Global Journals LaTeX JournalKaleidoscope™

Artificial Intelligence formulated this projection for compatibility purposes from the original article published at Global Journals. However, this technology is currently in beta. *Therefore, kindly ignore odd layouts, missed formulae, text, tables, or figures.*

Fuzzy Goal Programming Method for SolvingMulti-Objective Transportation Problems

S G Acharyulu[1] and Dr. K V V Chandra Mouli[2]

[1] GITAM Institute of Technology, GITAM University

Received: 13 January 2011 Accepted: 3 February 2011 Published: 15 February 2011

Abstract

The multi-objective transportation problem refers to a special class of vector minimum linear programming problem, in which constraints are of inequality type and all the objectives are non-commensurable and conflict with each other. A common problem encountered in solving such multi-objective problems is that to identify a compromise solution among a large number of non-dominated solutions, the decision maker has to develop a utility function for meeting the desired goal. In this paper, fuzzy membership functions are considered and deviation goals also taken for each objective function. Fuzzy max-min operator is implemented to show the effectiveness of the proposed methodology. LINGO software package is used to solve constrained optimization problem. To illustrate the proposed method, two numerical examples are solved and the results have been compared with interactive, fuzzy and deviation criterion approaches.

Index terms—

1 INTRODUCTION

he classical transportation problem is one of the sub classes of linear programming problem in which all constraints are inequality type. Hitchcock (1941) developed transportation model. Because of the complexity of the social and economic environment requires explicit consideration of criteria other than cost, the single objective transportation problems in real world cases can be formulated as multi-objective models. Charnes and Cooper (1961) first discussed on various approaches to solutions of managerial level problems involving multiple conflicting objectives. Ignizio (1978) applied goal programming for multiobjective optimization problems and solved twoobjective optimization problem. Some of the authors (see Garfinkl & Rao 1971; Swaroop et al., 1976) have solved the two objective problem by giving high and low priorities to the objectives. Belenson and Kapur (1973) presented two person-zero sum game approach consists of a p x p pay off matrix and solved each objective function individually finally developed best compromise solution using proper weights to the objective functions. Jimmenez and Vudegay (1999) solved a multi-criteria transportation problem using parametric approach by developing auxiliary solutions. Rakesh Varma et al., (1997) used fuzzy min operator approach to develop a compromise solution for the multi-objective problem. Ringuest and Rinks (1987) proposed two interactive algorithms for generating all non-dominated solutions and identified minimum cost solution as a best compromise solution. Gen et al., (1998) solved a bi-criteria transportation problem using hybrid genetic algorithm adopting spanning tree based prufer number to generate all possible basic solutions. Waiel. (**??**001) developed all non-dominated solutions and defined family of distance function to arrive a compromise solution.

The existing procedures in the literature (see Deb, 2003;Rao, 2003) for solving multi-objective transportation problems can be divided into two categories. First category of those are generating all the sets of efficient solutions (see Ringuest and Rinks, 1987;Gen et al., 1997) and the second category represents the procedure of using an additional criterion to obtain the best compromise solution among the set of efficient solutions (see Rakesh Varma & Biswas, 1997; **??**en et al., 1998;Bit et al., 1992; and Sy-Ming Gun & Yan -Kuen Wu, 1999) developed various functions to achieve direct compromise solution without developing and testing all the Pareto solutions.

Although several researchers have been proposed various advances in transportation problems (see Bit et al.,1993 In this paper, authors propose membership functions and goal deviation functions from Pareto solutions for each objective, and these functions are added as constraints. By introducing a max-min operator ? an auxiliary variable, then the equivalent fuzzy interactive goal programming problem is formulated to maximize ? and the solution is obtained by using LINGO software. The remaining of the paper is organized as follows: in section 2 we give a mathematical model of the multi-objective transportation problem (MOTP) and formulation with fuzzy max-min operator and goal deviations. Section 3 ( ) [ ] ( ) L U X F U X F k k k k k k ? ? ? ? ? = µ

represents proposed methodology; while in section 4 two numerical examples are solved. Finally, in section 5 and 6 we discuss on the results and conclusions.

2 II.

3 MATHEMATICAL MODEL

In a typical transportation problem, a homogenous product is to be transported from several origins (or sources) to numerous destinations in such way that the total transportation cost is minimum. Suppose there are "m" origins (i=1,2,??,m) and "n" destinations (j=1,2,??.,n). The sources may be production facilities, warehouses etc and they are characterized by available supplies a 1 , a 2, ?,a m . The destinations may be warehouses and sales outlets etc, and they are characterized by demand levels b 1 , b 2 ,?.,b n . A penalty c ij is associated with transporting a unit of product from origin i to destination j. The penalty could represent transportation cost, delivery time, distance, quality of goods delivered under used capacity or many other criteria. A variable x ij is used to represent the unknown quantity to be transported from origin O i to destination D j . In the real life, however all transportation problems are not single objective. The transportation problems, which are characterized by multiple objective functions, are considered in this paper. The decision maker would like to minimize the set of K objectives simultaneously; a point will likely be reached where a further reduction of the value of any single objective function may only be obtained at the expense of increasing the value of at least one other objective function. Thus, in general, the objectives will also be conflicting. The mathematical model of the multi-objective transportation problem is written as follows:

(2.1) X C (X) F Min ij ij k n 1 j m 1 i k ? ? = = = subject to the (2.4) j and i all for , 0 x (2.3) n, ..., 1,2,...... j , b x (2.2) m, ..., 1,2,...... i , a x ij j m 1 i ij i n 1 j ij ? = = = ? ? ? = = Where, { } (X) F ....., ... (X), F (X), F (X) F k 2 1 k

4 =

is a vector of K objective functions and superscript on both F k (X) and C k ij are used to identify the number of objective functions (k=1,2,?.,K) without loss of generality it will be assumed in the whole paper that a i ?0 and b j ?0 for all i and j and ? i a i =? j b j , c ij >0 for all i and j.

5 a) Problem Formulation Using Fuzzy Max-Min Operator

Fuzzy set theory appears to be an ideal approach to deal with decision problems that are formulated as linear programming models with imprecision parameters. Two face fuzzy linear programming models are designated by Sy-Ming Gun & Yan-Kuen Wu (1999) for such problems. In the literature fuzzy linear programming has been classified into different categories, depending on how imprecise parameters are modeled by possibility distributions or subjective preference based membership functions. In this paper the net relative deviation is considered as fuzzy variable and converted into deterministic form using Zadeh's max-min operator as per Zimmermann (1985). We define a linear membership function by considering suitable upper and lower bounds to the objective function as given below.µ[F k (X)] = 1, if F k (X) ? U k ,if Lk<Fk (X) <Uk (2.5) µ[F k (X)] = 0 otherwise

By introducing a max-min operator ? an auxiliary variable, then, the equivalent fuzzy linear programming problem is as follows.

[X*] = Maximize ? (0? ? ? 1) where ? ? minimum ?k [Fk(X)], k=1,2,???.,K subject to the constraints (2.2) ?. **??**2.4) where, ?k [Fk(X)] is membership of the kth objective function and Lk,Uk are its lower and upper bound solutions.

6 b) Goal Deviations

The goals for each objective are considered for each objective functions namely under achievement and over achievement goals. Initially, the upper and lower bounds for each objective functions are estimated and then the goals are included as by adding the under achievement and removing the over achievement for each objective on the left hand side of the objectives as variables. After setting goals, an overall fuzzy operator ? has been introduced to identify the minimum value for each function and maximizing it subject to the constraints as per Zimmermann (1985).

7 III.

8 PROPOSED METHOD

For solving MOTP, the proposed method is summarized in the following steps) [ ] ( ) L U X F U X F k k k k k ? ? ? ? ? ? ? ? = µ

Step 3 (Membership function): Based on the interaction approach by Waiel (2001) between lower bound and upper bounds L k and U k of the K th objective function, membership functions are estimated for all the objective functions [F k (X)], (k=1,2,?,K) as follows µ[F k (X)] = 1, if F k (X) ? U k , if L k < F k (X) < U k (2.5) µ[F k (X)] = 0 otherwise

Step 4. Developing a goal deviation function by setting goals (over achievement and under achievement) for each objective based on the upper bounds (U k) and lower bounds (L k) add these goal deviation functions as constraints.

Step 5. By introducing a max-min operator ? an auxiliary variable, then the equivalent fuzzy linear goal programming problem is as follows.[X*] = Maximize ? (0 ? ? ? 1) where ? ? Minimum ? k [F k (X)] , k = 1,2??..,K. subject to the constraints (2.2) -(2.4).

Here, [F k (X)] is membership of the k th objective function and L k and U k are the lower and upper bounds for each objective function F k (X) (k=1,2,??,K).

9 IV.

10 ILLUSTRATIVE EXAMPLES

To illustrate the proposed method, consider the following two examples of MOTP taken from Ringuest and Rinks (1987). (2.4) [X 1*] = [0,0,5,0,0, 0,3,1,0,0, 1,1,0,0,0, 3,0,0,2,4] [X 2*] = [3,0,0,2,0, 1,0,0,0,4, 0,2,0,0,0, 1,2,6,0,0] [X 3*] = [3,2,0,0,0, 1,0,3,0,0, 0,2,0,0,0, 0,0,3,2,4] F 1 [X 1*] = 102 F 2 [X 2*] = 73 and F 3 [X 3*] = 64

(2) Determine k objective functions (k Pareto solutions, where k=1,2, ?.,K). Identify the its lower and upper bounds as L k and U k. F 1 [X 1] = 102, F 1 [X 2] = 164, and F 1 [X 3] = 134; hence, lower limit L 1 =102 and upper limitU 1 =164. F 2 [X 1] = 141, F 2 [X 2] = 73 and F 2 [X 3] = 122; hence, L 2 =73 and U 2 =141. F 3 [X 1] = 94, F 3 [X 2]

= 90 and F 3 [X 3] = 64; hence, L 3 =64 and U 3 =94.

(3) The membership function of F 1 (X), F 2 (X) and F 3 (X) are determined as follows( ) [ ] ( ) 102 164 X F 164 X F 1 1 1 ? ? µ ( ) [ ] ( ) 73 141 X F 141 X F 2 2 2 ? ? µ ( ) [ ] ( ) 64 94 X F 94 X F 3 3 3 ? ? µ (4)

The goal deviation functions of F 1 (X), F 2 (X) and F 3 (X) are determined as follows. The solution obtained as [X*] = [3,0,0,2,0,0,2,2,0,0,0,2,0,0,0,1,0,4,0,4] and ? = 0.54 The corresponding objective functions values are F 1 1) As the first step the ideal solutions obtained by solving of each objective function is F 1 (X 1 *) =143 and F 2 (X 2 *) =167 2) Determination of Pareto solutions For each objective function the corresponding Parato solutions at each feasible ideal solution and lower and upper bounds are obtained as follows:F 1 (X) + d 1 + -d 2 -? 164 F 2 (X) + d 3 + -d 4 -? 141 F 3 (X) + d 5 + -

F 1 (X 1) =143 and F 1 (X 2) = 208 hence, lower limit L 1 =143 and upper limit U 1 =208 F 2 (X 1) =265 and F 2 (X 2) = 167 hence, lower limit L 2 =167 and upper limit U 2 =265

3) The membership functions of F 1 (X) and F 2 (X) are determined as follows:( ) [ ] ( ) 143 208 X F 208 X F 1 1 1 ? ? µ ( ) [ ] ( ) 167 265 X F 265 X F 2 2 2 ? ? µ 4)

The goal deviation functions of F 1 (X) and F 2 (X) are F 1 (X) + d 1 + -d 2 -? 208 F 2 (X) + d 3 + -d 4 -?

11 RESULTS AND DISCUSSION

The work reported here for solving MOTP, results a compromise solution in five steps. Initially, a feasible ideal solution is obtained for each objective function, using these feasible solutions, upper and lower bounds values are identified for each objective function. From the upper and lower bounds, membership functions are estimated. Goal deviations are included for each membership functions by introducing under achievement and over achievement variables. By introducing a max-min operator ? an auxiliary variable, then an equivalent fuzzy linear goal programming is formulated and the solution obtained using LINGO software. The feasible ideal solutions obtained for the proposed method is exactly similar to the exiting methods in the literature (see Ringuest & Rinks, 1987;Waiel, 2001;Mouli et al., 2005), and the solutions obtained are compared with those in the literature.

For the example 1, the solution obtained using the proposed method as ?F(X *) = 307 (127,104 and 76) with overall satisfaction level of 0.54. This shows (Table 1), proposed method results exactly similar to the solutions obtained with interactive approach proposed by Ringuest and Rinks (1987), and net deviation approach proposed by Mouli et al., (2005) and better solution than the fuzzy approach proposed by Waiel (2001). For the numerical example 2, the solution for the proposed method obtained as ?F(X *) = 355 (160 and 195) with overall satisfaction level of 0.71. This indicates the solution obtained is much more superior to the existing interactive, net deviation and fuzzy approaches. Also, the fuzzy approach results ?F(X *) = 360 (170 and 190) with 7 number of allocations. The proposed approach generates the same number of allocations with much improved value at ?F(X *) = 355 (160 and 195). VI.

12 CONCLUSION

A common problem encountered in solving multi-objective optimization problems is that the decision maker has to identify a problem dependent compromise function among a large number of nondominated solutions. For the past 20 years, although many researchers have investigated compromise functions, there is still no compromise function among them is generating an optimal solution for all types of problems. In the absence of exact method for solving multi-objective transportation problems a reasonable method has some value. In this paper, a fuzzy goal deviation criterion is developed to determine compromise solution. The effectiveness of the proposed method is tested with fuzzy max-min operator and solved using LINGO software. Two numerical examples are presented and obtained results are compared with those reported in the literature. The results shows a great promise in developing an efficient solution for solving multi-objective optimization problems and this can be extended for all engineering applications in future to achieve global solution.

Fuzzy numbers. Computers and Industrial Engineering. 33 (3-4), 589-592. Computers and Industrial Engineering. 35 (1-2), 363-366. [1] [2] [3] [4]

Figure 1:

[1]

[2]Fuzzy Goal Programming Method for Solving Multi-Objective Transportation Problems

[3]Fuzzy Goal Programming Method for Solving Multi-Objective Transportation Problems

[4]Fuzzy Goal Programming Method for Solving Multi-Objective Transportation Problems

1

Here,d_1^+ and d_3^+ are over achievements and d_2^- and d_4^- -are under achievements of each function of $F_1(X)$ and $F_2(X)$ respec

Hence, the problem is written as follows

Maximize ? (x_{17})

subject to

$x_1 + x_2 + x_3 + x_4 = 8$

$x_5 + x_6 + x_7 + x_8 = 19$

$x_9 + x_{10} + x_{11} + x_{12} = 17$

$x_1 + x_5 + x_9 = 11$

$x_2 + x_6 + x_{10} = 3$

$x_3 + x_7 + x_{11} = 14$

$x_4 + x_8 + x_{12} = 16$

$x_1 + 2x_2 + 7x_3 + 7x_4 + x_5 + 9x_6 + 3x_7 + 4x_8 + 8x_9 + 9x_{10} + 4x_{11} + 6x_{12} + x_{13} - x_{14}$? 208

$4x_1 + 4x_2 + 3x_3 + 4x_4 + 5x_5 + 8x_6 + 9x_7 + 10x_8 + 6x_9 + 2x_{10} + 5x_{11} + x_{12} + x_{15} - x_{16}$? 265

Simplifying the above two constraints,

$0.48x_1 + 0.96x_2 + 3.37x_3 + 3.37x_4 + 0.48x_5 + 4.33x_6 + 1.44x_7 + 1.92x_8 + 3.85x_9 + 4.33x_{10} + 1.92x_{11} + 2.88x_{12} + 31.25x_{17}$? 100

$1.51x_1 + 1.51x_2 + 1.132x_3 + 1.509x_4 + 1.887x_5 + 3.018x_6 + 3.396x_7 + 3.77x_8 + 2.26x_9 + 0.75x_{10} + 1.886x_{11} + 0.377x_{12} + 36.98x_{17}$? 100

where, all x_i ? 0 and integers (i =1,2,??..,16) and x_{17} ? 1

The solution obtained as

[X*] = [4,3,1,0,7,0,12,0,0,0,1,16] and ? = 0.71

The corresponding objective functions values are $F_1(X^*)$ =160 and $F_2(X^*)$ =195.

Figure 2: Table 1 :

2

Ideal solution	Proposed Method	Net Deviation Approach Mouli et al., (2005)	Interactive Approach Ringuest and Rinks (1987)	Fuzzy Approach Waiel (2001)
F 1 [X] 143	160	186	186	170
F 2 [X] 167	195	171	174	190
? F[X]	355	357	360	360
? V.	0.71	0.7		

Figure 3: Table 2 :

[Mouli et al. ()] , Chandra K V V Mouli , K Venkatasubbaiah , K M Rao . 2005.

[Lau et al. ()] *A Fuzzy guided Multi-objective evolutionary algorithm model for solving Transportation problem*, H C W Lau , T M Chan , W T Tsui , F T S Chan , G T Ho , S Choy , KL . 2009.

[Waiel ()] *A Multi-objective transportation problem under fuzziness. Fuzzy Sets and Systems*, Waiel . 2001. 117 p. .

[Bit et al. ()] *An additive Fuzzy programming model for Multiobjective transportation problem. Fuzzy Sets and Systems*, A K Bit , M P Bismal , S S Alam . 1993. 57 p. .

[An efficient method for solving Multi-objective transportation problems Journal of the Computer Society of India] 'An efficient method for solving Multi-objective transportation problems'. *Journal of the Computer Society of India* 35 p. .

[Zimmermann ()] 'Application of Fuzzy Set theory to Mathematical Programming'. H J Zimmermann . *Information Sciences* 1985. 36 p. .

[Rao ()] *Engineering optimization theory and practice, New age International*, Singiresu S Rao . 2003. India.

[Expert Systems and Applications] *Expert Systems and Applications*, 36 p. .

[Sinha et al. ()] *Fuzzy programming approach to Mulit-objective Stochastic programming problems when b i ' s follow Joint Normal Distribution. Fuzzy Sets and Systems*, S B Sinha , S Hulsurkar , M P Biswal . 2000. 109 p. .

[Bit et al. ()] *Fuzzy programming approach to Multi criteria decision making transportation problem. Fuzzy sets and systems*, A K Bit , M P Biswal , S S Alam . 1992. p. .

[Hulsurkar et al. ()] *Fuzzy programming approach to Multi-objective Stochastic linear programming problems. Fuzzy Sets and Systems*, S Hulsurkar , M P Biswal , S B Sinha . 1997. 88 p. .

[Varma et al. ()] *Fuzzy programming technique to solve Multi-objective transportation problems with some Non-linear membership functions. Fuzzy Sets and Systems*, Rakesh Varma , M P Biswal , A Biswas . 1997. 91 p. .

[Ignizio ()] 'Goal programming: A tool for multi-objective analysis'. J P Ignizio . *Journal of Operations Research Society* 1978. 29 p. .

[Gen et al. ()] *Improved Genetic algorithm for solving Multiobjective solid transportation problems with*, M Gen , K Ida , Y Li . 1997.

[Ringuest and Rinks ()] 'Interactive solutions for the linear multi-objective transportation problem'. J L Ringuest , D B Rinks . *European Journal of Operations Research* 1987. 32 p. .

[Belenson and Kapur ()] 'Linear programming problems'. S M Belenson , K C Kapur . *Operations research* 1973. 24 (1) p. .

[Charnes and Cooper ()] *Management models and industrial applications of linear programming*, A Charnes , W W Cooper . 1961. Wiely, NY.

[Deb ()] *Multi objective optimization using evolutionary algorithms*, K Deb . 2003. Singapore: John Wiley & Sons.

[Pramanik Senapati. Roy Tapan Kumar. ()] 'Multi objective transportation model into Fuzzy parameters: Priority based Fuzzy Goal programming approach'. *Journal of Transportation systems Engineering and Information Technology* Pramanik Senapati. & Roy Tapan Kumar. (ed.) 2008. 8 (3) p. .

[Kyung Sam Park. and Kwan Jae Kim. ()] 'Optimizing Multi Response Surface Problems: How to use Multi-Objective Optimization Techniques'. *IIE Transactions* Kyung Sam Park. and Kwan Jae Kim. (ed.) 2005. 37 p. .

[Zinmenoz and Vudegay ()] 'Solving fuzzy solid transportation problem by an evolutionary algorithm based parametric approach'. F Zinmenoz , J L Vudegay . *European Journal of Operations Research* 1999. 117 p. .

[Garfinkl and Rao ()] 'The bottleneck transportation problems'. R S Garfinkl , M R Rao . *Naval Research Logistics Quarterly* 1971. 18 p. .

[Hitchcock ()] 'The distribution of several sources to numerous localities'. F L Hitchcock . *Mathematical Physics* 1941. 20 p. .

[Swaroop et al. ()] 'Time cost trade of in a transportation problem'. K Swaroop , H L Bhatia , M C Puri . *Opsearch* 1976. 13 p. .

[Sy-Ming Gun Yan-Kuen Wu ()] *Two-face approach for solving the Fuzzy linear programming problems. Fuzzy Sets and Systems*, Sy-Ming Gun & Yan-Kuen Wu . 1999. 107 p. .