



GLOBAL JOURNAL OF RESEARCHES IN ENGINEERING: J  
GENERAL ENGINEERING

Volume 16 Issue 2 Version 1.0 Year 2016

Type: Double Blind Peer Reviewed International Research Journal

Publisher: Global Journals Inc. (USA)

Online ISSN: 2249-4596 Print ISSN:0975-5861

# Hyperbolic Metamaterial Interface: Propagation of Surface Waves

By T. Gric, M. Cada & J. Pistora

*Dalhousie University, Canada*

**Abstract-** Herein we study propagation of surface waves at a boundary of an isotropic media and a multilayered hyperbolic metamaterial. The structure dispersion is discussed for various cases of a hyperbolic metamaterial. It is demonstrated that it is possible to tune the frequency range of surface waves by varying the thickness of dielectric sheets. It is also shown that this frequency range can be broadened by decreasing the thickness of the dielectric in the metal-dielectric compound or by replacing the isotropic media with a metal. It is also shown that the mentioned effect can be achieved by increasing the doping concentration of the semiconductor if the metamaterial/semiconductor structure is considered. The effective medium approximation is used and its validity in the long-wavelength limit is justified by investigating the electromagnetic field variations over one period of the proposed structure.

**Keywords:** surface wave; metamaterial; drude metal; surface plasmons; terahertz.

**GJRE-J Classification :** FOR Code: 020301



*Strictly as per the compliance and regulations of :*



# Hyperbolic Metamaterial Interface: Propagation of Surface Waves

T. Gric<sup>α</sup>, M. Cada<sup>σ</sup> & J. Pistora<sup>ρ</sup>

**Abstract-** Herein we study propagation of surface waves at a boundary of an isotropic media and a multilayered hyperbolic metamaterial. The structure dispersion is discussed for various cases of a hyperbolic metamaterial. It is demonstrated that it is possible to tune the frequency range of surface waves by varying the thickness of dielectric sheets. It is also shown that this frequency range can be broadened by decreasing the thickness of the dielectric in the metal-dielectric compound or by replacing the isotropic media with a metal. It is also shown that the mentioned effect can be achieved by increasing the doping concentration of the semiconductor if the metamaterial/semiconductor structure is considered. The effective medium approximation is used and its validity in the long-wavelength limit is justified by investigating the electromagnetic field variations over one period of the proposed structure.

**Keywords:** surface wave; metamaterial; drude metal; surface plasmons; terahertz.

## I. INTRODUCTION

The waves at an interface between an isotropic medium and a uniaxial crystal characterized by its effective permittivities have been first demonstrated in [1]. The existence of these surface waves in some material examples is studied in [2], discussing the challenge posed by their experimental observation. The novelty and potential importance of Dyakonov waves for integrated optics applications were described in a stream of papers [3-5].

In near-infrared and visible wavelengths, the behavior of nanolayered metal-dielectric (MD) compounds is similar to plasmonic crystals. In this case a simplified description of the medium by using the long-wavelength approximation can be justified; the homogenization of the structured metamaterial can also be employed [6-8]. It is interesting to note that the second-rank tensor representing the medium permittivity includes elements of opposite signs, thus yielding extremely anisotropic metamaterials under certain conditions [9, 10]. This category of nanostructured media opens the wide avenues for the practical applications from biosensing to fluorescence engineering [11].

*Author α:* Dalhousie University, Halifax, Canada, Vilnius Gediminas Technical University, Vilnius, Lithuania.

*e-mails:* Tatjana.Gric@dal.ca, tatjana.gric@vgtu.lt

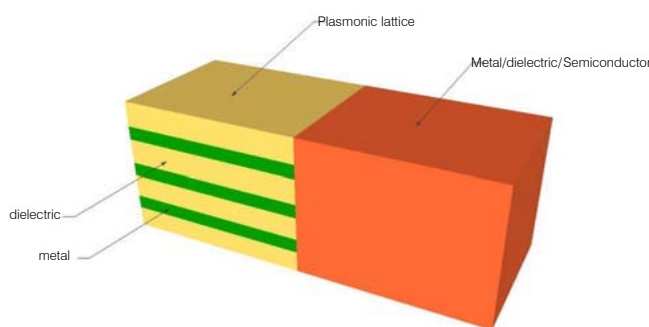
*Author σ:* Dalhousie University, Halifax, Canada Nanotechnology Centre, Technical University of Ostrava, the Czech Republic.

*Author ρ:* Nanotechnology Centre, Technical University of Ostrava, the Czech Republic.

The existence of surface waves when dealing with anisotropic media possessing the indefinite permittivity was reported for the first time in [12]. This study was dedicated mainly to surface waves enabling sub-diffraction imaging in magnifying superlenses [13], where the surface waves exist at an interface between a metal and an all-dielectric birefringent metamaterial. Dealing with hyperbolic media, however, the authors concluded only with an indefinable analysis of surface waves.

In this paper we retake the task and perform a thorough analysis of surface waves traveling in infinite MD lattices. Our approach is to use the effective-medium approximation.

## II. GEOMETRY OF THE PROBLEM



**Fig. 1 :** The model under study consisting of an infinite MD lattice ( $x < 0$ ) and an isotropic material ( $x > 0$ ). The periodic structure contains a Drude metal and a dielectric.

The investigated structure is shown in Fig. 1. It should be mentioned that an infinite periodic model consisting of alternating layers of a metal and a conventional dielectric is placed on the left of the homogeneous dielectric. The effective dielectric tensor components parallel ( $\epsilon_{\parallel}$ ) and perpendicular ( $\epsilon_{\perp}$ ) to the anisotropy axis are described in [14] as:

$$\epsilon_{\perp} = \frac{\epsilon_m d_m + \epsilon_d d_d}{d_m + d_d} \quad (1)$$

$$\frac{1}{\epsilon_{\parallel}} = \frac{d_m / \epsilon_m + d_d / \epsilon_d}{d_m + d_d} \quad (2)$$

where  $d_m$  ( $d_d$ ) is the thickness and  $\epsilon_m$  ( $\epsilon_d$ ) is the dielectric constant of the metallic (dielectric) component. The effective medium approximation in Eqs. (1) and (2) is

valid in the long-wavelength limit. However, the field varies significantly on the scale of one period due the excitation of surface plasmon (SP) polaritons at metal/dielectric interfaces. Therefore, the approximation in Eqs. (1) and (2) may not be applicable in some spectral ranges.

### III. NUMERICAL ANALYSIS OF DISPERSION CHARACTERISTICS

Herein we discuss the numerical solution to the dispersion relation [15], either obtained for metamaterial/air interface or for metamaterial/metal and metamaterial/semiconductor interfaces.

#### a) Metamaterial/Air interface

The dispersion relation [15] is graphically represented in Fig. 2 for the MD crystal displayed in Fig. 1 at  $x < 0$ , considering different widths of the dielectric  $d_d$  and assuming, that we are dealing with metamaterial/air interface.

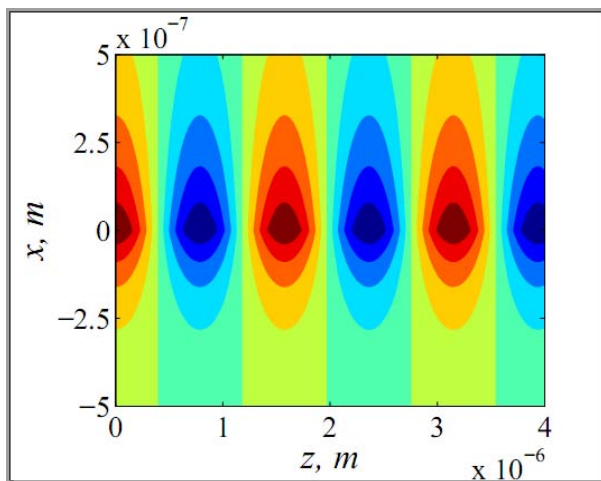


Fig. 2 : Dispersion curves of TM modes at a metamaterial/air interface for different widths of the dielectric layer  $d_d > 0$  (red dots) in a Drude metal/dielectric compound. The light line is shown (dashed line), and  $d_m = 35$  nm.

Fig. 2 shows the dispersion of surface waves at a metamaterial/air interface. The case ( $d_d > 0$ ) corresponds to classic surface waves well known for conductive interfaces [16]. Thus, this example illustrates the verification of our methodology [15]. In addition, the light line in vacuum is plotted in Fig. 2 as dashed line. Our calculated data corresponds to the data of [17] where also a complete discussion of the surface wave characteristics in this case can be found.

As can be seen from Fig. 2, the frequency range for surface waves can be controlled by the fill factors of the air and metal sheets in the MD compound.

If we decrease the thickness of the dielectric  $d_d$ , the dispersion curve moves to a lower frequency. The dependence of the frequency range for the surface

wave existence on the thickness of the dielectric layer provides additional degree of freedom to control surface waves.

Fig. 3 shows the profiles of the magnetic field along the  $x$  axis for the investigated case, i. e. metamaterial/air. The magnetic field has been obtained when  $\beta = 4 \cdot 10^6$  1/m. The wave field is tightly confined near  $x = 0$  for all the thicknesses of the dielectric  $d_d$ .

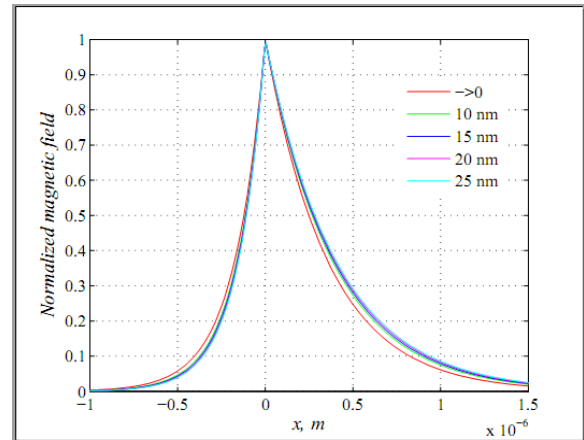


Fig. 3 : Magnetic field (normalized) for different values of parameter  $d_d$  for SP modes at metamaterial/air interface.

The conducted analysis shows, that the frequency range for the surface wave existence seems to be narrow. Thus, we shall leave the geometry unchanged but assume that instead of an interface between a metamaterial and air we will be dealing with an interface between a metamaterial and a metal. Doing so, we can extend the frequency range for the surface wave existence. It should be noticed, that each medium is capable of supporting propagating surface waves separately.

#### b) Metamaterial/Metal interface

The dispersion curves for the matamaterial/metal structure are presented in Fig. 4. The calculations were performed using the following parameters for bulk silver:  $\omega_p = 2297.09$  THz,  $\epsilon_\infty = 5.2$  [18]. As in previous cases we discuss the effects of the thickness of the dielectric  $d_d$  on the dispersion curve. It should be noticed that the upper and the lower limits move to lower frequencies when  $d_d$  is decreased. However the lower limit moves faster than the upper one. The mentioned issue leads to a broader frequency range for the surface wave existence.

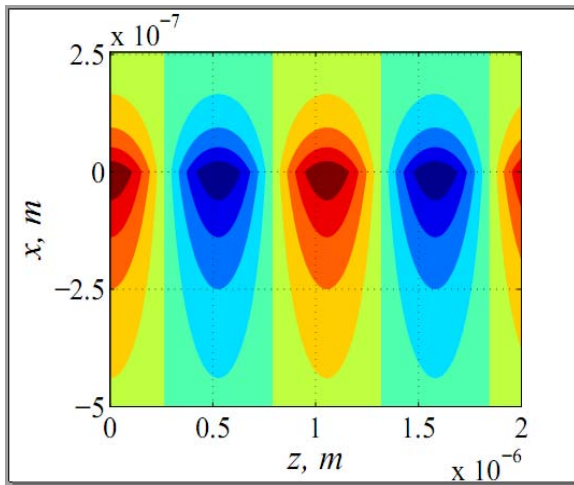


Fig. 4 : Dispersion curves of TM modes at a metamaterial/metal interface for different widths of the dielectric layer  $d_d > 0$  (red dots) in a Drude metal/dielectric compound. The light line is shown (dashed line), and  $d_m = 35$  nm.

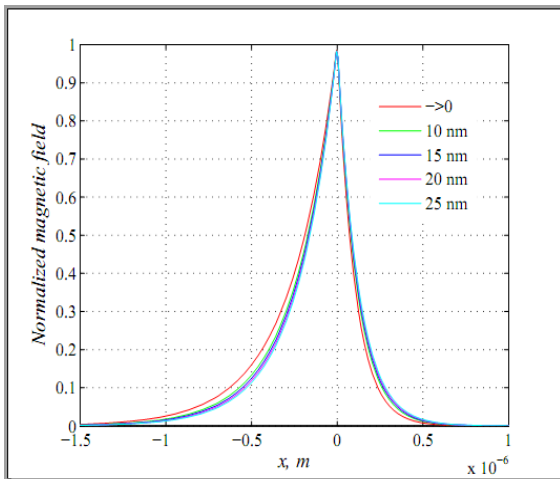


Fig. 5 : Magnetic field (normalized) for different values of parameter  $d_d$  for SP modes at metamaterial/air interface.

Fig. 5 shows the profile of the magnetic field along the  $x$  axis for the case under the study. It is interesting to point out, that  $\beta = 6 \cdot 10^6$  1/m.

It should be mentioned, that the wave field is tightly confined near  $x = 0$  for all the thicknesses of the dielectric  $d_d$ . It is interesting to note that in the case of a metamaterial/air interface (Fig. 3) the tighter confinement near the boundary  $x = 0$  is exhibited in the metamaterial while in the case of a metamaterial/metal interface the tighter confinement is in the metal.

c) *Metamaterial/Semiconductor interface*

It is interesting to notice, that the case of heavy doped Si is considered, assuming that the doping level is  $N_1 = 5 \cdot 10^{19}$  cm<sup>-3</sup> [19]. An average effective mass  $m_1$  for electrons is  $0.26m_0$  with  $m_0$  being the free-electron mass, and  $\epsilon_{\infty 1} = 11.68$ .

Figure 6 shows the longitudinal propagation constant  $\beta$  as a function of the propagation frequency, for different values of dielectric layers  $d_d$ , employed in the MD lattice.

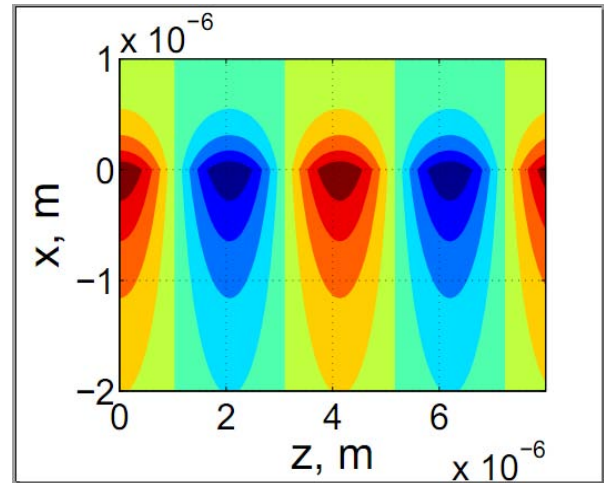


Fig. 6 : Dispersion curves of TM modes at a metamaterial/semiconductor interface for different widths of the dielectric layer  $d_d > 0$  (red dots) in a Drude metal/dielectric compound. The light line is shown (dashed line), and  $d_m = 35$  nm. Points A, B, C, D, E are used in Fig. 8.

It is interesting to notice, that the behavior of the dispersion curves in the investigated case is consistent with the effect of  $d_d$  on the frequency range of the negative  $\epsilon_{||}$  [20].

To date it is well known, that the presence of semiconductor in the structure can tune the properties of the investigated system in the easy way. Thus in Fig. 7 we present the dispersion curves of the surface wave dealing with the different concentrations of the Si.

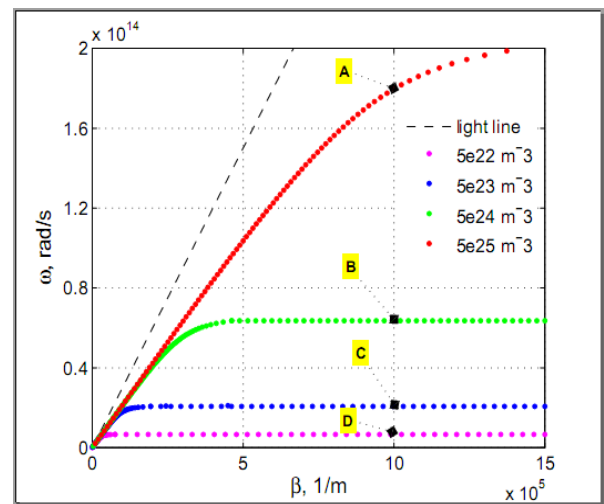


Fig. 7 : Dispersion curves of TM modes at a metamaterial/semiconductor interface for different concentration  $N_1$  of Si. The light line is shown (dashed line), and  $d_d = d_m = 35$  nm. Points A, B, C, D are used in Fig. 9.

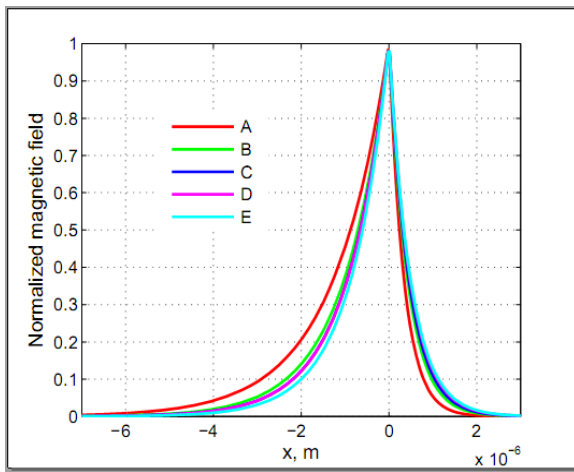


Fig. 8 : Magnetic field (normalized) of SP modes at metamaterial/semiconductor

Interface for the points A, B, C, D and E highlighted in Fig. 2, when that the doping level is  $N_1 = 5 \cdot 10^{19} \text{ cm}^{-3}$ .

Fig. 8 shows the profile of the magnetic field along the x axis for the investigated case, i. e. metamaterial/semiconductor. Fig. 8 shows the magnetic field for the points when  $\beta = 1.5 \cdot 10^6 \text{ 1/m}$ . The tight confinement of the wave field is observable near  $x = 0$  for all the thicknesses of the dielectric  $d_d$ . It is interesting to note that the tighter confinement near the boundary  $x = 0$  is exhibited in the semiconductor.

Moreover, it is of particular interest to plot the profile of the magnetic field along the x axis, when the doping level of the silicon changes. The results are depicted in Fig. 9 being  $d_d = d_m = 35 \text{ nm}$ .

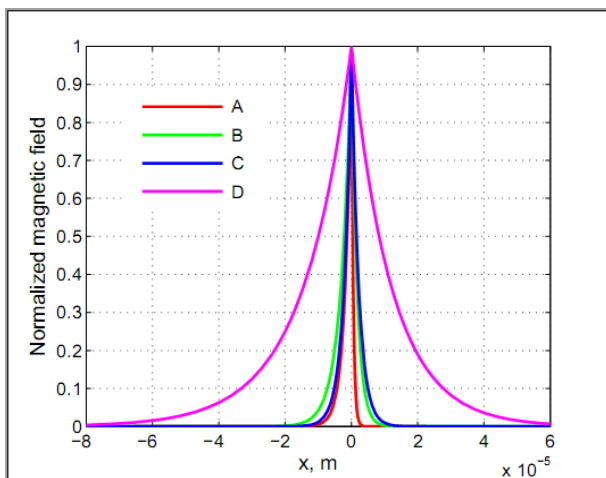


Fig. 9 : Magnetic field (normalized) of SP modes at metamaterial/semiconductor interface for the points A, B, C and D highlighted in Fig. 3, if  $d_d = d_m = 35 \text{ nm}$ .

It is interesting to notice, that the lowest confinement in this case is produced for the lowest doping concentration of silicon.

It is valuable to confirm the presence of SP waves for all the investigated cases. In the insets of Figs. 2, 4 and 6 we show the nature of the magnetic fields for all the investigated cases.

#### IV. CONCLUSIONS

The surface waves at the boundary of hyperbolic metamaterial have been studied for three different cases: metamaterial/ air, metamaterial/metal, metamaterial/semiconductor. The dependences of the longitudinal propagation constant on the propagation frequency have been examined. It is established that one can tune the frequency range of surface waves by varying the thickness of dielectric sheets. It is also revealed that this frequency range can be broadened by three different manipulations:

- decreasing the thickness of the dielectric in the metal-dielectric compound;
- dealing with the metamaterial/metal structure;
- increasing the doping concentration of the semiconductor, if considering the metamaterial/semiconductor interface.

#### V. ACKNOWLEDGMENTS

The authors gratefully acknowledge the financial support provided to this study by NSERC (Natural Sciences and Engineering Research Council) and NSERC's CREATE (Collaborative Research Training Experience) Program ASPIRE (Applied Science in Photonics and Innovative Research in Engineering), both of Canada, also by the projects "New creative teams in priorities of scientific research" and the IT4Innovations Centre of Excellence project (CZ.1.05/1.1.00/02.0070), Czech Republic. We gratefully acknowledge helpful discussions with members of Cada's photonics research group.

#### REFERENCES RÉFÉRENCES REFERENCIAS

1. M. I. D'yakonov, Sov. Phys. JETP 67, 714-716 (1988).
2. O. Takayama, L.-C. Crasovan, S. K. Johansen, D. Mihalache, D. Artigas, and L. Torner, Electromagnetics 28, 126-145 (2008).
3. L. Torner, J. P. Torres, F. Lederer, D. Mihalache, D. M. Baboiu, M. Ciumac, Electron. Lett. 29, 1186-1188 (1993).
4. L. Torner, J. P. Torres, D. Mihalache, IEEE Photon. Technol. Lett. 5, 201-203 (1993).
5. L. Torner, J. P. Torres, C. Ojeda, D. Mihalache, J. Lightwave Technol. 13, 2027-2033 (1995).
6. S. M. Rytov, Sov. Phys. JETP 2, 466 (1956).
7. A. Yariv and P. Yeh, J. Opt. Soc. Am. 67, 438-448 (1977).
8. S. M. Vukovic, I. V. Shadrivov, Y. S. Kivshar, Appl. Phys. Lett 95, 041902 (2009).

9. D. R. Smith, D. Schurig, Phys. Rev. Lett. 90, 077405 (2003).
10. I. I. Smolyaninov, E. Hwang, E. Narimanov, Phys. Rev. B 85, 235122 (2012).
11. Y. Guo, W. Newman, C. L. Cortes, Z. Jacob, Advances in OptoElectronics 2012, 452502 (2012).
12. Z. Jacob, E. E. Narimanov, Appl. Phys. Lett. 93, 221109 (2008).
13. I. I. Smolyaninov, Y.-J. Hung, C. C. Davis, Science 315, 1699–1701 (2007).
14. V. M. Agranovich, V. E. Kravtsov, Solid State Commun. 55, 85–90 (1985).
15. M. Cada, J. Pistora, "Optical plamonics in semiconductors," ISMOT conference, June 20–23, 2011.
16. S. A. Maier, *Surface plasmon polaritons at metal/insulator interfaces, in Plasmonics: Fundamentals and Applications*, Springer, New York (2007).
17. M. Nakayama, J. Phys. Soc. Jpn., 36(2), 393 (1974).
18. V. V. Vodnik, D. K. Božanić, N. Bibić, Zo. V. Šaponjić, and J. M. Nedeljković, Journal of Nanoscience and Nanotechnology 8, 1–5 (2008).
19. J. Jung, T. G. Pedersen, J. Appl. Phys. 113, 114904 (2013).
20. Y. Xiang, J. Guo, X. Dai, S. Wen, D. Tang, Opt. Express 22, 3054-3062 (2014).