements. This 20% threshold was also adapted for determining the modulation period and the time-frequency localization (both are described below).

For modulation bandwidth determination, the 20% threshold value was included in the time-frequency plot algorithms so that the threshold could be applied automatically during the plotting process. From the threshold plot, the modulation bandwidth was manually measured (see Figure 4). Distance from highest frequency value of signal (at a threshold of 20% maximum intensity) to lowest frequency value of signal (at same threshold) in X-direction (time). automatically during the plotting process. From the threshold plot, the modulation period was manually measured (see Figure ??).

For modulation period determination, the 20% threshold value was included in the time-frequency plot algorithms so that the threshold could be applied. Measure of the thickness of a signal component (at a threshold of 20% maximum intensity on each side of the component) -converted to % of entire X-Axis, and % of entire Y-Axis.

For time-frequency localization determination, the 20% threshold value was included in the timefrequency plot algorithms so that the threshold could be applied automatically during the plotting process. From the threshold plot, the time-frequency localization was manually measured (see Figure 6). plot algorithms so that the thresholds could be applied automatically during the plotting process. From the threshold plot, the signal was declared a detection if any portion of each of the signal components was visible. The lowest SNR level for which the signal was declared a detection is the lowest detectable SNR (see Figure ??). Year 2017

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For lowest detectable SNR determination, these threshold values were included in the time-frequency ?? : Lowest detectable SNR. This plot is an frequency vs. time (x-y view) of the Spectrogram of a triangular modulated FMCW signal (256 samples, with SNR= -3dB) with threshold value automatically set to 60%. From this threshold plot, the signal was declared a (visual) detection because at least a portion of each of the 4 signal components (the 2 legs for each of the 2 triangles of the triangular modulated FMCW) was visible. Note that the signal portion for the 60% max intensity (just above the 'x' in 'max') is barely visible, because the threshold for the Spectrogram is 60%. For this case, any lower SNR would have been a non-detect. Compare to Figure 2, which is the same plot, except that it has an SNR level equal to 10dB.

The data from all 50 runs for each test was used to produce the actual, error, and percent error for each of these metrics listed above.

The metrics from the Spectrogram were then compared to the metrics from the Scalogram. By and large, the Spectrogram outperformed the Scalogram, as will be shown in the results section.

7 III. Results

Table 1 presents the overall test metrics for the two classical time-frequency analysis techniques used in this testing (Spectrogram versus Scalogram).

8 Discussion

This section will elaborate on the results from the previous section.

From Table 1, the Spectrogram outperformed the Scalogram in every category. The Spectrogram's reduction of cross-term interference is grounds for its better plot time. Average percent detection and lowest the Time-Frequency representation. Figure 8 and Figure 9 show clearly that the signals in the Spectrogram plots are more readable than those in the Scalogram plots, which account for the Spectrogram's better average percent detection and lowest detectable SNR. At relatively low frequencies (as in this paper), wavelets (Scalograms), because of their multi-resolution analysis basis, are better resolved (localized) in frequency and more poorly resolved (localized) in time. Therefore for relatively low frequencies, the best waveforms to be analyzed by wavelets (Scalograms) are tonals. In addition, the irregularity of the wavelet (Scalogram) basis lends itself to analysis of signals with discontinuities (such as frequency hopping signals (tonals)). Also, since the wavelet is irregular shape and compactly supported, it makes it an ideal tool for analyzing signals of transient nature (such as the frequency hopping signals (tonals)). Therefore as the signal goes from being 'flat' (i.e. a tonal) signal, to more 'upright' (i.e. a triangular modulated FMCW) signal, the Scalogram of this signal becomes more poorly resolved (localized), i.e. 'fatter', accounting for the Scalogram's poorer metrics in the categories of modulation bandwidth, modulation period, chirp rate, carrier frequency, time-frequency localization (x), and timefrequency localization (y). Future plans include continuing to analyze low probability of intercept radar waveforms (such as the frequency hopping and the triangular modulated FMCW), using additional time-frequency analysis techniques.

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³F © 2017 Global Journals Inc. (US) detectable SNR are both based on visual detection in

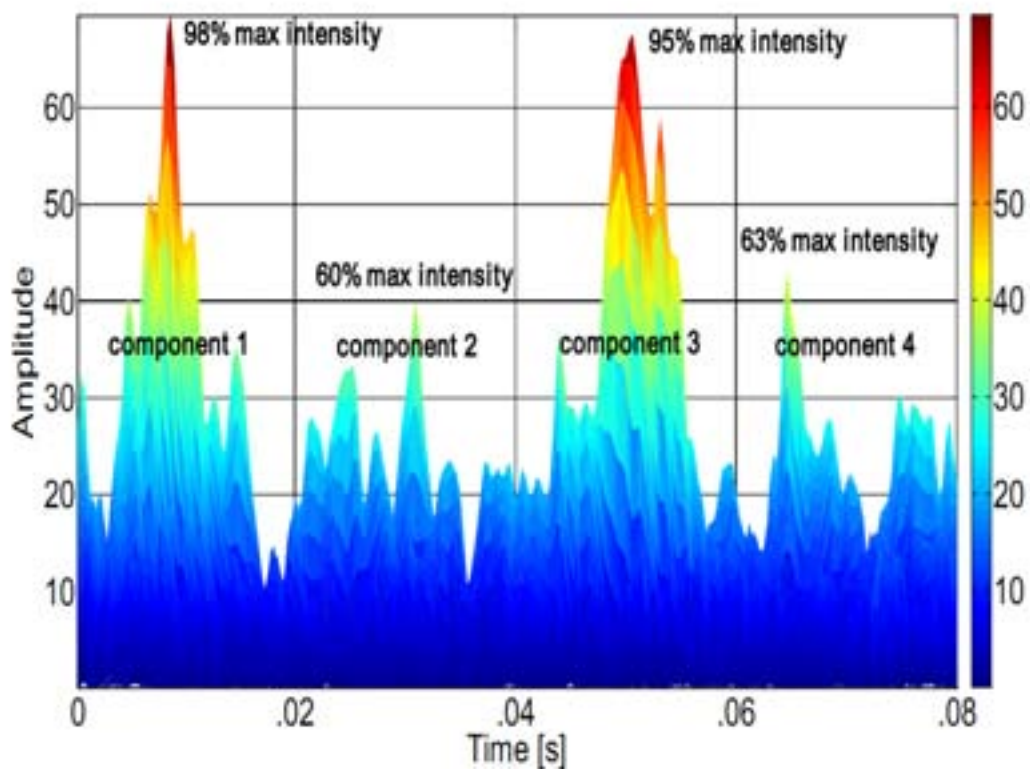
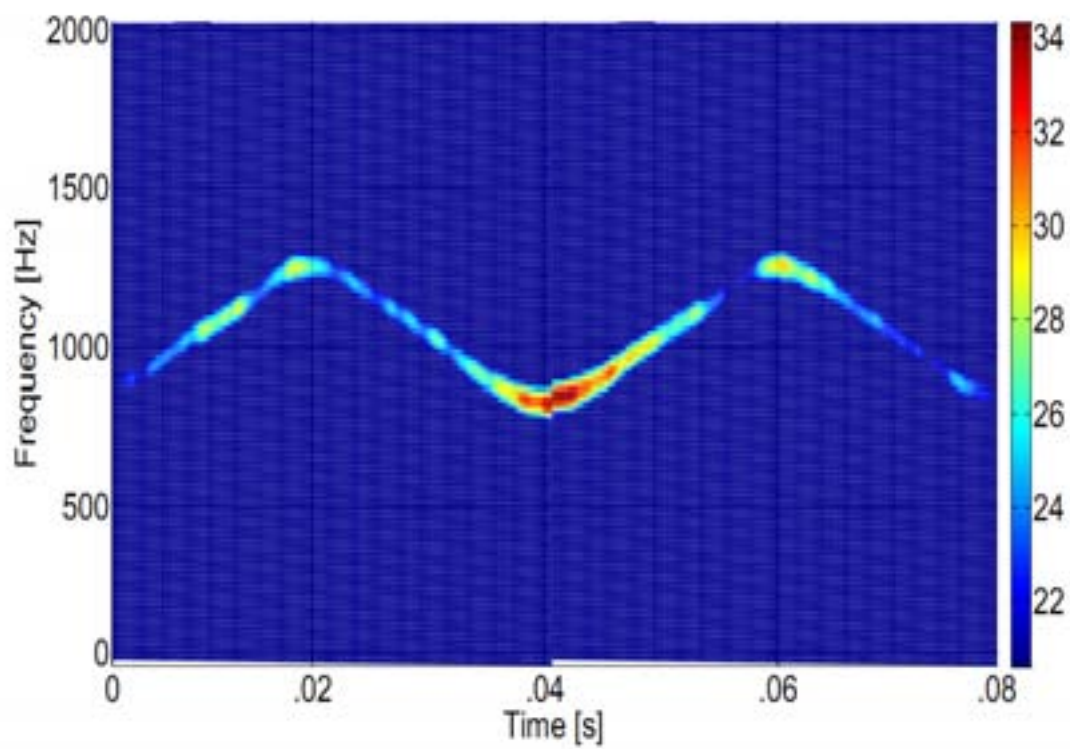
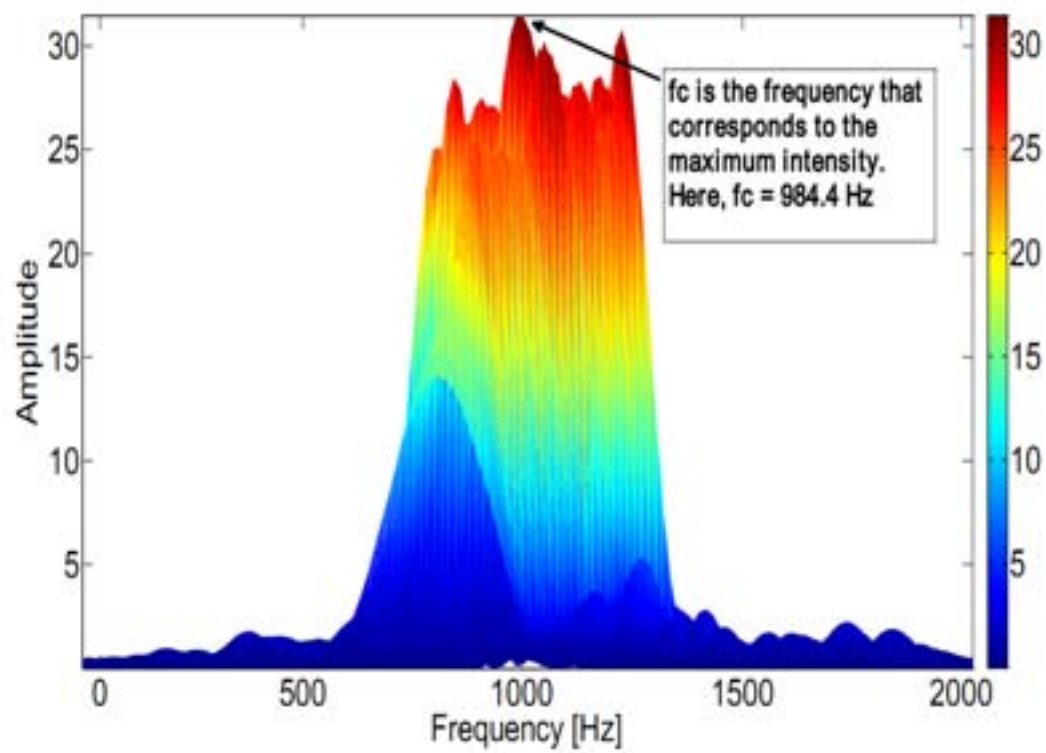


Figure 1:



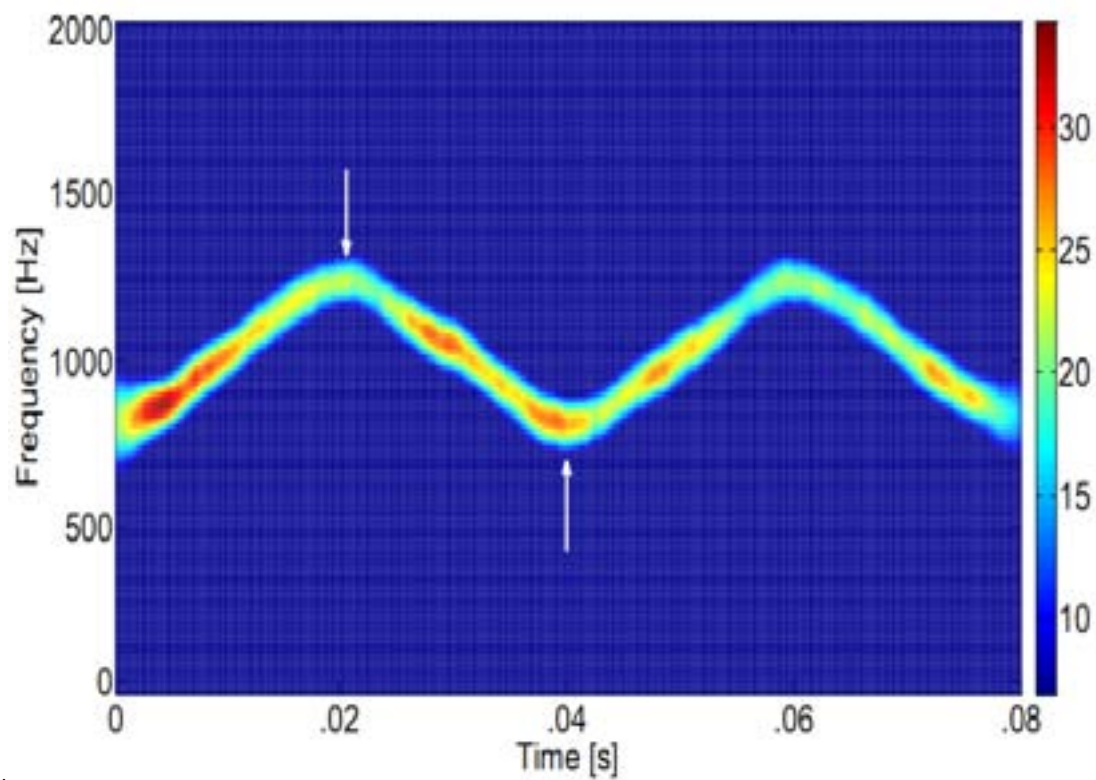
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Figure 2: Figure 2 :



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Figure 3: Figure 3 :



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Figure 4: Figure 4 :

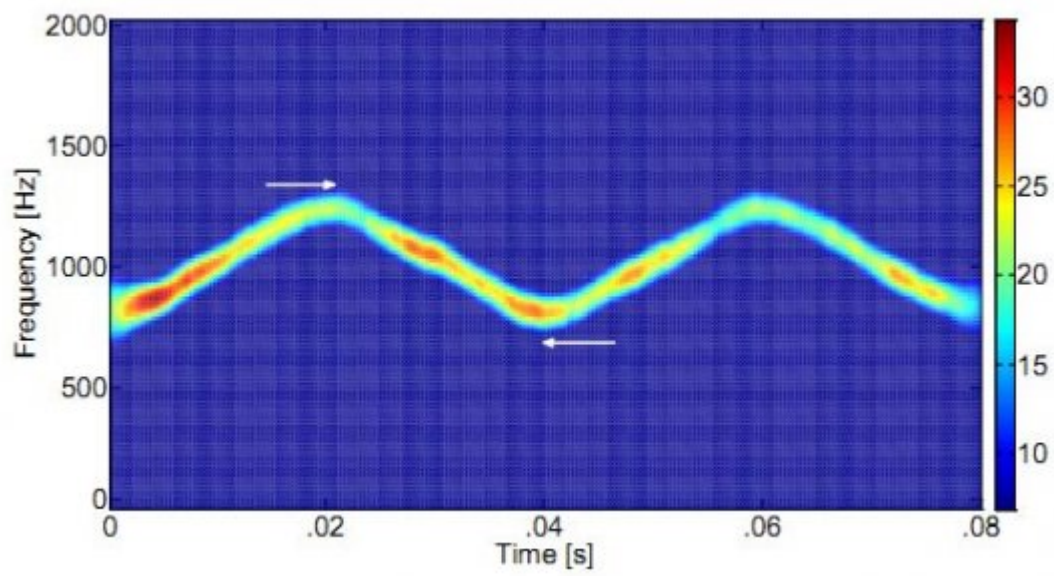
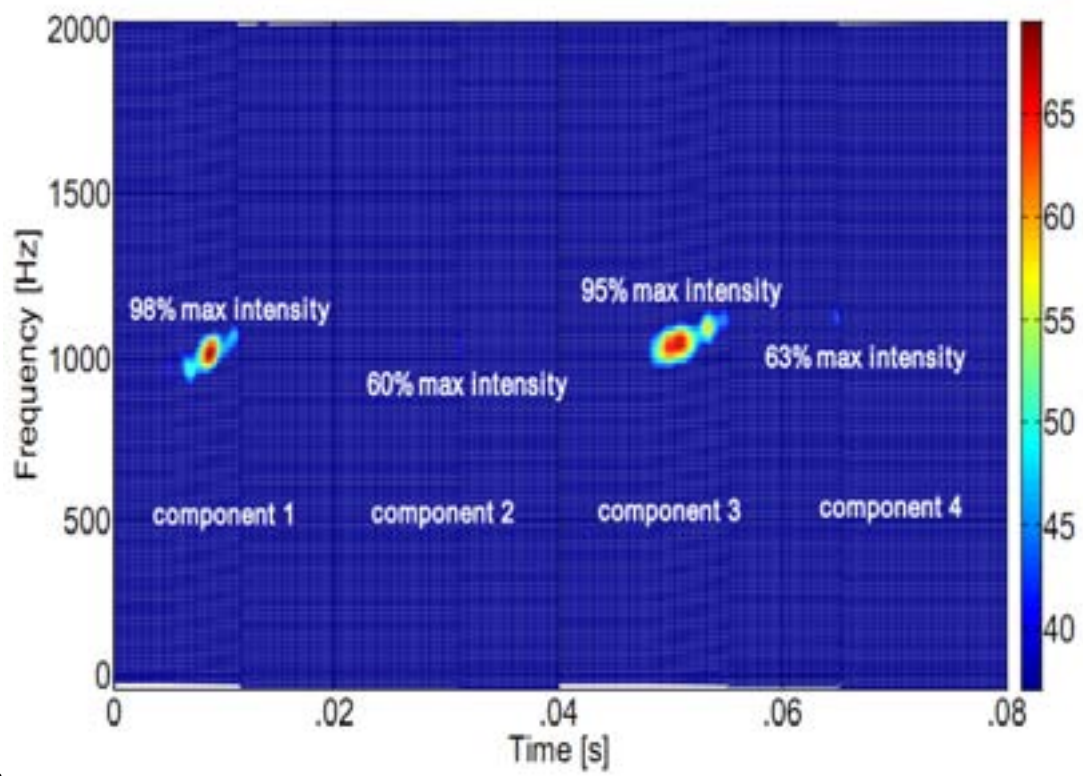


Figure 5: GlobalF



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Figure 6: Figure 6 :

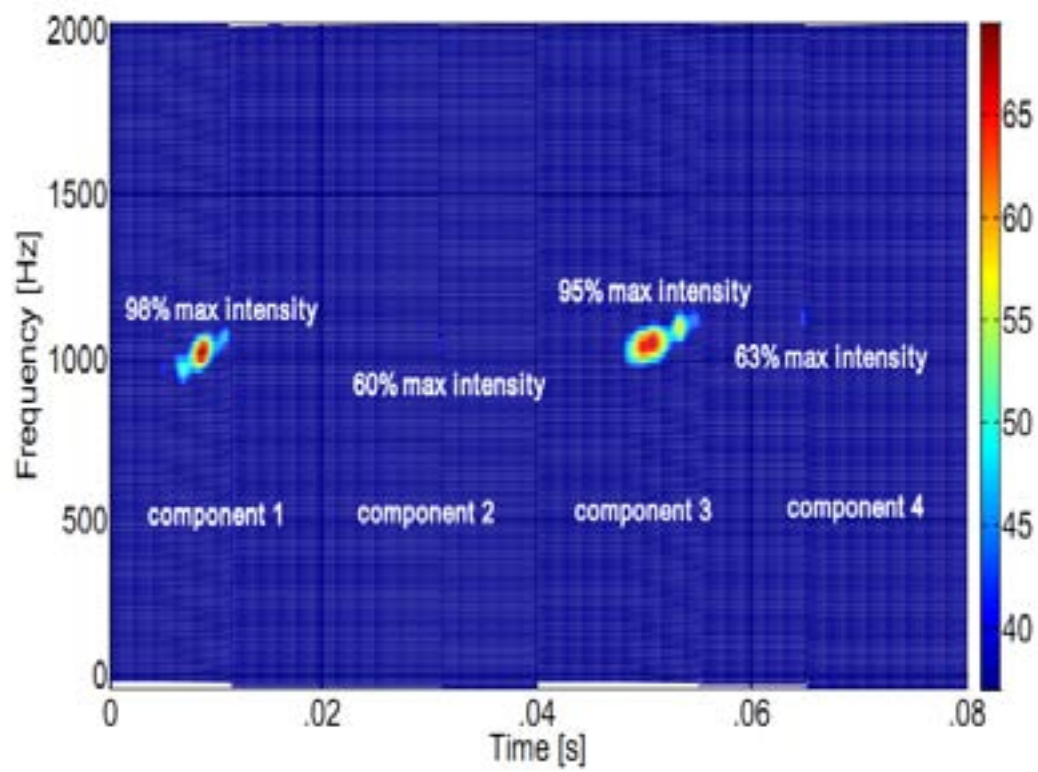


Figure 7:

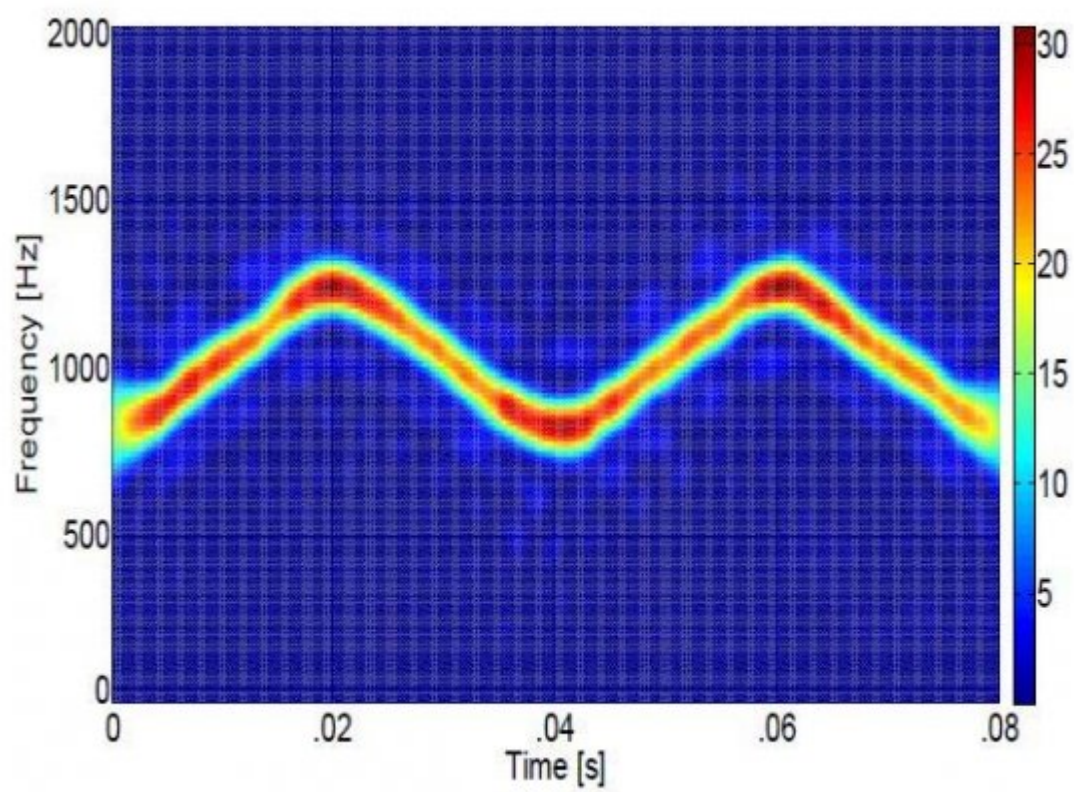
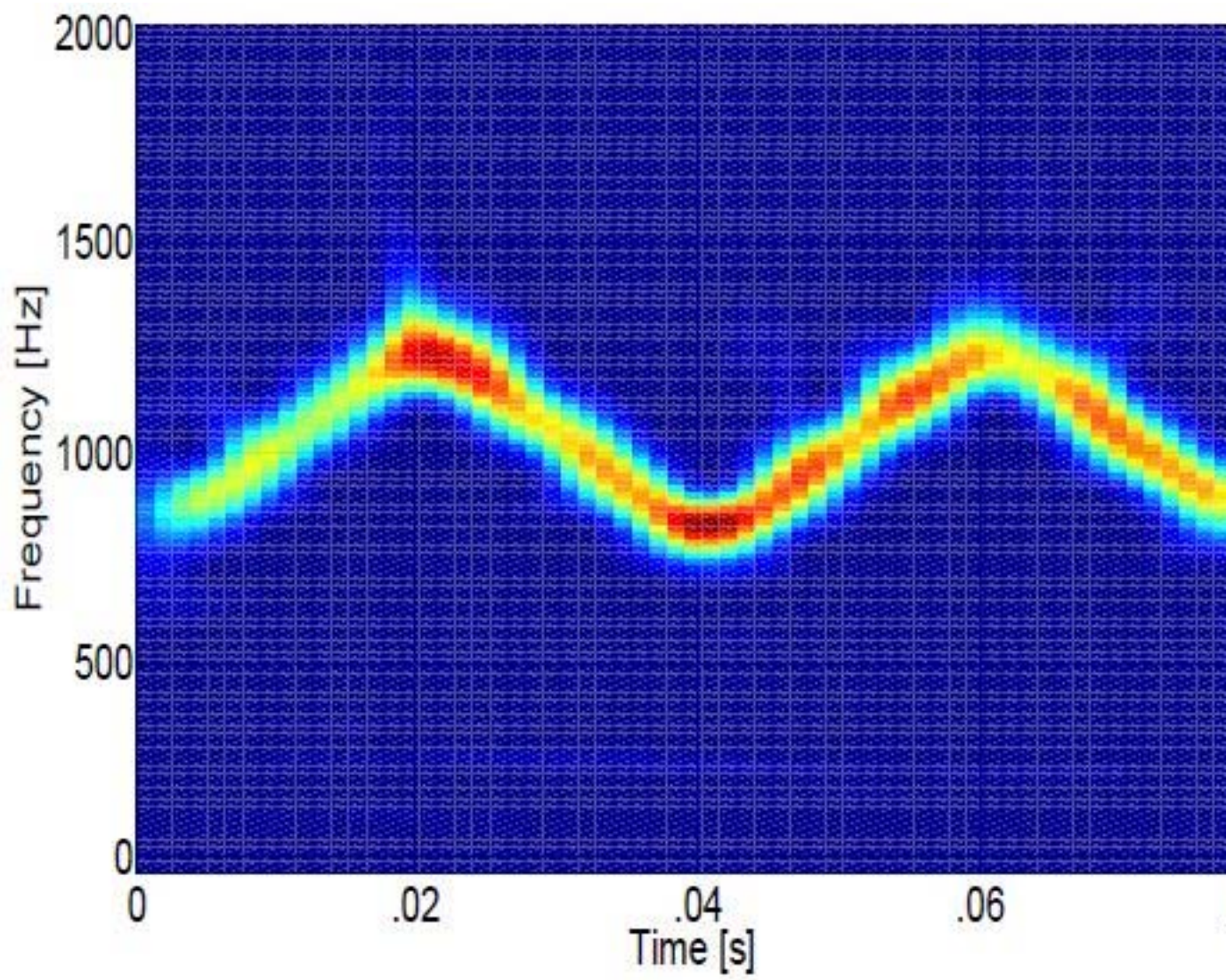
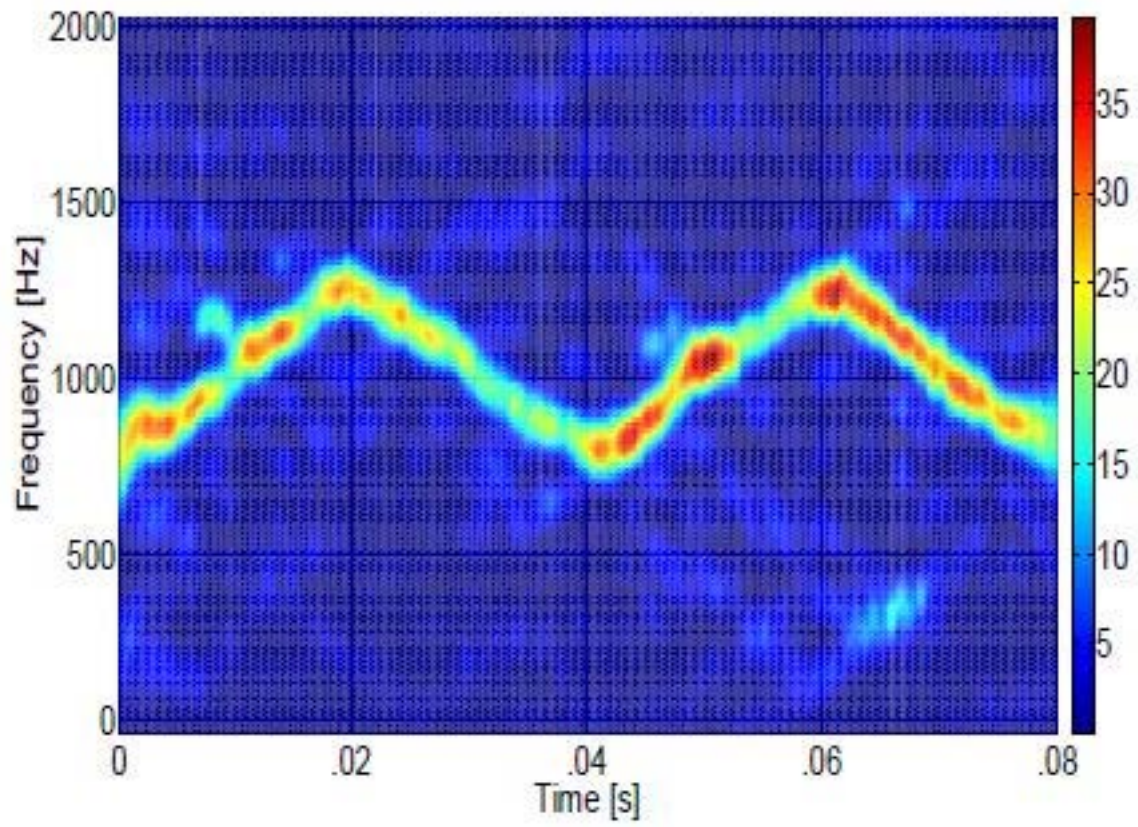


Figure 8: Figure



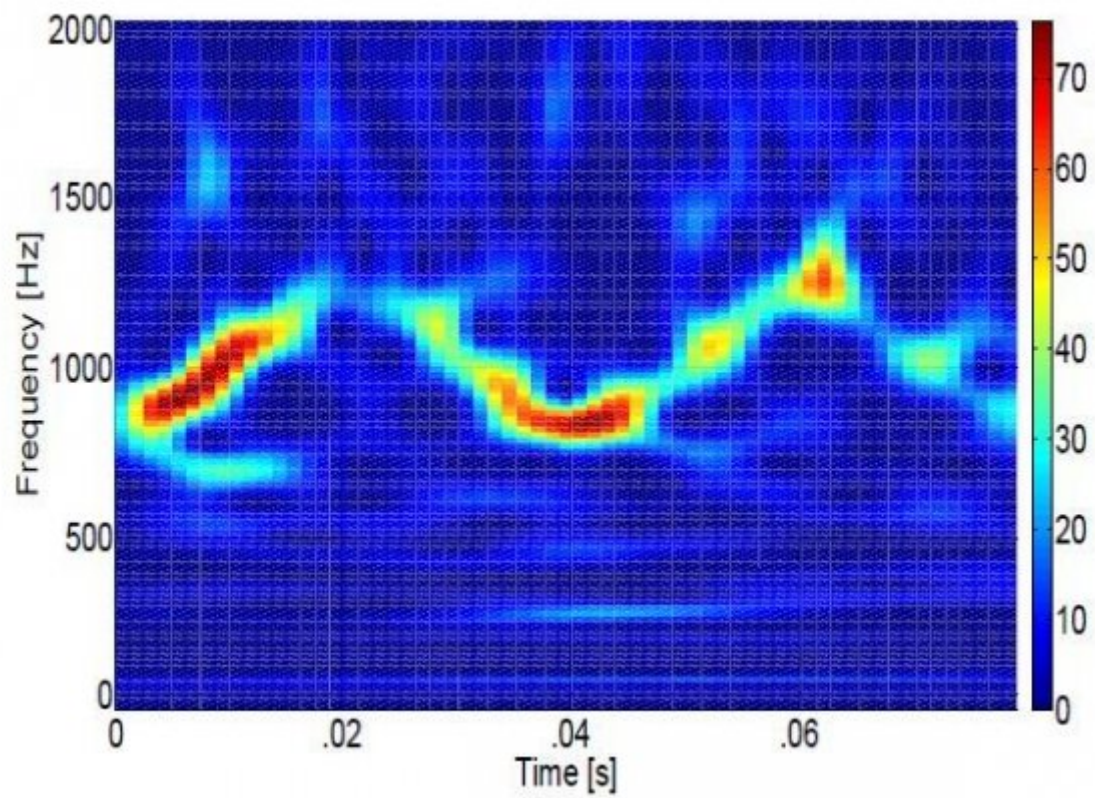
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Figure 9: Figure 8



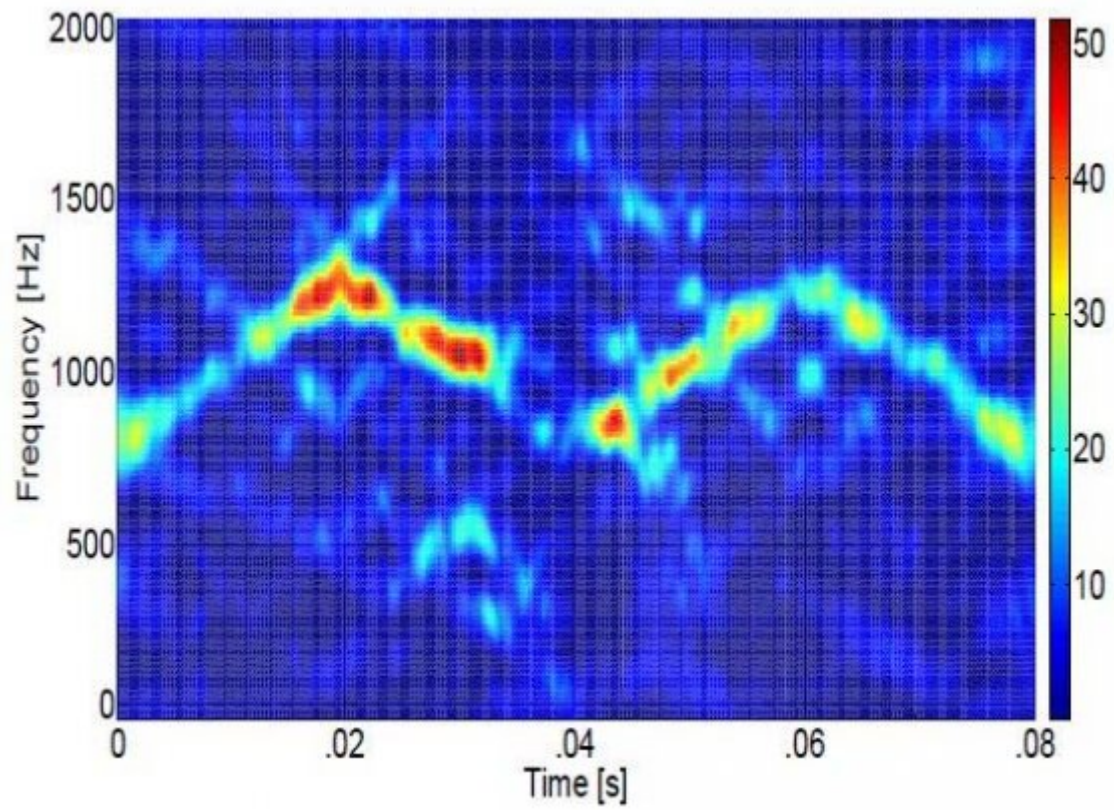
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Figure 10: Figure 8 :



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Figure 11: Figure 9



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Figure 12: Figure 9 :

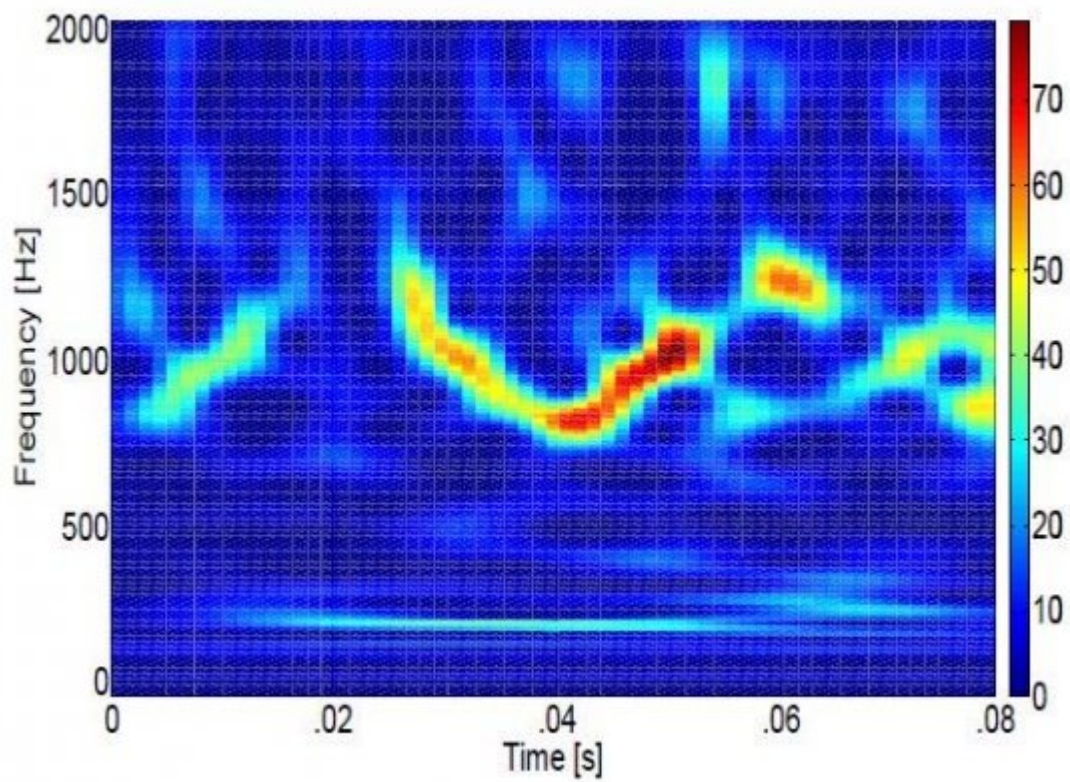


Figure 13: Conclusions

[Note: 1 Approved for Public Release; Distribution Unlimited: CaseNumber: 88ABW-2017-1873, 25-Apr-2017]

Figure 14:

1

parameters	Spectrogram	Scalogram
carrier frequency	6.83%	8.26%
modulation bandwidth	16.60%	28.17%
modulation period	0.68%	0.72%
chirp rate	16.25%	28.47%
percent detection	70.0%	62.22%
lowest detectable snr	-3.67db	-2.67db
plot time	3.28s	4.16s
time-frequency localization-x	2.88%	4.51%
time-frequency localization-y	5.75%	9.0%

From Table 1, the Spectrogram outperformed the Scalogram in every metrics category.

Figure 15: Table 1 :

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