Modeling of the Transfer Function Characteristics of an Electrical Power Distribution System

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Abstract- This study seeks to determine the mathematical model for the transfer function characteristics of a single-input-single-output (SISO) electrical power distribution system with a view to understanding better, how the variation in input variables affects the overall output of electrical power distribution. It involved taking input-output data from a real life electrical power distribution system over a given period of four months and developing a transfer function model for the system so as to determine its operations efficiency using SPSS software. From the analysis carried out, the value of $\alpha$ (coefficient of performance) in the month 1-2 was found to be 0.670 while the value was 1.065 in the month 3-4. The higher the value of $\alpha$, the more efficient is the power distribution system.

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Modeling of the Transfer Function Characteristics of an Electrical Power Distribution System

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Abstract - This study seeks to determine the mathematical model for the transfer function characteristics of a single-input-single-output (SISO) electrical power distribution system with a view to understanding better, how the variation in input variables affects the overall output of electrical power distribution. It involved taking input-output data from a real life electrical power distribution system over a given period of four months and developing a transfer function model for the system so as to determine its operations efficiency using SPSS software. From the analysis carried out, the value of $\omega_0$ (coefficient of performance) in the month 1-2 was found to be 0.670 while the value was 1.065 in the month 3-4. The higher the value of $\omega_0$ the more efficient is the power distribution system. The system performance was found to be more effective in the month 3-4 than in the month 1-2 because it has a higher value of (COP). This shows that the transfer function parameters could be used as performance indicators. In addition to this, effective determination of the capability of a process will lead to better monitoring, improved operations and maintenance of the distribution system.

Keywords: transfer function, SISO, mathematical model, coefficient of performance (COP).

I. Introduction

The focus of this research is to address the challenges of electrical power distribution systems in Nigeria. The relationship between the input and output of a single-input-single-output electrical power distribution system although fairly known, but it is least understood. A major reason for this is that electrical power distribution system transfer function which serves as a guide for measuring and monitoring electrical power distribution output is often never determined and factored into the distribution control system. The decision problem is which analytical technique to adopt to clarify the nature of the relationship. In this study, the transfer function model is employed. Transfer function modeling is an integral part of control and process monitoring which is used to determine the causal relationship between input and output of a process. Efforts at relating the input to output of a system statistically started with regression analysis (Lai, 1979). Hence, regression analysis formed the basis of the traditional statistical method of modeling the relationship between the input and output to systems. Regression analysis has many deficiencies, for example; regression analysis is inappropriate in situations where the output lags the input when there is a significant amount of noise in the system regression analysis cannot accommodate noise in the filter (Box and Jenkins, 1994).

In 1970, Box and Jenkins introduced an improved statistical method of modeling the relationship between the input and output to a system. This method was named Box-Jenkins transfer function modeling methodology (Lai, 1979). Box et al did a very significant work in transfer function modeling by introducing ARIMA and noise models into transfer function modeling. This significantly improved the efficiency and reliability of transfer function models. In addition to this, transfer function model forecasts usually have smaller forecasting errors than the forecasts based on univariate models which are based on the output, and gives good forecasts of the future output from a process which is very significant. Since the introduction of transfer function modeling in 1970, efforts have been made by various researchers to improve and extend its application in various fields of life. For example Lai (1979) applied it extensively in modeling geographical systems. DerLurgio (1998) applied it extensively in econometrics and economic forecasting. Nwobi-Okoye and Igboanugo (2011, 2012) applied transfer function modeling to better understand the complex relationship between the input and output of a production system. Albarbar et al. (2008) carried out research to compare the performance of MEMS accelerometers for condition monitoring using input-output transfer function analysis based on frequency responses, as well as other performance indicators. They investigated the performances of three of the MEMS accelerometers from different manufacturers and compared them to a well calibrated commercial accelerometer which was used as a reference for MEMS sensors performance evaluation.

This work is therefore conceived to explore transfer function modeling as a possible monitoring and control tool for improving the operational efficiency of an electrical power distribution system. The hub of our...
investigation is Shell Forcados Terminal located Southwest of Warri, Delta State Nigeria.

II. THE GENERAL TRANSFER FUNCTION MODELING PROCEDURE

A discrete transfer function model applicable to a distribution process has been developed by Box et al. We shall assume the model as stated in equation (3.1) as follows:

\[ Y_t = \delta^{-1}(B) \omega(B)X_t - b + N_t \]  

(3.1)

The noise term, \( N_t \), is represented by an ARIMA (p,d,q) process such that:

\[ N_t = \varphi^{-1}(B)\theta(B)a_t \]  

(3.2)

Here \( a_t \) is the white noise. Substituting equation (3.2) into (3.1), gives

\[ Y_t = \delta^{-1}(B) \omega(B)X_t - b + \varphi^{-1}(B)\theta(B)a_t \]  

(3.3)

III. METHODOLOGY

The basic model used in this research is the transfer function modeling. The transfer function modeling procedure consists of the following steps:

1. Model Identification
2. Model Estimation
3. Model Diagnostics
4. Forecasting

a) Transfer Function Modeling Procedure: Electrical Power Distribution

In order to realize the transfer function model based on equation (3.3), a plot of the 4-month input-output data was done using SPSS software. After the plot, the data was investigated for stationarity, using the plots of the autocorrelation functions (ACF) and Pearson’s autocorrelation functions (PACF). The input and output series derived from the plots were found not to be stationary, hence differencing was used to achieve stationarity. Stochastic regularity was achieved after the second differencing. Following the achievement of stationarity of the input \( Y_t \) and output \( X_t \), univariate model was individually fitted to \( X_t \) and \( Y_t \) in order to respectively estimate pre-whitened input and output series namely \( \beta_t \) and \( \alpha_t \). Calculation of the cross correlation function was used to identify \( r \), \( s \) and \( b \) parameters of the transfer function model.

Furthermore, the transfer function was estimated using \( Y_t \) and \( X_t \). The residual of the transfer function was used to identify the noise term \( N_t \) of the transfer function model. Finally, the model adequacy check and optimization was done using genetic algorithm.

As transfer function model parameters are continuous variables, the genetic algorithm method was the continuous version. The model parameters are: \( b, \delta, \omega, \theta \) and \( \phi \) as shown in equation (3.3).

IV. RESULTS

Transfer Function Modelling: The data gotten from transformer (TR-102) at switchgears SG-101 (Input) and SG-102 (Output) at Shell Forcados Power Terminal was analyzed in line with the theory and procedure developed and described earlier in the methodology.

The abscissas of Figures 1 to 4 are in days-of-the-month (30). The distribution station is supplied with 33kv from the national grid and it is stepped down to 11kv for distribution to consumers. The power input depends on availability of power in the national grid which is fed by the generation stations. Thus as shown in Figures 1 and 3, the power supply does not follow any particular pattern. The output power depends on demand and the conditions of the distribution facilities. Hence, the output time series of Figures 2 and 4 follow the pattern described in the foregoing.
Analysis of Input Series

Figure 2: Output Series for month 1-2

Figure 3: Input Series for month 3-4

Figure 4: Output Series for month 3-4

Figure 5: ACF of the input series
The input series upon analysis was found to be stationarity, hence differencing was not used. Examination of the ACF and PACF in Figures 4 and 5 are indicative that auto regression one (AR (1)) model is the appropriate model to use. The formula for AR (1) models [2, 14 and 15] is given by equation (4):

\[ X_t = \theta_0 + \phi X_{t-1} + e_t \]  

But for AR (1) models, we have:

\[ \text{ACF}(1) = \phi = 0.427 \]  
\[ \theta = (1 - \phi) \mu \]  
\[ \theta = (1 - 0.427) 297.42 \]  
\[ \theta = 170.42 \]  

Fitting the coefficients \( \theta_0 \) and \( \theta_1 \) into the formula for AR (1) models, equation (9) is obtained.

\[ X_t = 170.42 + 0.427 X_{t-1} + e_t \]  

But

\[ e_t = \alpha \]  

In forecasting form, equation (9) is transformed to equation (11):

\[ \hat{X}_t = 170.42 + 0.427 X_{t-1} \]  

a) Analysis of output series

The output series upon analysis was found to be stationarity, hence differencing was not used. Examination of the ACF and PACF in Figures 5 and 6 are indicative that auto regression one (AR (1)) model is the appropriate model to use.

\[ \text{Figure 6: PACF of the input series} \]

\[ \text{Figure 7: ACF of the output series} \]
The formula for AR (1) models [2, 14 and 15] is given by equation (12):

\[ Y_t = \theta_0 + \phi Y_{t-1} + \varepsilon_t \]  

But for AR (1) models, we have:

\[ \text{ACF}(1) = \phi = 0.427 \]  

\[ \theta_0 = (1 - \phi) \mu \]  

\[ \theta = (1 - 0.427)278.69 \]  

\[ \theta = 159.69 \]  

Fitting the coefficients \( \theta_0 \) and \( \phi_1 \) into the formula for AR (1) models, equation (18) is obtained.

\[ Y_t = 159.69 + 0.427 Y_{t-1} + \varepsilon_t \]  

But

\[ \varepsilon_t = \beta t \]  

In forecasting form equation (18) is transformed to equation (20):

\[ \hat{Y}_t = 159.69 + 0.427 Y_{t-1} \]  

The CCF between \( \beta_t \) and \( \alpha_t \) is shown in Figure 5.3.2. It has one significant CCF at lag zero (0). Hence, according to [14], the parameters \( r, s \) and \( b \) of the transfer function that supports such CCF pattern are 0, 0 and 0 respectively. In view of this fact, the CCF supports the following transfer function model:

\[ y_t = \omega x_t + N_t \]  

Based on Ljung-Box statistics shown in Table 5.3 and analysis of the residuals, the transfer function was found to have white noise residuals, hence we disregarded the noise term \( N_t \), to obtain equation (22).

\[ y_t = \omega x_t \]  

As shown by [1] and [4],

\[ \omega_0 = \omega_0 \]  

Figure 8: PACF of the output series

Figure 9: CCF of the pre-whitened series
But
\[ v = \frac{\gamma \alpha \beta (0) S}{\alpha x} \]  \hspace{1cm} (24)

\( \gamma \alpha \beta (0) \) is the cross correlation between \( \alpha \) and \( \beta \) at lag zero (0).

But
\[ X - \mu = x \]  \hspace{1cm} (25)
And
\[ Y - \mu = y \]  \hspace{1cm} (26)

Substituting equation (26) into equation (22), equation (27) is obtained.

\[ Y = \mu + \omega x \]  \hspace{1cm} (27)
In forecasting form equation (27) is transformed to equation (28).

\[ \hat{Y} = \mu + \omega x \]  \hspace{1cm} (28)

b) Analysis for Month 1-2

The lag of 0 in the transfer function model shows that the average gas flow in the month is used for generation the same month. The model has intuitive and theoretical appeal. The model fit and statistics are good as shown for month 1-2 in Tables 1 and 2 respectively.

**Table 1: Model fit for Month 1-2**

<table>
<thead>
<tr>
<th>Fit Statistic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stationary R-squared</td>
<td>.712</td>
</tr>
<tr>
<td>R-squared</td>
<td>.712</td>
</tr>
<tr>
<td>RMSE</td>
<td>12.362</td>
</tr>
<tr>
<td>MAPE</td>
<td>3.407</td>
</tr>
<tr>
<td>MaxAPE</td>
<td>12.212</td>
</tr>
<tr>
<td>MAE</td>
<td>9.300</td>
</tr>
<tr>
<td>MaxAE</td>
<td>30.309</td>
</tr>
<tr>
<td>Normalized BIC</td>
<td>5.234</td>
</tr>
</tbody>
</table>

**Table 2: Model Statistics for Month 1-2**

<table>
<thead>
<tr>
<th>Model</th>
<th>Number of Predictors</th>
<th>Model Fit statistics</th>
<th>Ljung-Box Q(18)</th>
<th>Number of Outliers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transfer Function Model</td>
<td>1</td>
<td>.712</td>
<td>9.592</td>
<td>17</td>
</tr>
</tbody>
</table>

For month 1-2 operations of the Power Station we obtained:

\[ \gamma \alpha \beta (0) = 0.759 \]

\[ S_\beta = 19.12 \]

\[ S_\alpha = 21.68 \]

Hence,
\[ v = \frac{0.759 \times 19.12}{21.68} = 0.67 \]

Hence from equation (22)
\[ y = 0.67 x \]

Since \( \omega = 0.67 \) for the month 1-2 operation of the power station, the transfer function is given by:
\[ \hat{Y} = \mu + 0.67 x \]  \hspace{1cm} (29)
c) Analysis for Month 3-4

The lag of 0 in the transfer function model shows that the average gas flow in the month is used for generation the same month. The model has intuitive and theoretical appeal. The model fit and statistics are good as shown for the month 3-4 in Tables 2 and 3 respectively.

Table 3: Model fit for Month 3-4

<table>
<thead>
<tr>
<th>Fit Statistic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stationary R-squared</td>
<td>.587</td>
</tr>
<tr>
<td>R-squared</td>
<td>.067</td>
</tr>
<tr>
<td>RMSE</td>
<td>31.836</td>
</tr>
<tr>
<td>MAPE</td>
<td>7.766</td>
</tr>
<tr>
<td>MaxAPE</td>
<td>80.056</td>
</tr>
<tr>
<td>MAE</td>
<td>18.033</td>
</tr>
<tr>
<td>MaxAE</td>
<td>131.997</td>
</tr>
<tr>
<td>Normalized BIC</td>
<td>7.123</td>
</tr>
</tbody>
</table>

Table 4: Model Statistics for Month 3-4

<table>
<thead>
<tr>
<th>Model</th>
<th>Number of Predictors</th>
<th>Model Fit Statistics</th>
<th>Ljung-Box Q(18)</th>
<th>Number of Outliers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Stationary R-squared</td>
<td>Statistics</td>
<td>DF</td>
</tr>
<tr>
<td>Transfer Function Model</td>
<td>1</td>
<td>.712</td>
<td>9.592</td>
<td>17</td>
</tr>
</tbody>
</table>

From August to November 2012 operations at Shell Forcados Power Terminal, we obtained:

\[
\gamma_{\alpha\beta}(0) = 0.954
\]

\[
S_{\beta} = 33.57
\]

\[
S_{\alpha} = 30.08
\]

Hence,

\[
v = \frac{0.954 \times 33.57}{30.08} = 1.065
\]

\[
\omega(0) = 1.065
\]

Hence from equation (22)

\[
y(t) = 1.065x_t
\]

Since \(\omega(0) = 1.065\) for the month 3-4 operation of the Power station, the transfer function is given by:

\[
\hat{Y} = Y + 1.065x_t
\]

Table 5: Transfer Function Models of the Power Station

<table>
<thead>
<tr>
<th>Months</th>
<th>Transfer Function Model (v(B))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>(\hat{y} = \mu + 0.67x_t)</td>
</tr>
<tr>
<td>3-4</td>
<td>(\hat{y} = Y + 1.065x_{t-1})</td>
</tr>
</tbody>
</table>

Table 6: Energy Transformed Vs Coefficient of Performance of the Power Station

<table>
<thead>
<tr>
<th>Months</th>
<th>Total Energy Output (MWH)</th>
<th>Coefficient of Performance (\omega(0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>401.32</td>
<td>0.670</td>
</tr>
<tr>
<td>3-4</td>
<td>432.73</td>
<td>1.065</td>
</tr>
</tbody>
</table>
V. Discussion

The transfer function parameter \( \omega_0 \) is a measure of how effective the available gas is converted to electric energy, and could be regarded as the coefficient of performance of the Power Station’s yearly operations. The higher the value of \( \omega_0 \), the more efficient is the power distribution facility and the lower the value of \( \omega_0 \); the power distribution facility is less effective in transforming the input power to suitable output state. Hence, \( \omega_0 \) is analogous to the intercept \( m \) of the equation of a straight-line. The results indicate that the month 3-4 had the highest coefficient of performance (COP) in the 4-month sample studied. On the other hand the month 1-2 had the least COP. As shown in Table 3.10, the value of \( \omega_0 \) in the month 1-2 was 0.670 while the value was 1.065 in the month 3-4.

VI. Conclusion

In this research study, the relationship between the input and output of a single-input-single-output electrical power distribution system was analyzed using the transfer function modeling technique. From the analysis carried out, the system performance of transformer (TR-102) was more effective in the month 3-4 than in the month 1-2. This is in conformity with the theoretical proposal that the transfer function parameters could be used as performance indicators. The lower energy output in the month 1-2 was partly because operations efficiency was poorer in those months when compared to the month 3-4.

References Références Referencias
