

# Using Fuzzy Goal Programming Technique to Obtain the Optimum Production of Vehicle Spare Parts, A Case Study

Jasem Alrajhi<sup>1</sup>, Hilal A. Abdelwali<sup>2</sup>, Elsayed E.M. Ellaimony<sup>3</sup>, Yousef M. Alhouli<sup>4</sup> and Ahmed E. Murad<sup>5</sup>

<sup>1</sup> College of Technological Studies, PAAET, Kuwait

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## Abstract

This paper studies the vehicle spare parts production problem to obtain the optimum production rate under fuzzy capital budget. We applied the integer goal programming technique to determine the best compromise solution. There are two goals in our case study. These goals are minimization of the uncertain capital budget and maximization of the uncertain expected profits. The case study is a factory which produces different types of vehicle heat exchangers. The results indicate that the problem solution depends on the membership function and the  $\alpha$ -cut. The optimum quantities of heat exchangers production are found to be biased to the lower limit of production.

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*Index terms—*

## 1 Introduction

Due to huge changes in vehicle prices, fuels, oils, and spare parts' prices, it is important for transportation companies to study their fleet of vehicles from the opinion of vehicle operations economics. The vehicle operations economics field studies the maximization of profit and/or minimization of costs. One of the topics that is needed to be studied and applied to the real world applications is finding the optimum production rate of vehicle spare parts.

The capital budgeting in automobile firms is introduced by AbouelNour [1]. The optimal distribution of a certain amount of capital budgeting in production of vehicle spare parts had been obtained using integer programming technique in formulation and solution. Mohan A. et al [2] developed a decision support system for budget allocation of an R&D organization.

Multiple criteria decision making (MCDM) refers to making decisions in the presence of multiple, usually, conflicting objectives. These problems can be solved either directly [10] or using different secularization forms (SOP) [11]. Most investigators in the general area of multiobjective mathematical programming agree that goal programming technique represents the work horse of multiobjective mathematical programming. Lee S.M. et al [3] introduced capital budgeting for multiple objectives. Zamfirescu, L. et al [4] prepared a goal programming as a decision model for performancebased budgeting.

Goal programming is found to be useful in real life situations, for many problems it may not be possible to satisfy certain specified goals within given constraints. Then problem then becomes one of maximizing the degree of attainment of these goals. Using goal programming for marketing decisions with a case study is illustrated by Lee, S. et al [5]. Lee, S. [6] introduced goal programming for decision analysis. Dauer, J.P. et al [7] introduce a finite iteration algorithm for solving general goal programming problems. The approach enables one to solve linear, no linear, integer and other goal programming problems using the corresponding optimization technique in an iterative manner.

In the real life systems, the uncertainties and vagueness accomplishing the determination of cost of production, the expected selling price and upper and lower bounds of the production of the spare parts make the problem fuzzy or stochastic rather than deterministic one.

44 Hu, C.F. et al [8] introduced a fuzzy goal programming approach to multi-objective optimization problem with  
 45 priorities. Khalili, K. et al [9] presented a paper about solving multi-period project selection problems with fuzzy  
 46 goal programming beased on TOPSIS and fuzzy preference relation. Eid, M.H. [12] gives methods of solving  
 47 integer multi criteria decision making problems with fuzzy parameters.

48 This paper introduces the analysis of the optimum vehicle spare parts production, where the problem of  
 49 distribution of fuzzy capital budget is applied.

50 The technique is introduced in Sakawa [13] for transforming fuzzy problems to non-fuzzy form is combined  
 51 with the interactive approach to goal programming [12] to develop the method of solution of such problem, where  
 52 the method is through two goals is used. The first goal is the fuzzy capital budgeting minimization, while the  
 53 second one is fuzzy profit maximization.

54 Our study is applied on a company which produces different types of vehicles' heat exchangers. The aim  
 55 of this study is to obtain the optimum number of heat exchangers which maximizes profits and minimizes the  
 56 production cost under fuzzy budget.

## 57 2 II.

### 58 3 Concept of Goal Programming a) General

59 Goal programming is a modification and extension of linear programming technique. The goal programming  
 60 approach allows a simultaneous solution of a system of complex objectives rather than a single objective. In  
 61 other words, goal programming is a technique that is capable of handling decision problems that deal with a  
 62 single goal and multiple sub-goals, as well as problems with multiple goals and multiple subgoals. In addition, the  
 63 objective function of a goal programming model may be composed of non-homogeneous units of measure, such  
 64 as pounds and dollars, rather than one type of units. Often, multiple goals of management are in conflict or are  
 65 achievable only at the expense of other goals. Furthermore, these goals are incommensurable. Thus, the solution  
 66 of the problem requires an establishment of a hierarchy of importance among these incompatible goals so that  
 67 the low-order goals are considered only after the higherorder goals are satisfied or have reached the point beyond  
 68 which no further improvements are desired. In goal programming, instead of trying to maximize or minimize the  
 69 objective criterion directly as in linear programming, deviations between goals and what can be achieved within  
 70 the given set of constraints are to be minimized [6].

### 71 4 b) Formulation of the Goal Programming Problem

72  $z = c_1x_1 + c_2x_2 + \dots + c_nx_n$   
 73  $s.t. \quad a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$   
 $a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$   
 $\vdots$   
 $a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m$   
 $x_1, x_2, \dots, x_n \geq 0$

74 Subject to: is the set representing the system constraints.  $\sum_{i=1}^n a_{ij}x_j = b_i, i = 1, 2, \dots, m$   
 75  $x_j \geq 0, j = 1, 2, \dots, n$

### 76 5 c) Methods of Solving Goal Programming Problems

77 There are more than one method for solving goal programming problems. From these methods, the most common  
 78 are:

- 79 1. The graphical method of goal programming.
  - 80 2. The simplex method of goal programming.
  - 81 3. The interactive approach of goal programming. These methods are illustrated in [6,7].
- 82 In this work the interactive approach of goal programming was used.

## 83 6 III.

### 84 7 Vehicle Spare Parts Production Formulation

85 The mathematical model of vehicle spare parts production in fuzzy environment which minimizes the total fuzzy  
 86 capital budgeting and maximizes the total expected fuzzy profit is formulated as an integer goal programming  
 87 problem with fuzzy parameters. There are two goals here, fuzzy capital budgeting goal and fuzzy profit goal.  
 88 Also, there are a set of constraints represent upper and lower bounds of the quantities which should be produced  
 89 from the different types of spare parts.

## 90 8 Let:

91  $x_i$ : is the quantity of production from spare parts  $i, i = 1, 2, \dots, n$ .  $c_i$ : is the production cost of the spare part  
 92  $i, i = 1, 2, \dots, n$ .

93  $u_i$ : is fuzzy upper bound of the production of spare parts  $i, i = 1, 2, \dots, n$ .  $p_1 \gg p_2$ : is  
 94 the priority structure.  $d_1^-, d_2^-$ : are the under achievements of the deviational variables.

95  $d_1^+, d_2^+$ : are the upper achievements of the deviational variables.

96 Then the problem takes the following form:  $\min z = c_1x_1 + c_2x_2 + \dots + c_nx_n$

97 Subject to:  $\sum_{i=1}^n x_i + \sum_{i=1}^n x_i = 1000000 + 1000000$ ,  $i = 1, 2, \dots, n$   
 98  $\sum_{i=1}^n x_i \geq 1000000$ ,  $i = 1, 2, \dots, n$   
 99  $\sum_{i=1}^n x_i \leq 1000000$ ,  $i = 1, 2, \dots, n$

100 **IV. Deterministic (crisp) form of the Problem**

101 The mathematical formulation of spare parts production in fuzzy environment problem can be transformed to  
 102 the following non-fuzzy form (fuzzy integer goal programming problem):  
 103  $\sum_{i=1}^n x_i + \sum_{i=1}^n x_i = 1000000 + 1000000$ ,  $i = 1, 2, \dots, n$

104 Subject to:  $\sum_{i=1}^n x_i + \sum_{i=1}^n x_i = 1000000 + 1000000$ ,  $i = 1, 2, \dots, n$   
 105  $\sum_{i=1}^n x_i \geq 1000000$ ,  $i = 1, 2, \dots, n$   
 106  $\sum_{i=1}^n x_i \leq 1000000$ ,  $i = 1, 2, \dots, n$   
 107  $\sum_{i=1}^n x_i = 1000000$ ,  $i = 1, 2, \dots, n$

108 **9 Vehicle Spare Parts Production Application a) Data Collection**

109  
 110 The application was carried out in a vehicle heat exchangers production factory which produce different different  
 111 types of heat exchangers used for different types of vehicles which operates with diesel engines.

112 Table (1) illustrates the variable name, company part number, minimum limit, maximum limit of production,  
 113 production cost and selling price. Where:  $x_1$  : is the number of heat exchangers for 3.5t transporter.  $x_2$  : is  
 114 the number of heat exchangers for 2.8t transporter.  $x_3$  : is the number of heat exchangers for 2t transporter.  $x_4$  :  
 115 is the number of heat exchangers for 2.5t transporter.  $x_5$  : is the number of heat exchangers for 5 cylinder  
 116 microbus.  $x_6$  : is the number of heat exchangers for 4 cylinder microbus.  $x_7$  : is the number of heat exchangers  
 117 for 5 cylinder microbus.  $x_8$  : is the number of heat exchangers for 5 cylinder microbus.  $x_9$  : is the number of  
 118 heat exchangers for 6t transporter.  $x_{10}$  : is the number of heat exchangers for 2.5t transporter.

119 The fuzzy budget allocated to the production (LE) = LE 4000000 + 1000000

120 The fuzzy expected profit (LE) = LE 200000 + 500000

121 **10 b) Mathematical Formulation of Applied Problem**

122 The applied problem takes the following form:  
 123  $\sum_{i=1}^n x_i + \sum_{i=1}^n x_i = 1000000 + 1000000$ ,  $i = 1, 2, \dots, n$   
 124  $\sum_{i=1}^n x_i \geq 1000000$ ,  $i = 1, 2, \dots, n$   
 125  $\sum_{i=1}^n x_i \leq 1000000$ ,  $i = 1, 2, \dots, n$

126 By taking the cut  $\alpha = 0.5$ , then table 2 illustrates values of  $S_j, T_j, Q_j, R_j, M_j, N_j$ .  
 127  $S_j = 250, T_j = 0.35, Q_j = 1, R_j = 0.7, M_j = 0.5, N_j = 0.3$   
 128  $S_j = 250, T_j = 0.35, Q_j = 1, R_j = 0.7, M_j = 0.5, N_j = 0.3$   
 129  $S_j = 250, T_j = 0.35, Q_j = 1, R_j = 0.7, M_j = 0.5, N_j = 0.3$   
 130  $S_j = 250, T_j = 0.35, Q_j = 1, R_j = 0.7, M_j = 0.5, N_j = 0.3$   
 131  $S_j = 250, T_j = 0.35, Q_j = 1, R_j = 0.7, M_j = 0.5, N_j = 0.3$   
 132  $S_j = 250, T_j = 0.35, Q_j = 1, R_j = 0.7, M_j = 0.5, N_j = 0.3$   
 133  $S_j = 250, T_j = 0.35, Q_j = 1, R_j = 0.7, M_j = 0.5, N_j = 0.3$

134 The problem is solved by a mixed integer linear programming package using the iterative approach of goal  
 135 programming. The optimum solution which minimizes the allocated capital budget and maximizes the profit is  
 136 given in Table (3). The minimum production cost is \$1544680.5 The maximum profit is \$666825.5 The  
 137 optimum quantities of heat exchangers are biased to the lower limits of the production as a result of solution of  
 138 the problem with the minimization of total capital budget goal with a higher priority than the maximization of  
 139 the expected profit goal.

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x 7	630 501 01 01	600	2750	80.3	130
x 8	655 501 86 01	1000	5500	71.7	150
x 9	350 501 19 01	600	2100	359	400
x 10	560 501 22 01	40	450	149.5	200

types of vehicle heat exchangers, for a group of ten

1) : Collected Data

Variable Name	Company Part Number	Minimum Limit of Production	Maximum Limit of Production	Production Cost (in \$)	Selling Price (in \$)
x 1	450 501 22 01	30	100	277	366
x 2	450 501 70 01	500	12000	175	255
x 3	450 501 24 01	4600	15000	144	203.2
x 4	450 501 19 01	600	1500	168	240
x 5	620 50591 04	700	1500	100	160
x 6	620 50594 04	50	1200	41.4	90

Figure 1: Table (

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Where:

? 200000 + 50000 ?? ?2 ?? ?1, ?? ?2, ?? ? ?? , ?? ?

Where:

? ?? , i= 1. 2, ?, n are fuzzy parameters. 30 + 4 ?

The fuzzy parameters are represented by the following membership function.

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Figure 2: 2 ?

(

Subject to:

	??	??	??	??	??	??	??	S	T	Q <sub>j</sub>	R <sub>j</sub>	M <sub>j</sub>	N <sub>j</sub>
	??	??	??	??	??	??	??	j	j				j
?? 1	3	4	7	10				3.5	8.5				
?? 2	1	2	3	4				1.5	3.5				
?? 1	1	2	4	6						1.5	5		
?? 2	1	3	4	6						2	5		
?? 3	1	3	6	8						2	7		
?? 4	1	2	3	5						1.5	4		
?? 5	1	2	3	5						1.5	4		
?? 6	1	2	4	6						1.5	5		
?? 7	1	2	3	5						1.5	4		
?? 8	2	4	7	8						3	7.5		
?? 9	1	2	3	5						1.5	4		
?? 10	1	2	5	7						1.5	6		
?? 1	1	2	3	5								1.5	4
?? 2	3	5	7	9								4	8
?? 3	2	4	7	9								3	8
?? 4	3	5	7	9								4	8
?? 5	2	4	4	6								3	5
?? 6	2	4	5	7								3	6
?? 7	1	3	7	8								2	7.5
?? 8	3	4	5	7								3.5	6
?? 9	2	4	4	6								3	5
?? 10	1	2	3	5								1.5	4

Then the problem takes the following non-fuzzy (Crisp) form:

$$?????. ?? = ?? 1 ?? 1 + + ?? 2 ?? 2 ?$$

Figure 3: Table ( 2

(

the membership functions determination and the ?-cut.

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Variable Name	Company Part Number	Quantity	Optimum
x 1	501 22 01 450 501 70 01 450	36	900
x 2	501 24 01 450 501 19 01 620	4900	750
x 3	50591 04 620 50594 04 630 501	805	65 750
x 4	01 01	1600	
x 5	655 501 86 01	750	
x 6	350 501 19 01	55	
x 7	560 501 22 01		

VI.

[Note: Conclusion]

Figure 4: Table ( 3

- 
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