From Hertz-Heaviside Electrodynamics to the Trans-Coordinate Electrodynamics

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I. **Introduction**

In the initial form the system of equations of classical electrodynamics was recorded by Maxwell in his famous treatise \([1]\) with the use of quaternion calculation within the framework of classical ideas about the space and time, that allow the Galileo conversions upon transfer from the examination of electromagnetic field in one inertial reference system to the examination of the same field in another inertial reference system. However, it was immediately explained that the apparatus for quaternion calculation in mathematics was developed not so well so that physics they could it successfully apply to the wide circle of the tasks of electrodynamics. In order to draw into the electrodynamics the simpler and more effective means of mathematical physicists, Hertz and Heaviside reformulated Maxwell equations from the language of quaternion calculation to the language of vector analysis.

At that time it seemed that the formulation of Hertz-Heaviside is equivalent to the initial formulation of Maxwell, but now already it is possible to establish that the equations, obtained by Hertz and Heaviside, are essential simplification in Maxwell equations in the quaternions, moreover this simplification relates not only to their mathematical form, but also (that most important!) to their physical content, since in this case equations were deprived naturally Galileo-invariance of inherent in them. Nevertheless for the concretely undertaken inertial reference system (but not their totality) the equivalence of formulations occurred, by virtue of which the formulation of Hertz-Heaviside it obtained the deserved acknowledgement of scientific association it extruded in the theoretical and applied research the formulation of Maxwell himself.

Despite the fact that Maxwell equations both in the formulation of Maxwell himself and in the formulation of Hertz-Heaviside, are obtained within the framework classical ideas about the space and of time, who use Galileo conversions, subsequently precisely of Maxwell equation they became the theoretical prerequisite of the creation of the special theory of relativity (SR). As convincingly shown, for example, in \([2]\), be SR it consists of the identification of the natural geometry of the electromagnetic field, described by Maxwell equations, with the geometry of world physical space-time. And now already in the contemporary works on the electrodynamics (typical example – the work \([3]\) of Maxwell they are examined in the four-dimensional pseudo-Riemann space-time).

Is it possible to return to Maxwell equations the original Galileo-invariance within the framework of certain new, its kind of neoclassical ideas about the space and the time, without rejecting the use of an apparatus of vector analysis during writing of equations? In this work we will show that the answer to this question is affirmative.

II. **Conceptual Approach**

In the classical mechanics particle dynamics is described by the differential equations for its radius-vector, which use usual derivative of the second order on the time. Specifically, its use ensures the Galileo-invariance of equations. If we connect the set of massive material points by weightless elastic threads into the united string, i.e. fluctuation will be described by the Galileo-invariant system of differential equations. But if
we complete passage to the limit, after fixing the number of material points to infinity, and their mass and the length of separate threads – to zero, then we will obtain the one-dimensional wave equation (equation of vibrations of string), not invariant relative to the Galileo conversions, but invariant relative to the group of pseudo-orthogonal conversions (hyperbolic turnings, which preserve pseudo-Euclidean certificate). The culprit of this strange and unexpected metamorphosis upon transfer from “material- point mechanics to continuous medium” – this passage to the limit with the substitution by usual derivative to the quotient, which, generally speaking, is analytically legal [4], but it narrows the region of the physical applicability of equation. The real wave process of mechanical vibrations of string remains Galileo- invariant, but its equation is already deprived of the mathematical means of the description of passage from one inertial reference system to another, and completely ties process to one concrete frame of reference, attaching in it the ends of the string. So classics- field natural-science paradigm revealed fundamental contradiction between the continuity and the discretion [5-6], not overcome, until now, but led to the celebration in theoretical physics of the doubtful principle of the geometrization [7].

The discovery wave equation in the mechanics did not lead to the revision of ideas about the space and the time, but to this led the discovery the same equation in the electrodynamics. In the theory of relativity the corresponding group of pseudo-orthogonal conversions for the electromagnetic waves in the vacuum (Lorenz conversion) obtained status of the subgroup of the motion of the certificate of united world physical space-time. But appears doubt about the justification of the use of traditional equations of electrodynamics, in particular, wave equation, for the adequate extraction of them of ideas about the space and the time. Easily to assume that these equations, using partial derivatives of field functions on the time, similar to the equation of mechanical fluctuations, are simply deprived of the mathematical means of the adequate description of passage from one inertial reference system to another and so completely they tie process to one concrete frame of reference. The question of the possibility of the suitable refinement or generalizing the equations of electrodynamics so arises, beginning from the equations of the induction of electric field by magnetic and magnetic – electrical. The thorough study of this problem in [8] led to the appearance of an idea about the fact that this improvement of electrodynamics must assume existence of the dependence of electromagnetic field on the speed of the motion of observer, caused not by the geometry of space-time, but by physical nature of field.

In the theory of relativity the electromagnetic field also depends on the speed of the motion of observer, but it is only defined by example through the dependence on it of the intervals of time and spatial distance (Lorenz conversion), the relativistic invariance of electric charge occurs result of which. However, the more fundamental (direct) dependence of field on the speed is combined with the dependence even absolute value of electric charge. Until recently this not invariance of charge was confirmed only by indirect empirical data, which were being consisted in the appearance of an electric potential on the superconductive windings and the tori during the introduction in them of direct current, or in the observation of the electric pulse of nuclear explosions [9].

In particular, 9 July 1962 with the explosion in space above Pacific Ocean of H-bomb with the TNT equivalent 1.4 Mt. according to the program of the USA «Starfish » the tension of electrical pour on she exceeded those forecast by Nobel laureate Bethe in 1000 once. with the explosion of nuclear charge according to the program “Program K”, which was realized into the USSR, the radio communication and the radar installations were also blocked at a distance to 1000 km. It was discovered, that the registration of the consequences of space nuclear explosion was possible at the large (to 10 thousand kilometers) distances from the point of impact. The electric fields of pulse led to the large focusings to the power cable in the lead shell, buried at the depth about 1 m, which connects power station in Akmola with Alma-Ata. Focusings were so great that the automation opened cable from the power station.

However, 2015 was marked by the already direct experimental confirmation of this phenomenon as a result of detection and study of the pulse of the electric field, which appears with the warming-up of the plasma as a result of the discharge through the dischargers of the capacitors of great capacity [9]. It turned out that in the process of the warming-up of plasma with an equal quantity in it of electrons and positive ions in it the unitary negative charge of free electrons, not compensated by slower positive ions, is formed.

This fact contradicts not only the classical, but also relativistic conversions of electromagnetic field upon transfer from one inertial reference system to another, testifying about the imperfection not only of classical, but also relativistic ideas about the space and the time. Idea about the fact that the promising electrodynamics must assume existence of the dependence of electromagnetic field on the speed of the motion of observer, caused not by the geometry of space-time, and by physical nature of field, which does not assume the invariance of electric charge, was developed in a number of the work of F. F. Mende, beginning [8]. In these works, in particular, in [8, 9] is given the substantiation of introduction into the electrodynamics instead of the classical and relativistic
new conversions of electromagnetic field, which was called the Mende conversions.

However, the sequential development of this radical idea, as not the invariance of charge, requires the deep revision of the mathematical apparatus for electrodynamics, called to the creation of the mathematical means of the more adequate description of passage from one inertial reference system to another. Approach to precisely this development of the mathematical apparatus for electrodynamics was proposed by A. S. Dubrovin in [10]. This approach lies within the framework the sequential revision of ideas about the space and the time with the failure of the relativistic and the passage to the new ideas, which we call gipercontinual.

The concept of time–space gipercontinuum is introduced in [11] as a result the joint study of the algebraic and geometric structures of the commutative algebras with one, elements of which are the functions of sine waves. The hypothesis of gipercontinuum (about the hierarchical gipercontinual structure of world physical space-time) is starting point of scientific studies, directed toward the generalization of ideas about the structure of space and time in the course of passage from the contemporary quantum scientific paradigm to the new system, that simultaneously structurally connecting up its framework continuity and the discretion, dynamicity and static character, and also globality and the locality [5, 6, 12]. The hierarchical quality of gipercontinuum limits the applicability of the conventional principle of geometrization in physics and the connected with it ideas of symmetry in the geometry due to the introduction into theoretical physics of the ideas of hierarchical quality [7,13], effectiveness of which have approved we with the creation of the standard model of the protected automated system (EMZAS) and the mathematical apparatus of the EMZAS-networks [14].

In [10] is proposed new approach to the development of the mathematical apparatus for electrodynamics in the direction of the more adequate description of passage from one inertial reference system to another on the basis of giperkontinualnykh ideas about the space and in the time due to the improvement of differential calculus of the field functions under the assumption of their dependence on the speed of the motion of observer. Let us accept for the basis this approach.

III. Mathematical Apparatus for the Transcoordinate Electrodynamics

Two inertial reference systems with the time united for them will examine \( t \in \mathbb{R} \), one of them (with the system of rectangular Cartesian space coordinates \( OXYZ \)) we name laboratory (not strikhovannoy) and we will interpret it as relatively fixed. The second (with the system of rectangular Cartesian space coordinates \( O’X’Y’Z’ \)) we name substantive (strikhovannoy) and we will interpret it as connected with the certain moving real or imaginary medium. Let us assume that with \( t = 0 \) the system of space coordinates of both frame of references they coincide.

Let us introduce the indices \( \alpha = 1,3 \quad \beta = 1,3 \). Coordinates along the axes \( OX, OY \) \( OZ, O’X’, O’Y’, O’Z’ \) we will assign by variables \( x^\alpha \) and \( x’^\alpha \) respectively. Unit vectors along the axes \( OX \) and \( O’X’ \) the axes \( OY \) and \( O’Y’ \), the axes \( OZ \) \( O’Z’ \) let us designate through \( e_\beta = (e_\beta^\alpha) \), moreover

\[
e_\beta^\alpha = \delta_{\alpha\beta}, \quad \text{where} \quad \delta_{\alpha\beta} = \text{Kronecker's symbol.}
\]

Through \( \mathbf{v} = (v^\alpha) \mathbf{v} \) let us designate the velocity vector of the motion of substantive frame of reference relative to laboratory and the module of this vector. Directing a unit vector \( \mathbf{e}_1 \mathbf{v} \), we lengthwise have:

\[
\mathbf{v} = \mathbf{v}_1 \mathbf{e}_1 = (v^\alpha) \mathbf{v}. \quad v^\alpha = v_0 \delta_{\alpha 1}.
\]

Event in the data two frame of references takes the form \( \mathbf{x} = (r,t) = (x^\alpha, t) ; \quad \mathbf{x’} = (r’, t) = (x’^\alpha, t), \) where \( r = (x^\alpha), \quad r’ = (x’^\alpha) \) – the radius-vectors. We will consider that the physical equivalence of events \( \mathbf{x} = \mathbf{x’} \) indicates the validity of the Galileo conversion

\[
r = r’ + tv, \quad (1)
\]
or, otherwise, substituting vector idea by the component,

\[
x^\alpha = x’^\alpha + tv_0 \delta_{\alpha 1} \quad (2)
\]

Classical physical field is described in the laboratory and substantive frame of references by its field functions \( \Phi(r, t) \) and \( \Phi’(v, r’, t) \), moreover \( \Phi(0, r’, t) = \Phi(r’, t) \), and equality \( \mathbf{v} = 0 \) indicates \( v^\alpha = 0 \). Their values are called field variables. For pour on different physical nature they can be suitable the different mathematical ideas of field functions, so that field variables can be, for example, scalar or vector with the material or complex values of their most variable or vector components. If in the role of this field electric field comes out, then in this role can come out the functions of its tension \( \mathbf{E} = \Phi(r, t) \), \( \mathbf{E’} = \Phi’(v, r’, t) \), and in the case of magnetic field we have functions of the magnetic induction \( \mathbf{B} = \Phi(r, t) \), \( \mathbf{B’} = \Phi’(v, r’, t) \).
In the classical nonrelativistic field theory it is considered that the equality occurs
\[ \Phi(r' + tv, t) = \Phi'(v, r', t), \]  
(3)

mathematically expressing the physical concept of the invariance of field relative to the speed of the motion of observer. In the theory of relativity (3) no longer it is carried out, but the Lorenz conversions are used instead of the Galilean conversions. But this not invariance of field does not have fundamental, that not connected with the geometry of the space-time of physical nature, but it occurs simply the consequence of the effects of the reduction of lengths and time dilation in the moving frame of references. The proposed by us gipercontinual ideas about the space and the time [11] provide for the great possibilities of the invariance of various physical processes relative to various transformation groups of coordinates with the fact that separate continua.

Invariance of field relative to the speed of the motion of observer, i.e., with fulfillment condition (3), we have:
\[ \frac{d \Phi'(r', t)}{dt} = \lim_{\Delta t \to 0} \frac{\Phi(r' + (t + \Delta t)v, t + \Delta t) - \Phi(r' + tv, t)}{\Delta t} \]
(4)

But it is possible to examine also the derivative (let us name its Galileo derivative), whose arguments will coincide with the arguments of field function no longer in the laboratory, but in the substantive frame of reference:
\[ \phi'(v, r', t) = \lim_{\Delta t \to 0} \frac{\Phi(r' + (t + \Delta t)v, t + \Delta t) - \Phi(r' + tv, t)}{\Delta t} \]
(5)

If the arguments of the Lagrange and Galileo derivatives are connected with equality (1), that their corresponding values are equal and are decomposed into one and the same sum of quotient on the time and the convective derivative of field function in the laboratory frame of reference:
\[ \phi'(v, r', t) = \frac{d \Phi(r, t)}{dt} \partial v + (v \cdot \nabla) \Phi(r' + tv, t) \]
(6)

Let us explain a difference in the physical sense of the Lagrange and Galilean derivatives of field function. Lagrange's derivative (4) is complete time derivative of the function of field in the laboratory frame of reference, measured at the point of space, which in the laboratory frame of reference at the moment of time \( t \) has a radius-vector \( r \), determined by the equality (1). But Galileo derivative (5) is complete time derivative of the function of field in the laboratory frame of reference, measured at the point of space, which in the substantive frame of reference has a radius-vector \( r' \).

The concepts of Lagrange and Galilean derivatives (4)-(6) naturally are generalized to the case derivative of higher order \( (n = 1, \infty) \):
\[ \phi'(v, r', t) = \frac{d^1 \Phi(r, t)}{dt^1} = \frac{d^{n+1} \Phi(r, t)}{dt^{n+1}}, \quad \frac{d^n \Phi(r, t)}{dt^n} = \frac{d^{n+1} \Phi(r, t)}{dt^{n+1}} \]

Within the framework concepts of the invariance of field relative to the speed of the motion of observer, i.e., with fulfillment condition (3), we have:
\[ \frac{\partial^n \Phi(v, r', t)}{dt^n} = \frac{d \Phi'(v, r', t)}{dt} = \frac{d \Phi'(v, r', t)}{dt} = \frac{\partial^n \Phi'(v, r', t)}{dt^n} \]
i.e., Galileo derivative of field in the laboratory frame of reference is not distinguished from the particular time derivative of the function of field in the substantive frame of reference. Therefore, introduction within the framework of this concept of the Galileo derivative as some new mathematical object with its independent physical sense, is superfluous. However, within the framework relativistic ideas examination by Galileo’s derivative is empty because of the emptiness of very Galileo conversions (in contrast to the Lorentz conversions). But giperkontinual ideas about the space and the time make Galilean derived completely by that claimed, and equality (7) – to false.

This view on the space, the period and the electromagnetic field in conjunction with the application of Galileo’s derivative leads to the new, trans-coordinate formulation of the electrodynamics 10. It generalizes the conventional formulation of Hertz-Heaviside, which will be examined below.

IV. MATHEMATICAL MODELS OF THE ELECTROMAGNETIC FIELD

Electromagnetic field in the isotropic homogeneous medium without the dispersion is described in the laboratory and substantive frame of references by its variables (tension of electric field \( \mathbf{E} = \left( E^\alpha \right) \), magnetic induction \( \mathbf{B} = \left( B^\alpha \right) \), by constants (electrical \( \varepsilon_0 \) and magnetic \( \mu_0 \), and also expressed as them speed of light in the vacuum \( c = 1/\sqrt{\varepsilon_0 \mu_0} \)), by the

\[
\oint \mathbf{E} \cdot ds = Q/(\varepsilon_0) ; \oint \mathbf{B} \cdot ds = 0 ; \oint \mathbf{E} \cdot dl = -\frac{d}{dt} \oint \mathbf{B} \cdot ds ; \oint \mathbf{B} \cdot dl = \frac{\mu_0}{\varepsilon_0} \oint \mathbf{E} \cdot ds .
\]

(10)

where \( s \), \( l \) – the arbitrary two-dimensional closed (for the first two equations) or open (for the second two equations) surface and its limiting locked outline, which not not compulsorily coincides with the electric circuit. If we on Wednesday put the even additional condition of the absence of free charges and currents, then last two equations (10) will take the form:

\[
\oint \mathbf{E} \cdot dl = -\frac{d}{dt} \oint \mathbf{B} \cdot ds ; \oint \mathbf{B} \cdot dl = \frac{\mu_0}{\varepsilon_0} \oint \mathbf{E} \cdot ds .
\]

(11)

They are the integral form of the law of the induction of Faraday and circulation theorem of magnetic field in the laboratory frame of reference for this special case of medium.

These two laws take the mutually symmetrical form with an accuracy to of scalar factor, by virtue of which their analysis it is identical. Let us examine the first law in more detail, for example. In Faraday parameters (dielectric and magnetic constant \( \varepsilon \) and \( \mu \), and also the density of strange electric charge \( \rho \), \( \rho' \) the electric current density of conductivity \( j = \left( j^\alpha \right) \), \( j' = \left( j'^\alpha \right) \), electric charge \( Q \), \( Q' \), electric current \( I \), \( I' \), by field functions \( \mathbf{E} = \mathbf{E}(r, t) = \left( E^\alpha(r, t) \right) \), \( \mathbf{B} = \mathbf{B}(r, t) = \left( B^\alpha(r, t) \right) \), \( \mathbf{E}' = \mathbf{E}'(v, r', t) = \left( E'^\alpha(v, r', t) \right) \), \( \mathbf{B}' = \mathbf{B}'(v, r', t) = \left( B'^\alpha(v, r', t) \right) \), moreover

\[
\mathbf{E}'(0, r', t) = \mathbf{E}(r', t) ; \mathbf{B}'(0, r', t) = \mathbf{B}(r', t) .
\]

(8)

In the classical nonrelativistic electrodynamics it is relied:

\[
\mathbf{E}(r' + tv_1, t) = \mathbf{E}'(v, r', t) ; \mathbf{B}(r' + tv_1, t) = \mathbf{B}'(v, r', t) .
\]

(9)

what is the application of a general formula (3) of the invariance of field relative to the speed of the motion of observer for the case of electromagnetic field. The proposed by us giperkontinualnye ideas about the space and the time [11] exceed the scope of this concept, but is explained nature of this not invariance not by the geometry of united space-time similar to the theory of relativity, but by the fundamental properties of field.

The integral form of Maxwell equations in the idea of Hertz-Heaviside with the above-indicated conditions (isotropy, the uniformity of medium, the absence in it of dispersion) is the following system of four integral equations of the electrodynamics:

\[
\oint \mathbf{E} \cdot ds = Q/(\varepsilon_0) ; \oint \mathbf{B} \cdot ds = 0 ; \oint \mathbf{E} \cdot dl = -\frac{d}{dt} \oint \mathbf{B} \cdot ds ; \oint \mathbf{B} \cdot dl = \frac{\mu_0}{\varepsilon_0} \oint \mathbf{E} \cdot ds .
\]

(10)

experiences it is experimentally established that in the outline the identical currents appear regardless of the fact, this outline relative to the current carrying outline does move or it rests, and the current carrying outline moves, provided their relative motion in both cases was identical (Galilean invariance of Faraday law). Therefore the flow through the outline can change as a result of a change of the magnetic field with time, and the position of its boundary also because with the displacement of outline changes [15]. The corresponding generalization of laws (11) to the case of the outline, which moves in the laboratory and which is rested in the substantive frame of reference, takes the form:

\[
\oint \mathbf{E}' \cdot dl = -\frac{d}{dt} \oint \mathbf{B} \cdot ds ; \oint \mathbf{B}' \cdot dl = \frac{\mu_0}{\varepsilon_0} \oint \mathbf{E} \cdot ds .
\]

(12)

where \( \mathbf{E}' \), \( \mathbf{B}' \) are described fields in the element \( dl \) in the substantive frame of reference, i.e., in such inertial
reference system, in which $dl$ it rests; specifically, such electric field causes the appearance of a current in the case of the presence of real electric circuit in this place. Equations (12) are completely interesting and uncommon from a mathematical point of view, since they mutually connect field variables in the different inertial reference systems (let us name such equations trans-coordinate). Specifically, the use of trans-coordinate equations makes it possible to adequately describe physical fields in giperkontinuum. At the same time in this case the discussion deals not simply about the trans-coordinateawen of equations (12), and with their global trans-coordinateawen, ensured by use by the Galilean derivative (connected by them inertial reference

$$\oint E \cdot ds = Q/\varepsilon_0; \oint B \cdot ds = 0; \oint E' \cdot dl = -\frac{d}{dt} \oint B' \cdot ds; \oint E \cdot dl = \frac{\varepsilon_0}{\varepsilon_0} + \frac{d}{dt} \oint E \cdot ds$$ (13)

If the transcoordinate idea of the equations of Maxwell (both in that examined by integral and in that examined lower than the differential forms) to interpret in the context of the description of electromagnetic field in time-spatial gipercontinuum, then it is necessary to consider that the equalities (8) are always carried out, but (9) – in the general case no.

Equations (12) (13) are known in the classical electrodynamics [15, 16]. Arises question, as to pass

$$\nabla \cdot E = \rho/\varepsilon_0; \nabla \cdot B = 0; \nabla \times E = -\partial B/\partial t; \nabla \times B = \mu_0 j + \left(\varepsilon_0/c^2\right) \partial E/\partial t$$ (14)

In accordance with the given traditional approach, in [16] is introduced the operation of differentiation with respect to time in the moving (substantive) frame of reference, designated through $\partial'/\partial t'$. In this case it is secretly assumed that at the point of space, which in the substantive frame of reference has a radius-vector $r'$, measurement by field variable in the laboratory frame of reference equivalent to its measurement in the same substantive frame of reference. But these measurements are not equivalent out of the concept of the invariance of physical field relative to the speed of the motion of observer. Therefore measurement must be limited by laboratory frame of reference, not perenosya its results for the substantive. Thus, we come to the Galilean derivative (5), of the electrodynamics in the differential form leaving equations globally transcoordinate.

Unknown globally transcoordinate differential equations of electrodynamics, which correspond to integral equations (12) and which use the Galileo derivative:

$$\nabla \times E' = -\frac{\partial B}{\partial t}, \nabla \times B' = \frac{\varepsilon_0}{c^2} \frac{\partial E}{\partial t}.$$ (15)
They are generalization to the case of the noncoincidence of the laboratory and substantive frame of references \( (\mathbf{v} \neq \mathbf{0}) \) of the known differential equations of Maxwell:

\[
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \times \mathbf{B} = \varepsilon \mu \frac{\partial \mathbf{E}}{\partial t}.
\]

(16)

The differential form of Maxwell equations in the trans-coordinate idea for the case of isotropic, homogeneous medium without the dispersion is the following system of four new globally trans-coordinate differential equations of the electrodynamics:

\[
\frac{\partial \mathbf{E}}{\partial t}(\mathbf{r}', t) = \varepsilon \mu \frac{\partial \mathbf{E}}{\partial t}(\mathbf{r}', t) + (\mathbf{v}_1 \cdot \nabla)\mathbf{E}(\mathbf{r}', t + v_1 t)
\]

(19)

\[
\frac{\partial \mathbf{B}}{\partial t}(\mathbf{r}', t) = \varepsilon \mu \frac{\partial \mathbf{B}}{\partial t}(\mathbf{r}', t) + (\mathbf{v}_1 \cdot \nabla)\mathbf{B}(\mathbf{r}', t + v_1 t)
\]

(20)

With \( \mathbf{v} = 0 \) (17)-(18) it passes in (14).

In the particular case the absences of free charges and currents of equation (17)-(18) will take the form:

\[
\nabla \times \mathbf{E}'(\mathbf{r}', t) = -\frac{\partial \mathbf{B}}{\partial t}(\mathbf{r}', t) \nabla \times \mathbf{B}'(\mathbf{r}', t) = \frac{\varepsilon \mu}{c^2} \frac{\partial \mathbf{E}}{\partial t}(\mathbf{r}', t)
\]

(22)

Galilean derivative of field functions and generalizing equations (24) in the case \( \mathbf{v} \neq \mathbf{0} \):

\[
c^2 \nabla^2 \mathbf{E}' = \varepsilon \mu \frac{\partial^2 \mathbf{E}}{\partial t^2}, \quad c^2 \nabla^2 \mathbf{B}' = \varepsilon \mu \frac{\partial^2 \mathbf{B}}{\partial t^2}.
\]

(25)

We investigate in more detail the equation of form (25) in connection with to arbitrary field functions \( \Phi(x, t) \), also, \( \Phi'(v, x', t) \) for the case of plane wave with the wave vector, collinear to vector \( \mathbf{v} = (v, 0, 0) \) and to axes \( OX, \quad O'X' \), coordinates along which are assigned by the variables \( x, \quad x' \). In this case the equation proves to be one-dimensional, and field functions – scalar:

\[
c^2 \frac{\partial^2 \Phi}{\partial x^2} \Phi'(v, x', t) = \varepsilon \mu \frac{\partial^2 \Phi}{\partial t^2}(v, x', t) = \varepsilon \mu \frac{\partial^2}{\partial t^2} \Phi(x' + vt, t)
\]

(26)

If we differentiate in the right side (26), this equation of signs the form:

\[
c^2 \frac{\partial^2}{\varepsilon \mu \partial x^2} \Phi'(v, x', t) = \left( \frac{\partial^2}{\partial t^2} + 2v \frac{\partial}{\partial t} + v^2 \frac{\partial^2}{\partial x^2} \right) \Phi(x' + vt, t) = \left( \frac{\partial}{\partial t} + v \frac{\partial}{\partial x} \right)^2 \Phi(x' + vt, t)
\]

(27)
With \( v = 0 \) \((26)\) \((27)\) it degenerates into the one-dimensional version of the wave equation of the form \((24)\):

\[
c^2 \frac{\partial^2}{\partial x^2} \Phi(x,t) = \varepsilon \mu \frac{\partial^2}{\partial t^2} \Phi(x,t). \tag{28}
\]

Any solution \((28)\) is determined by the proper superposition of the simple harmonic waves

\[
\Phi(x,t) = A \cos(\omega t - k_x x + \varphi) \tag{29}
\]

\(k'_x(v), \varphi'(v)\):

\[
\Phi(v,x',t) = A'(v) \cos(\omega'(v)t - k'_x(v)x' + \varphi'(v))
\]

\(\Phi(0,x',t) = \Phi(x',t), A'(0) = A, \omega'(0) = \omega, k'_x(0) = k_x, \varphi'(0) = \varphi\). Let us substitute \((29)-(30)\) in

\[
c^2 k_x^2(v) A'(v) \cos(\omega'(v)t - k'_x(v)x' + \varphi'(v)) = \varepsilon \mu (\omega - k_x v)^2 A \cos(\omega t - k_x (x' + vt) + \varphi).
\]

Equalizing the similar parameters of wave on the left side \((31)\) and in the right, we have:

\[
A'(v) = \left(\frac{\text{sgn} k_x - \frac{\sqrt{\varepsilon \mu}}{c} v}{1 - \frac{\sqrt{\varepsilon \mu}}{c} v \text{sgn} k_x}\right)^2 A, \quad \omega'(v) = \left|\omega - k_x v\right| = \left|1 - \frac{\sqrt{\varepsilon \mu}}{c} v \text{sgn} k_x\right| \omega
\]

\[
k'_x(v) = k_x \text{sgn}(\omega - k_x v), k'(v) = \left|k'_x(v)\right| = k, \varphi'(v) = \varphi \text{sgn}(\omega - k_x v), \left|\varphi'(v)\right| = |\varphi|.
\]

Thus, upon transfer from the laboratory to the substantive frame of reference change amplitude and frequency \((32)\) of simple harmonic wave, and its wave number and module of initial phase \((33)\) remain constant. In this case the frequency changes in such a way that phase wave velocity in the substantive frame of reference is obtained according to the classical summation rule of speeds from its phase speed in the laboratory frame of reference and speed of substantive frame of reference relative to the laboratory:

\[
\omega'(v)/k'_x(v) = \omega'(v)/k_x = \omega/k_x - v, \omega'(v)/k'(v) = \left|\omega/k - v \text{sgn} k_x\right| = \left|\frac{\sqrt{\varepsilon \mu}}{c} - v \text{sgn} k_x\right| \omega.
\]

From \((32)-(34)\) it is evident that if the vector of phase wave velocity in the laboratory frame of reference coincides with the velocity vector of substantive frame of reference in it \((k_x > 0, \ v = \omega/k)\), that in the substantive frame of reference wave generally disappears \((A'(v) = 0)\). Thus, in contrast to the theory of relativity, in the theory of gipercontinuum this wave can always be destroyed by the suitable selection of frame of reference. But if relative to laboratory frame of reference substantive frame of reference outdistances wave, then upon transfer from the laboratory frame of reference to the substantive the direction of propagation of wave changes by the opposite. If in the laboratory frame of reference wave is propagated in the positive direction, then upon transfer into the substantive it will satisfy wave equation \((35)\), while if in the negative, then to the equation \((36)\):

\[
\left(\frac{\sqrt{\varepsilon \mu}}{c} - v\right)^2 \frac{\partial^2}{\partial x^2} \Phi(v,x',t) / \partial x'^2 = \frac{\partial^2}{\partial t^2} \Phi(v,x',t) / \partial t^2
\]

\[
\left(\frac{\sqrt{\varepsilon \mu}}{c} + v\right)^2 \frac{\partial^2}{\partial x^2} \Phi(v,x',t) / \partial x'^2 = \frac{\partial^2}{\partial t^2} \Phi(v,x',t) / \partial t^2
\]

The selection of inertial reference system to the role of laboratory is, generally speaking, conditional. Thus, substantive frame of reference it is possible in turn to accept for the laboratory, and in the role of substantial to examine certain by third (twice prime) inertial reference system with that directed to the same side, that also \(OX\), \(O'X'\), by attitude reference \(O''X''\), the coordinate along which is assigned by the variable.

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Let, for example, the point $O''$ move in the positive direction of axis $O'X'$ with the speed $\Delta v$. Wave in the new laboratory and substantive frame of references will have an identical wave number and a module of initial phase and will be described by field functions

$$
\left(\frac{c}{\sqrt{\varepsilon \mu}} - (v + \Delta v)\right)^2 \frac{\partial^2 \Phi'(v + \Delta v, x'', t)}{\partial x''^2} = \frac{\partial^2 \Phi'(v + \Delta v, x'', t)}{\partial t^2}
$$

(37)

$$
\left(\frac{c}{\sqrt{\varepsilon \mu}} + (v + \Delta v)\right)^2 \frac{\partial^2 \Phi'(v + \Delta v, x'', t)}{\partial x''^2} = \frac{\partial^2 \Phi'(v + \Delta v, x'', t)}{\partial t^2}
$$

(38)

For (37) the role of equalities (32), (33) play the following transformations of the parameters of the wave:

$$
A''(v + \Delta v) = \left(\frac{\varepsilon \mu}{c - \sqrt{\varepsilon \mu} \cdot v}\right)^2 A'(v), \quad \omega''(v + \Delta v) = |\omega'(v) - k_x'(v)| \Delta v,
$$

(39)

$$
k_x''(v + \Delta v) = k_x'(v) \text{sgn}(\omega'(v) - k_x'(v) \Delta v), \quad \Phi''(v + \Delta v) = \Phi'(v) \text{sgn}(\omega'(v) - k_x'(v) \Delta v)
$$

(40)

For (38) the corresponding (39)-(40) conversions of the parameters are determined analogously.

Sequential passage from not shtrikhovannoy to shtrikhovannoy and is further to the twice shtrikhovannoy frame of reference equivalent to direct passage from not shtrikhovannoy to twice shtrikhovannoy. For example, which is obtained also upon direct transfer to the twice

$$
\left(\frac{c}{\sqrt{\varepsilon \mu}} - v\right)^2 \frac{\partial^2 \Phi'(v + \Delta v, x'', t)}{\partial x''^2} = \frac{\partial^2 \Phi'(v, x'' + \Delta vt, t)}{\partial t^2} + \left(2 \frac{\partial^2}{\partial t^2} + \Delta v^2 \frac{\partial^2}{\partial x^2}\right) \Phi'(v, x'' + \Delta vt, t).
$$

(42)

For the derivatives of arbitrary $n$ of order

$$
\frac{\partial^n \Phi'(v + \Delta v, x'', t)}{\partial x''^n} \quad \frac{\partial^n \Phi'(v, x', t)}{\partial x^n}
$$

it is possible to use a united designation

$$
\frac{\partial^n \Phi'(v + \Delta v, x, t)}{\partial x^n} \quad \frac{\partial^n \Phi'(v, x, t)}{\partial x^n}
$$

$$
\left(\frac{c}{\sqrt{\varepsilon \mu}} - v\right)^2 \frac{\partial^2}{\partial x^2} \left(\Phi'(v + \Delta v, x, t) - \Phi'(v, x + \Delta vt, t)\right) = \left(2 \frac{\partial^2}{\partial t^2} + \Delta v^2 \frac{\partial^2}{\partial x^2}\right) \Phi'(v, x + \Delta vt, t)
$$

(43)

Let $\Delta v \to 0$. Let us introduce one additional new derivative, which let us name trans-coordinate, and which in the case of the one-dimensional system of space coordinates takes the form:

$$
\frac{\partial^{\prime\prime} \Phi'(v, x, t)}{\partial v^{\prime\prime}} = \lim_{\Delta v \to 0} \frac{\Phi'(v + \Delta v, x, t) - \Phi'(v, x + \Delta vt, t)}{\Delta v}.
$$

(44)

In the determination (44) of value

$$
\Phi'(v, x + \Delta vt, t) \quad \Phi'(v + \Delta v, x, t)
$$

is described physical field at one and the same point of space, but in the different frame of references (shtrikhovannoy and moving relative to it with speed $\Delta v$ twice prime respectively). Within the framework they are equal to the

$$
\Phi'(v, x', t) \quad \Phi'(v + \Delta v, x'', t)
$$

respectively. The role of equation (28) plays (35) or (36), the role of the function of wave (29) – function (30), while the role of equations (35), (36) – the following wave equations:

$$
\frac{\partial^2 \Phi'(v, x', t)}{\partial t^2} = \frac{\partial^2 \Phi'(v + \Delta v, x'', t)}{\partial t^2}
$$

(41)

With $\text{sgn} k_x'(v) = \text{sgn} k_x = 1$ from (32), (39) it is possible to obtain

$$
A''(v + \Delta v) = \left(1 - \sqrt{\varepsilon \mu} (v + \Delta v)/c\right)^2 A,
$$

(41)

shtrikhovannoy frame of reference, since (41) it is obtained from (32) by replacement $v$ on $v + \Delta v$. In this case the role of equation (27) plays

$$
\frac{\partial^n \Phi'(v + \Delta v, x', t)}{\partial x'^n} \quad \frac{\partial^n \Phi'(v, x, t)}{\partial x^n}
$$

$$
\left(\frac{c}{\sqrt{\varepsilon \mu}} - v\right)^2 \frac{\partial^2}{\partial x^2} \left(\Phi'(v + \Delta v, x', t) - \Phi'(v, x + \Delta vt, t)\right) = \left(2 \frac{\partial^2}{\partial t^2} + \Delta v^2 \frac{\partial^2}{\partial x^2}\right) \Phi'(v, x + \Delta vt, t)
$$

(43)

concept of the invariance of field relative to the speed of the motion of observer:

$$
\Phi'(v, x + \Delta vt, t) = \Phi'(v + \Delta v, x, t)
$$

(45)

the equalities (3) (45) making identical physical sense, but in connection with to the different pairs of frame of references. However, out of the framework of the indicated concept upon transfer from shtrikhovannoy to the twice shtrikhovannoy frame of reference the field function at the particular point of space experiences the increase, the limit of relation of which $k \Delta v$ with $\Delta v \to 0$ gives the trans-coordinate derivative (44). It is possible to generalize it to the case of the higher orders ($n = 1, \infty$):
Using trans-coordinate derivatives of the first two orders (46), it is possible to represent increase in the field function of in the form corresponding partial summation of series of Taylor:

$$\Phi'(v + \Delta v, x, t) - \Phi'(v, x + \Delta vt, t) \approx \frac{\partial\Phi'(v, x, t)}{\partial v} \Delta v + \frac{1}{2} \frac{\partial^2\Phi'(v, x, t)}{\partial v^2} \Delta v^2$$ (47)

Substituting (47) in (43), equalizing between themselves members with the identical degrees $\Delta v$ in the left and right sides of the received equality, fixing $\Delta v \to 0$, taking into account that the fact that in this case $\Phi'(v, x + \Delta vt, t) \to \Phi'(v, x, t)$ and by adding equality (35) in the new form of record (with the use by variable $x$ instead of $x'$), we will obtain the following system of three equations:

$$\left\{ \begin{array}{l}
\left( \frac{c}{\sqrt{\varepsilon \mu}} - v \right)^2 \frac{\partial^2\Phi'(v, x, t)}{\partial x^2} = \frac{\partial^2\Phi'(v, x', t)}{\partial t^2}, \\
\left( \frac{c}{\sqrt{\varepsilon \mu}} - v \right)^2 \frac{\partial^2\Phi'(v, x, t)}{\partial x \partial v} = 2 \frac{\partial\Phi'(v, x, t)}{\partial t}, \\
\left( \frac{c}{\sqrt{\varepsilon \mu}} - v \right)^2 \frac{\partial^2\Phi'(v, x, t)}{\partial v^2} = 2 \Phi'(v, x', t)
\end{array} \right.$$ (48)

System of equations (48) can be written down in following that indexed on $\alpha$ the form:

$$\left( \frac{c}{\sqrt{\varepsilon \mu}} - v \right)^2 \frac{\partial^2\alpha}{\partial x^2} - 2 \text{sgn} \alpha \frac{\partial^2\alpha}{\partial t^2} \Phi'(v, x', t) = 0, \quad \alpha = 0, 2$$ (49)

or in the operator form

$$\bigotimes\Phi'(v, x', t) = 0$$ (50)

where $\bigotimes\alpha = (\bigotimes^\alpha); \quad \bigotimes^\alpha = \left( \frac{c}{\sqrt{\varepsilon \mu}} - v \right)^2 \frac{\partial^2\alpha}{\partial x^2} - 2 \text{sgn} \alpha \frac{\partial^2\alpha}{\partial t^2}$

– the suitable version of the one-dimensional (case of one axis of space coordinates) differential Dubrovin operator, which generalizes d’Alembert’s operator $\Box$, who occurs one of his three (zero) components for the laboratory frame of reference, i.e., $\alpha = 0, v = 0$.

Differential equation (49) or (50) is the gipercontinual one-dimensional homogeneous wave equation, which generalizes, similar to differential equation (26) or (27), the known one-dimensional homogeneous wave equation (28). The vital difference between them (26)-(27) is lies in the fact that the globally trans-coordinate form of gipercontinual wave equation, and (49)-(50) – by its locally trans-coordinate form. Local transcoordinate means that the equation connects the inertial reference systems, which move relative to each other with the infinitely low speed.

The transcoordinate of giperkontinualnykh wave equations is ensured by the use in them of the suitable derived field functions. Namely, use by Galileo’s derivative reports to equation global trans-coordinate, and by trans-coordinate derivative – local.

Thus, is proposed the new approach to the development of the mathematical apparatus for electrodynamics in the direction of the more adequate description of passage from one inertial reference system to another on the basis of gipercontinual ideas about the space and in the time due to the introduction into the examination of the globally and locally trans-coordinate equations, which use new Galilean and trans-coordinate derivatives of field functions, and also the new differential of Dubrovin operator, which generalizes d’Alembert operator. This approach leads to the reformulation of electrodynamics with the passage from the traditional formulation of Hertz-Heaviside to the new trans-coordinate. In this case immediately arise the question about what form they have conversions of electromagnetic field upon transfer from one inertial...
The convective derivatives of field functions in (19)-(20) can be written down in the form:

\[
(ve_1 \cdot \nabla)E(r' + tve_1, t) = \nabla \cdot E(r' + tve_1, t) e_1 - \nabla \times (ve_1 \times E(r' + tve_1, t)) \tag{51}
\]

\[
(ve_1 \cdot \nabla)B(r' + tve_1, t) = \nabla \cdot B(r' + tve_1, t) e_1 - \nabla \times (ve_1 \times B(r' + tve_1, t)) \tag{52}
\]

We have in view of the first two (22) equations taking into account (1)-(2):

\[
\nabla \cdot E(r' + tve_1, t) = 0; \nabla \cdot B(r' + tve_1, t) = 0. \tag{53}
\]

After substituting (53) in (51)-(52), we will obtain equalities for the convective derivatives:

\[
(ve_1 \cdot \nabla)E(r' + tve_1, t) = -\nabla \times (ve_1 \times E(r' + tve_1, t)) \tag{54}
\]

\[
(ve_1 \cdot \nabla)B(r' + tve_1, t) = -\nabla \times (ve_1 \times B(r' + tve_1, t)). \tag{55}
\]

After substitution (54)-(55) in (19)-(20) we take another form of the Galilean derivatives:

\[
\frac{\partial E}{\partial t}(v, r', t) = \frac{\partial E(r' + tve_1, t)}{\partial t} - \nabla \times (ve_1 \times E(r' + tve_1, t)) \tag{56}
\]

\[
\frac{\partial B}{\partial t}(v, r', t) = \frac{\partial B(r' + tve_1, t)}{\partial t} - \nabla \times (ve_1 \times B(r' + tve_1, t)) \tag{57}
\]

The substitution of Galilean derivatives (56)-(57) into the last two equalities (22) gives:

\[
\nabla \times E'(v, r', t) = -\partial B(r' + tve_1, t)/\partial t + \nabla \times (ve_1 \times B(r' + tve_1, t)), \tag{58}
\]

\[
\nabla \times B'(v, r', t) = \left(\varepsilon\mu/c^2\right)\partial E(r' + tve_1, t)/\partial t - \nabla \times (ve_1 \times E(r' + tve_1, t)). \tag{59}
\]

After substituting last two equations (23) in (58)-(59), we will obtain:

\[
\nabla \times E'(v, r', t) = \nabla \times E(r' + tve_1, t) + \nabla \times (ve_1 \times B(r' + tve_1, t)), \tag{60}
\]

\[
\nabla \times B'(v, r', t) = \nabla \times B(r' + tve_1, t) - \left(\varepsilon\mu/c^2\right)\nabla \times (ve_1 \times E(r' + tve_1, t)). \tag{61}
\]

Let us omit the operation of rotor both parts of the equalities (60)-(61):

\[
E'(v, r', t) = E(r' + tve_1, t) + ve_1 \times B(r' + tve_1, t), \tag{62}
\]

\[
B'(v, r', t) = B(r' + tve_1, t) - \left(\varepsilon\mu/c^2\right)ve_1 \times E(r' + tve_1, t). \tag{63}
\]

Besides the shtrikhovannoy frame of reference, which moves relative to laboratory with speed \(v\) let us introduce also relatively mobile frame of reference – twice shtrikhovannuyu, that moves in the same direction with another speed \(v + \Delta v\) relative to laboratory. Thus, the twice shtrikhovannaya frame of reference moves with relatively shtrikhovannoy with speed \(\Delta v\), the so that shtrikhovannuyu frame of reference can be accepted for the new laboratory (relatively fixed), and twice shtrikhovannuyu – for the new substantive.

Equalities (62)-(63) for them let us write down taking into account the replacement of radius-vector \(r'\) on \(r''\):

\[
E'(v + \Delta v, r'', t) = E'(v, r'' + t\Delta ve_1, t) + \Delta ve_1 \times B'(v, r'' + t\Delta ve_1, t), \tag{64}
\]

\[
B'(v + \Delta v, r'', t) = B'(v, r'' + t\Delta ve_1, t) - \left(\varepsilon\mu/c^2\right)\Delta ve_1 \times E'(v, r'' + t\Delta ve_1, t). \tag{65}
\]

Let us write down equalities (64)-(65) in the following form:
In (66)- (67) the values \( \mathbf{E}'(v, r'' + t\Delta v \mathbf{e}_1, t) \), \( \mathbf{B}'(v, r'' + t\Delta v \mathbf{e}_1, t) \) and \( \mathbf{E}'(v + \Delta v, r'', t) \), \( \mathbf{B}'(v + \Delta v, r'', t) \) is described the electromagnetic field at one and the same point of space (medium), but in the different frame of references (shtrikhovannoy and by twice shtrikhovannoy). Within the framework they are equal to the concept of the invariance of field relative to the speed of the motion of observer:

\[
\mathbf{E}'(v, r'' + t\Delta v \mathbf{e}_1, t) = \mathbf{E}'(v + \Delta v, r'', t), \quad \mathbf{B}'(v, r'' + t\Delta v \mathbf{e}_1, t) = \mathbf{B}'(v + \Delta v, r'', t). 
\]  

(68)

The equalities (9) (68) making identical physical sense, but in connection with to the different pairs of frame of references. However, out of the framework of the indicated concept upon transfer from shtrikhovannoy to the twice shtrikhovannoy frame of reference the field function at the particular point of space experiences the increase, the limit of relation of which \( k \Delta v \) gives that for the first time introduced into 10 the trans-coordinate derivative of the field function:

\[
\frac{\partial \mathbf{E}'(v, r'', t)}{\partial' v} = \lim_{\Delta v \to 0} \frac{\mathbf{E}'(v + \Delta v, r'', t) - \mathbf{E}'(v, r'' + t\Delta v \mathbf{e}_1, t)}{\Delta v}, 
\]

(69)

\[
\frac{\partial \mathbf{B}'(v, r'', t)}{\partial' v} = \lim_{\Delta v \to 0} \frac{\mathbf{B}'(v + \Delta v, r'', t) - \mathbf{B}'(v, r'' + t\Delta v \mathbf{e}_1, t)}{\Delta v}. 
\]

(70)

Equalities (66)-(67) with \( \Delta v \to 0 \) taking into account (69)-(70) after replacement \( r'' \) on \( r \) take the form:

\[
\frac{\partial \mathbf{E}'(v, r', t)}{\partial' v} = \mathbf{e}_1 \times \mathbf{B}'(v, r', t), \quad \frac{\partial \mathbf{B}'(v, r', t)}{\partial' v} = -\frac{\varepsilon \mu}{c^2} \mathbf{e}_1 \times \mathbf{E}'(v, r', t). 
\]  

(71)

If equations (22) are the globally trans-coordinate differential equations of electrodynamics for the case of isotropic homogeneous medium without the dispersion in the absence of free charges and currents, then equations (71) are the locally trans-coordinate differential equations of electrodynamics for the same case. The locality of transcoordinate, ensured by use by trans-coordinate derivative, means that the connected by differential equations inertial reference systems (conditionally speaking, prime and twice prime) they move relative to each other with the infinitely low speed \( \Delta v \). Equations (71) form the system, by solving which, it is possible to obtain the conversions of electromagnetic field upon transfer of one inertial reference system into another.

Let us use system of equations (71) for obtaining the conversions of electromagnetic field upon transfer from the laboratory frame of reference to the substantive.

Lowering the arguments of functions, let us write down vector products in (71) in the form:

\[
\mathbf{e}_1 \times \mathbf{B}' = \mathbf{e}_1 \times \left( B'^1 \mathbf{e}_1 + B'^2 \mathbf{e}_2 + B'^3 \mathbf{e}_3 \right) = B'^2 \mathbf{e}_3 - B'^3 \mathbf{e}_2 
\]

(72)

\[
\mathbf{e}_1 \times \mathbf{E}' = \mathbf{e}_1 \times \left( E'^1 \mathbf{e}_1 + E'^2 \mathbf{e}_2 + E'^3 \mathbf{e}_3 \right) = E'^2 \mathbf{e}_3 - E'^3 \mathbf{e}_2 
\]

(73)

Taking into account (72)-(73) the system of equations (71) is divided off into two independent systems of two equations each and two additional independent equations:
We differentiate the first equations of systems (74) and will substitute them the secondly:

\[
\begin{align*}
\frac{\partial^2 E^2}{\partial v^2} &= \frac{\varepsilon \mu}{c^2} E^2; \\
\frac{\partial^2 B^3}{\partial v^2} &= \frac{\varepsilon \mu}{c^2} E^3;
\end{align*}
\]  

(75)

The general solution of equations (75) is expressed as the arbitrary constants \( C_1, \ldots, C_{10} \):

\[
\begin{align*}
E^1 &= C_1; \quad E^2 = C_2 \cosh \frac{\sqrt{\varepsilon \mu} v}{c} + C_3 \sinh \frac{\sqrt{\varepsilon \mu} v}{c}; \quad E^3 = C_4 \cosh \frac{\sqrt{\varepsilon \mu} v}{c} + C_5 \sinh \frac{\sqrt{\varepsilon \mu} v}{c}; \\
B^1 &= C_6; \quad B^2 = C_7 \cosh \frac{\sqrt{\varepsilon \mu} v}{c} + C_8 \sinh \frac{\sqrt{\varepsilon \mu} v}{c}; \quad B^3 = C_9 \cosh \frac{\sqrt{\varepsilon \mu} v}{c} + C_{10} \sinh \frac{\sqrt{\varepsilon \mu} v}{c}.
\end{align*}
\]

(76, 77)

Since we search for the conversions of electromagnetic field upon transfer from the laboratory frame of reference, then the desired particular solutions of equations (75) must with \( v = 0 \) describe electromagnetic field in the laboratory frame of reference, i.e., satisfy equalities (8) and (74), and the, which means, following totality of the equalities:

\[
\begin{align*}
E^1(0, r', t) = E^1(r', t) &\quad E^2(0, r', t) = E^2(r', t); \quad E^3(0, r', t) = E^3(r', t); \\
B^1(0, r', t) = B^1(r', t) &\quad B^2(0, r', t) = B^2(r', t); \quad B^3(0, r', t) = B^3(r', t); \\
\frac{\partial^2 E^2}{\partial v^2}(0, r', t) = -B^3(r', t) &\quad \frac{\partial^2 E^3}{\partial v^2}(0, r', t) = B^2(r', t); \\
\frac{\partial^2 B^2}{\partial v^2}(0, r', t) = \frac{\varepsilon \mu}{c^2} E^3(r', t) &\quad \frac{\partial^2 B^3}{\partial v^2}(0, r', t) = -\frac{\varepsilon \mu}{c^2} E^2(r', t).
\end{align*}
\]

(78-81)

By substitution (76)-(77) in (78)-(81) let us find the values of constants \( C_1, \ldots, C_{10} \), as a result what after the substitution of these constants in (76)-(77) we will obtain the resultant expression in the component form for the desired conversions of electromagnetic field upon transfer from the laboratory frame of reference to the substantive:

\[
\begin{align*}
E^1(v, r', t) = E^1(r', t) &\quad B^1(v, r', t) = B^1(r', t); \\
E^2(v, r', t) = E^2(r', t) \cosh \frac{\sqrt{\varepsilon \mu} v}{c} - \frac{c}{\sqrt{\varepsilon \mu}} B^3(r', t) \sinh \frac{\sqrt{\varepsilon \mu} v}{c}; \\
E^3(v, r', t) = E^3(r', t) \cosh \frac{\sqrt{\varepsilon \mu} v}{c} + \frac{c}{\sqrt{\varepsilon \mu}} B^2(r', t) \sinh \frac{\sqrt{\varepsilon \mu} v}{c}; \\
B^2(v, r', t) = B^2(r', t) \cosh \frac{\sqrt{\varepsilon \mu} v}{c} + \frac{\varepsilon \mu}{c} E^3(r', t) \sinh \frac{\sqrt{\varepsilon \mu} v}{c}; \\
B^3(v, r', t) = B^3(r', t) \cosh \frac{\sqrt{\varepsilon \mu} v}{c} - \frac{\varepsilon \mu}{c} E^2(r', t) \sinh \frac{\sqrt{\varepsilon \mu} v}{c}.
\end{align*}
\]

(82-85)
\[ B'^3(v', r', t) = B^3(r', t) \cosh \frac{\sqrt{\varepsilon \mu v}}{c} - \frac{\sqrt{\varepsilon \mu}}{c} E^2(r', t) \sinh \frac{\sqrt{\varepsilon \mu v}}{c} \] (86)

In the vector form the same conversions take the following form:

\[ \mathbf{E}'(v', r', t) = \mathbf{E}(r', t) \cosh \frac{\sqrt{\varepsilon \mu v}}{c} + \frac{c}{\sqrt{\varepsilon \mu}} \mathbf{e}_1 \times \mathbf{B}(r', t) \sinh \frac{\sqrt{\varepsilon \mu v}}{c} \] (87)

\[ \mathbf{B}'(v', r', t) = \mathbf{B}(r', t) \cosh \frac{\sqrt{\varepsilon \mu v}}{c} - \frac{c}{\sqrt{\varepsilon \mu}} \mathbf{e}_1 \times \mathbf{E}(r', t) \sinh \frac{\sqrt{\varepsilon \mu v}}{c} \] (88)

It is easy to see that the conversions (82)-(88) are known Mende conversions.

V. Conclusion

Thus, the Mende conversions obtain a sufficient theoretical substantiation within the framework of the trans-coordinate formulation of electrodynamics, connected with the gipercontinual ideas about the space and the time, and also with the concept not of the invariance of electric charge relative to the speed of the motion of observer. Together with that been in [9] direct experimental confirmation of the concept not of the invariance of electric charge, this is convincing evidence of their larger adequacy of physical reality on the comparison not only with the classical, but also with the relativistic conversions of electromagnetic field, or the convincing evidence of the justification of the transfer of electrodynamics from the traditional formulation of Hertz-Heaviside to the trans-coordinate. The sequential development of transcoordinate electrodynamics is capable of not only deriving on the new qualitative level of idea about the space and the time, but also of opening the fundamentally new horizons of the development engineering and technologies due to the discovery and the mastery of new physical phenomena and effects.

References Références Referencias