# Lateral-Torsional Buckling of FRPI-Section Beams

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## Abstract
This paper presents the outcome of an experimental and theoretical investigation into the loadcarrying capacity of Fiber Reinforced Polymer (FRP) I-section beams subjected to four-point loading. The overall lateral-torsional buckling, web and flange local buckling as well as material rupture load estimates are also made using the American Society of Civil Engineers? Load and Resistance Factor Design (ASCELRFD) Pre-Standard for FRP Structures. Lateral-torsional buckling failure mode is found to govern for each of the beams studied. The study also revealed that the height of applied loads relative to the shear center has a very significant influence on lateral-torsional buckling load of a beam thus making ASCELRFD buckling load estimates over-conservative in a vareity of cases.

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***Index terms***— lateral-torional buckling, I-section FRP, ASCE-LFRD pre-standard for FRP structures.

# 1 I. Introduction

Fiber-Reinforced Polymer (FRP) beam subjected to inplane bending moments about its crosssectional strong axis can develop lateral-torsional buckling. Such a beam will initially deflect normal to the strong axis until the critical value of the bending moment is reached after which lateral and torsional deflections develop. Mamadou and Razzaq [1] investigated the failure modes for I-section Glass Fiber Reinforced Polymer (GFRP) beams with single mid-span web brace in which theoretical predictions were made based on ASCE-LFRD Pre-Standard for Pultruded Fiber Reinforced Polymer (FRP) Structures [2]. It was found that for small and medium I-sections, lateral-torsional buckling failure mode governed while the larger Isections reached their peak capacity associated with material rupture.

Sirjani, Bondi and Razzaq [3] presented the outcome of an experimental and theoretical study on FRP beams with an I-shaped cross section subjected to four-point loading with and without applied torsion. The focus of that study was to identify the significance of lateral bending and warpingstrains due to practical imperfections.

The present paper addresses the influence of vertical location of applied loads with respect to the shear center when estimating the beam lateral-torional buckling strength. Three different applied load locations are considered, namely, when the loads act above, below and at the shear center. In addition, load-carrying capacity predictions are made for various failure modes using the ASCE-LRFD Pre-Standard, and the buckling load estimates compared to those observed experimentally as well as obtained using the buckling formula presented by Razzaq, Prabhakaran, and Sirjani [4].

# 2 II. Experimental Study

Figure ?? shows a FRP beam of length L with an I-shaped cross section, and subjected to a pair of gradually increasing applied loads each of magnitude P. Figure 2 shows the experimental test setup. The beam ends were simply supported both flexurally and torsionally. The test procedure, Lateral-Torsional Buckling of FRPI-Section Beams

The experimental and theoretical maximum loads P e and P t , respectively, are presented in Table 1 in addition to their ratios for a 4x2x0.25 in. I-shaped FRP cross section withlength L equal to 60, 72, 84, 96 and 108 inches, respectively. The value of (L -2a), that is, the distance between the two applied loads P and P shown in Figure ?? was kept constant at 24 inches. The Young's (E 11 ) and shear (G 12 ) modulus values of the FRP beam material were 2,550 ksi and 420 ksi, respectively.

Figure 3, shows the applied loading mechanism in which a pair of steel tie rods are used to apply upward vertical load (P/2 per tie rod) placed symmetrically about the shear center, S. the resultant load P is transmitted to a steel bar which pushes a steel shaft against an aluminum loading plate mounted on to the FRP beam. The resultant force P acts at a distance y o * below the xaxis but passes through S. The value of y o * defines the vertical location of the applied loads. It should be noted that the downward load pair (P, P) shown in Figure ?? was applied in the upward direction by means of two separate sets of the loading mechanism schematically depicted in Figure 3.

# 3 III. Theoretical Study and Results

For the beam shown in Figure ??, the lateraltorsional buckling load P cr can be found using the following formula presented by Razzaq, Prabhakaran, and Sirjani [4]:[ ] 1 3 1 2 2 2 4 5 . 0 f f f f f P cr + + ? = (1)

in which:2 2 2 2 1 ) ( 2 ) ( 16 1 ? ? ? ? ? ? ? ? ? = a g l a L a a f f ? ? (2) ? ? ? ? ? ? ? = L a y L I E f y ? ? 2 * 0 3 11 4 2 sin 4 (3) ? ? ? ? ? ? ? + = T w y K G L I E L I E f 12 2 11 2 4 11 6 3 16 ? ? (4) ? ? ? ? ? ? ? ? ? ? ? ? ? ? = L a L a L a a f ? ? ? 2 sin 2 sin ) ( (5) ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? = L a L a a g 2 1 sin 2 1 2 1 ) ( ? ? (6)

In the above expressions, the distances a and L are defined in Figure ??; I y is the minor-axis moment of inertia; K T is the St. Venant torsional constant; and I w is the warping moment of inertia of the cross section.

Table 2 presents the ASCE-LRFD theortical maximum load values with aresistancefactor of ø =0.80for flange or web local buckling, and ø =0.65 for rupture load. Also, this table presents the moment modification factorC b for unsupported spans with both ends braced corresponding to various L values of the beam shown in Figure 1 with (L -2a) kept constant at 24 inches. Table ?? pesents the critical load results for different distance y o * of applied load about the shear center. The last three columns in Table ?? present the load ratios r 1 , r 2 , and r 3 defined as P LT divided by P cr correspoinding to y o * = -2.00 in., 0.0 in., and +2.0 in., respectively.

# 4 Table 3: Critical Load for various applied load through shear center

# 5 IV. Conclusions

Expeimental results are in good agreement with the lateral-torsional buckling load formula presented [4]. Theoretical predications for various beam failure modes are also made using ASCE-LRFD Pre-Standard for FRP Structures. It is found that in all of the cases presented, the I-section beam failure mode was governed by lateral-torsional buckling. The study also clearly reveals that the height of the applied loads relative to the shear center has a very significant influence on the lateraltorsional buckling load of the beam thus making ASCE-LRFD buckling load estimates over-conservative in a number of cases. There are four nominal moments that are calculated based on the formulae [2] as summarized here. The nominal bending moment ?? ???? due to lateraltorsional buckling is given by:

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?? ???? = ?? ?? ? ?? 2 ?? ??,ð ??"ð ??"?? ?? ?? ?? ?? ?? 2 + ?? 4 ?? ??,ð ??"ð ??" 2 ?? ?? ?? ?? ?? ?? 4**(3)**

A resistance factor ? = 0.7 is used for ?? ???? . The other terms are defined as follows: ?? ?? = Moment modification factor for unsupported spans with both ends braced ?? ?? = Torsional rigidity of an open section =?? ???? ?

If ?? ???? = ?? ð ??"ð ??"???? = ?? ?????? = ?? ð ??"ð ??"ð ??"ð ??" = ?? ð ??"ð ??" is the loadcarrying capacity of the member, a LFRD approach is proposed as follows:

?? ð ??"ð ??" = ??? ?? (10) in which ? = 0.7, 0.8, and 0.65 depending whether the failure is due to lateral torsional buckling, local instability in the flanges and webs, and rupture of the materials.

The C b values in Table 2 were computed using the following expression: C b = 12.5M max /(2.5M max + 3M A + 4M B + 3M C ) ??11) in which Mmax is the maximum bending moment, and M A , M B , and M C are the values of quarter-point moments along the beam length.

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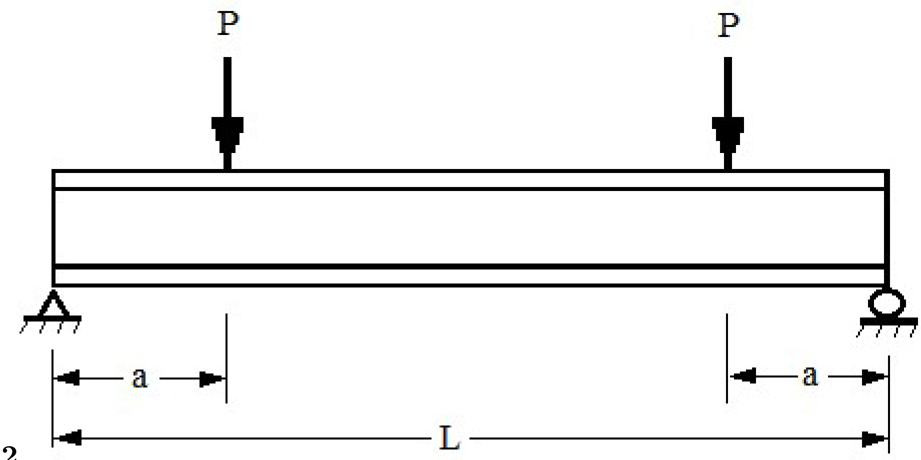


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Figure 1: Fig. 1 :Fig. 2 :

**3**

Figure 2: Figure 3 :

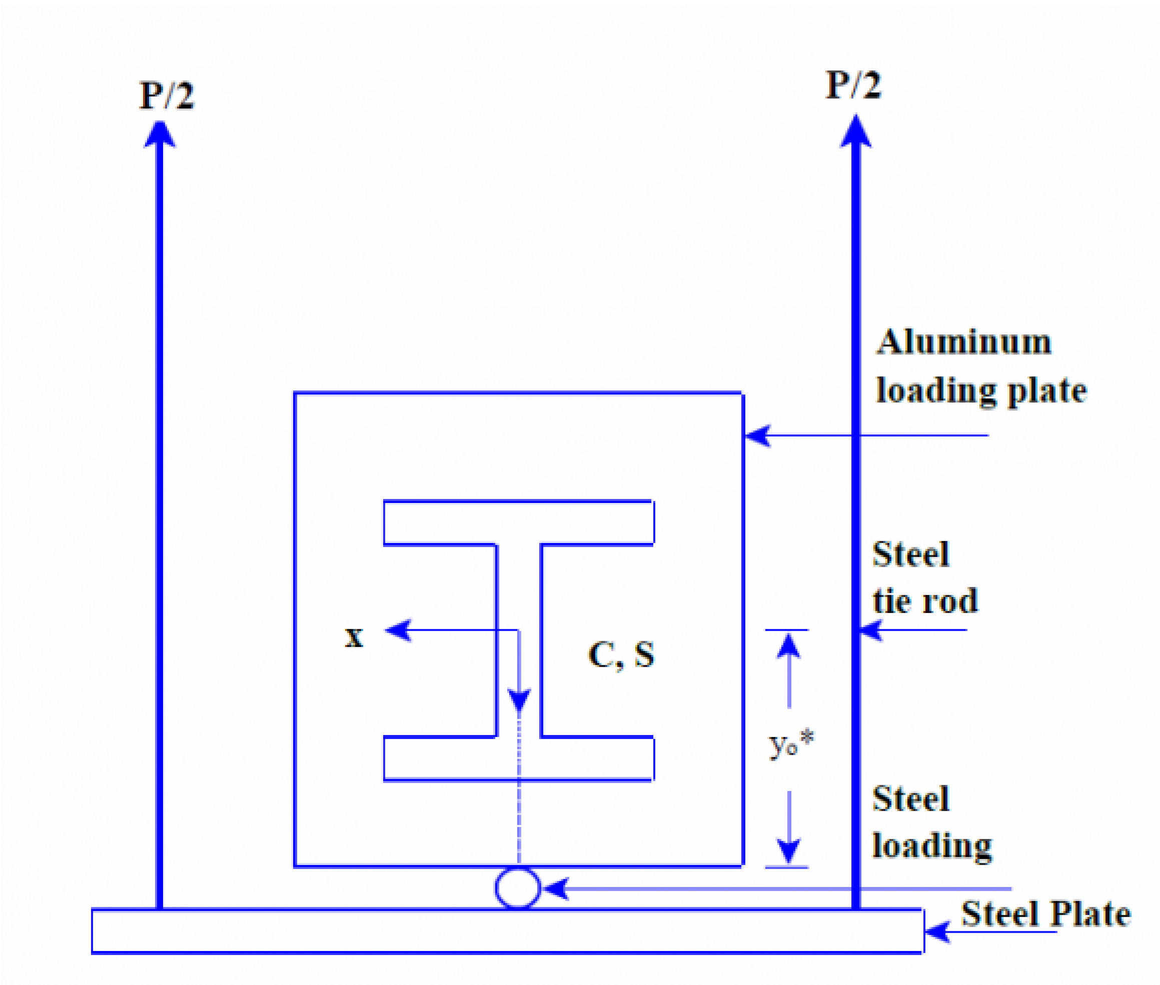


Figure 3:

**1**

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| L (in.) | P e (Lb.) (Experimental) | P t (Lb.) (Theoretical) | |
|---|---|---|---|
| 60 | 292 | 340 | 1.164 |
| 72 | 190 | 214 | 1.126 |
| 84 | 125 | 150 | 1.200 |
| 96 | 111 | 112 | 1.009 |
| 108 | 77 | 88 | 1.143 |



Figure 4: Table 1 :

**2**

| | | Based on ASCE-LRFD | | | |
|---|---|---|---|---|---|
| L | | P LT | øP fcr | øP wcr | øP rupture |
| (in.) | C b | (Lb.) | (Lb.) | (lb.) | (lb.) |
| 60 | 1.087 | 468 | 43749 | 13626 | 2057 |
| 72 | 1.136 | 288 | 43749 | 13626 | 1543 |
| 84 | 1.168 | 195 | 43749 | 13626 | 1244 |
| 96 | 1.190 | 141 | 43749 | 13626 | 1028 |
| 108 1.207 | | 107 | 43749 | 13626 | 881 |

Figure 5: Table 2 :

## .1 Appendix

This appendix summarizes the ASCE-LRFD Pre-Standard expressions used in arriving at those particular numerical results which were based on the ASCE-LRFD Pre-Standard [2]. The critical stress for the compression flange local buckling is given by:

## .2 This page is intentionally left blank

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