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A New Look into the Controllability and Observability of Lyapunov type Matrix Dynamical Systems on Measure Chains

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ANEWLOOKINTOTHECONTROLLABILITYANDOBSERVABILITYOFLYAPUNOVTYPEMATRIXDYNAMICALSYSTEMSONMEASURECHAINS

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A New Look into the Controllability and Observability of Lyapunov type Matrix Dynamical Systems on Measure Chains

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I. Introduction

Recently the concept of time scales(or measure chains) is growing very rapidly into many areas of research. One reason being it includes both continuous and discrete systems as special cases. Hence many researchers [1] – [4] and [8]-[11] explored different concepts on time scales. Stephan Barnett [12] studied the control theory for both the continuous and discrete cases for simple dynamical systems. Control theory has developed rapidly over the past two decades and is now established as an important area of contemporary Applied Mathematics. Many problems of great importance in the contemporary world require a quite different approach, the aim being to compel or control a system to behave in some desired fashion. Basically control theory has involved the study of analysis and control of any dynamical system. This theory has been successfully applied in a variety of branches in the disciplines of engineering and particularly it is receiving great impetus from Aerospace engineering. A fascinating fact is that all the widely different disciplines of applications depend on a common core of mathematical techniques of the modern control system theory.

In this paper we establish the concept of controllability and observability of dynamical systems on measure chains. The results presented in this chapter generalises the existing results on controllability and observability for continuous and discrete cases and includes them as a particular case. This paper is organised as follows; In section 2, we outline the salient features of time scales. In section 3, we briefly mention the necessary and sufficient conditions for the controllability and observability of vector dynamical systems on time scales.

$$S_1 \begin{cases} x^{\Delta}(t) = A(t)x(t) + B(t)u(t), \quad x(t_0) = x_0 \\ y(t) = C(t)x(t) \end{cases}$$

where A is $n \times n$, B is $n \times m$ and C is $r \times n$ matrices whose elements are rd-continuous on a time scale $T = [t_0, t_N]$, the control u is (mx1) and x is an (nx1)vector. Section 4 presents the controllability and observability criteria of the most general matrix Lyapunov differential system on measure chains

$$S_2 \begin{cases} X^{\Delta}(t) = A(t)X(t) + X(\sigma(t))B(t) + K(t)U(t), \quad X(t_0) = X_0 \\ Y(t) = C(t)X(t) \end{cases} \qquad (1.1)$$

where A(t), B(t), K(t) and C(t) are square matrices of orders n×n, nxn, nxs and rxn respectively. The control U(t) is an sxn matrix and the out put Y(t) is an rxn matrix whose elements are rd-continuous on a measure chain $T = [t_0, t_N]$. We firmly believe that these results will have a significant impact on control engineering problems.

II. Salient Features of Time Scales

A measure chain (or time scale) is an arbitrary closed subset of real numbers R and it is denoted by T throughout the paper. Time scales are not necessarily connected and this topological handicap is eliminated by introducing the notion of jump operators σ and ρ as follows:

Definition 2.1: Let T be a measure chain. For $t \in T$ define the forward jump operator $\sigma : T \to T$ by $\sigma(t) = \text{Inf}\{s \in T : s > t\}$ and the backward jump operator $\rho : T \to T$ by $\rho(t) = \text{Sup}\{s \in T : s < t\}$.

A point $t \in T$ is said to be right dense, right scattered, left dense and left scattered according as $\sigma(t) = t$, $\sigma(t) > t$, $\rho(t) = t$ and $\rho(t) < t$ respectively. The grainness $\mu : T \to [0, \infty)$ is defined by $\mu(t) = \sigma(t) - t$.

The set T^k which is derived from the measure chain T as follows:

If T has a left scattered maximum m, then $T^k = T - \{m\}$, otherwise $T^k = T$.

Definition 2.2: Let $f : T \to R$, $t \in T^k$. Then define $f^{\Delta}(t)$ to be the number (provided it exists) with the property that given any $\epsilon > 0$, there exists a neighbourhood $\cup$ of t such that

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$|[f(\sigma(t)) - f(s)] - f^{\Delta}(t)[\sigma(t) - s]| \leq \in |\sigma(t) - s|$ for all $s \in \cup$

Then $f^{\Delta}(t)$ is called the delta derivative of f at t.

If T = R, the delta derivative is same as that of ordinary derivative and for T = Z, $f^{\Delta}(t) = f(t+1) - f(t) = \Delta f(t)$, which is the forward difference operator.

Definition 2.3 : We say that f is delta differentiable on T^k, if $f^{\Delta}(t)$ exists for all $t \in T^k$.

Result 2.1 : Assume f, g : T→R are delta differentiable functions at $t \in T^k$, then

- f+g : T →R is delta differentiable at t with $(f+g)^{\Delta}(t) = f^{\Delta}(t) + g^{\Delta}(t)$.
- For any constant k, kf : T→ R is delta differentiable at t with $(kf)^{\Delta}(t) = k f^{\Delta}(t)$.
- fg : T x T→ R is delta differentiable at t with

$$(fg)^{\Delta}(t) = f^{\Delta}(t)\, g(t) + f(\sigma(t))\, g^{\Delta}(t)$$
$$= f(t)\, g^{\Delta}(t) + f^{\Delta}(t)\, g(\sigma(t)) \;.$$

Definition 2.4: A function F : $T^k \to R$ is called an antiderivative of f : $T^k\ ^{\Delta}R$, provided $F^{\Delta}(t) = f(t)$ holds for all $t \in T^k$. Then the delta integral of f is defined = F (t) – F (a)) ∀ t ∈T.

Definition 2.5: Let f : T→T be a function. We say that f is rd - continuous if it is continuous in right dense points and if limit f (s) exists as s → t - for all left-dense points t ∈ T.

Result 2.2: Rd - Continuous functions possess an anti derivative.

Proof: For the proof we refer [2].

Definition (Controllability) 2.6 : The linear time varying dynamical system S_1 on measure chain $T=[t_0, t_N]$ is completely controllable if "for any initial time t0 and any initial state $x(t_o) = x_o$ and any given final state xf, there exists a finite time tN > to and a control u(t), to ≤ t ≤ t_N such that $x(t_N) = xf$".

Definition 2.7 (Observability) : The linear time varying dynamical system defined by S_1 on measure chains is completely observable if and only if the knowledge of the control u(t) and the out put y(t) suffice to determine $x(t_0) = x_0$ uniquely for a finite time $t_N \geq t_0$.

III. Controllability and Observability of Vector Dynamical Systems on Measure Chains

Theorem 3.1: The system S_1 is completely controllable if and only if the n × n symmetric controllability matrix $W(t_o, t_N)$ =

$$\int_{t_o}^{t_N} (\phi(t_o,\sigma(s)) B(s) B^*(s) \phi^*(t_o,\sigma(s)))\Delta s$$

is non - singular where ϕ (t) is a fundamental matrix of $x^{\Delta}(t) = A(t)\, x(t)$. In this case the control

$$u(t) = - B^*(t)\, \phi^*(t_o,\sigma(t))\, W^{-1}(t_o\, t_N)\, [x_o - \phi(t_o, t_N)\, x_f] \quad (3.2)$$

defined on to ≤ t ≤ t_N transfers $x(t_o) = x_o$ to $x(t_N) = xf$. Here * denotes the transpose of the matrix.

Proof : Please refer [6]

Theorem 3.2 : Suppose $u^-(t)$ is any other control taking $X(t_0) = x_0$ to $x(t_N) = xf$.

$$\text{Then} \quad \int_{t_o}^{t_N} \|\bar{u}(s)\|^2 \Delta s \quad \int_{t_o}^{t_N} \|u(s)\|^2 \Delta s\,,$$

where $u(s) = - B^*(s)\, \phi^*\, to, \sigma(s))\, W^{-1}(t_o,t_N)\, [X_o - \phi(t_o,t_N) xf]$

provided $u(s) \neq u(s)\ \forall s \in [t_o, t_N]$.

Proof : Please refer [6]

Theorem 3.3 : The time varying dynamical system S_1 on measure chain $T=[t_0, t_N]$ is completely observable if and only if the symmetric observability matrix $V_1(t_0, t_N)$ = $\int_{t_o}^{t_N} \phi^*(s,t_0)\, C^*(s)\, C(s)\, \phi(s,t_0)\, ^{\Delta}s$ is non singular.

Proof : Please refer [6]

IV. Controllability and Observability of Lyapunov Type Matrix Dynamical Systems on Measure Chains

In this section we consider Lyapunov type matrix dynamical system on measure chains

$$X^{\Delta}(t) = A(t)X(t) + X(\sigma(t))B(t) + K(t)U(t), \quad X(t_0) = X_0 \quad (4.1)$$

and obtain necessary and sufficient conditions for the controllability and observability of S_2. Throughout this section $\phi_1(t)$ and $\phi_2(t)$ represent fundamental matrix solutions of $x_{\Delta}(t) = A(t)X(t)$ and $x_{\Delta}(t) = B^*(t)\, X(\sigma(t))$ respectively.

Theorem 4.1 : The solution of the equation (4.1) is given by

$$X(t) = \phi_1(t)\, \phi_1^{-1}(t_0) X_0 \phi_2^{*\,-1}(t_0)\, \phi_2^*(t) + \phi_1(t)$$
$$\left[\int_{t_0}^{t} \phi_1^{-1}(\sigma(s)) K(s) U(s) \phi_2^{*\,-1}(s)\Delta s\right] \phi_2^*(t) := \phi_1(t,t_0)$$
$$\left[X_0 + \int_{t_0}^{t} \phi_1(t_0,\sigma(s)) K(s) U(s) \phi_2^*(t_0,s)\Delta s\right] \phi_2^*(t,t_0)$$

where $\phi_i(t,t_0) = \phi_i(t)\phi_i^{-1}(t_0)$ (i=1,2)

Proof : Please refer [6]

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$$X(t) = \phi_1(t)\ \phi_1^{-1}(t_0)X_0\phi_2^{*-1}(t_0)\ \phi_2^*(t) + \phi_1(t)$$

$$\left[\int_{t_0}^{t}\phi_1^{-1}(\sigma(s))K(s)U(s)\phi_2^{*-1}(s)\Delta s\right]\phi_2^*(t) .= \phi_1(t,t_0)$$

$$\left[X_0 + \int_{t_0}^{t}\phi_1(t_0,\sigma(s))K(s)U(s)\phi_2^*(t_0,s)\Delta s\right]\phi_2^*(t,t_0)$$

where $\phi_i(t,t_0) = \phi_i(t)\phi_i^{-1}(t_0)$ (i=1,2)

Proof : Please refer [6]

Theorem 4.2 : The Lyapunov type matrix dynamical system on measure chains S_2 is completely controllable if and only if the symmetric controllability matrix $W_1(t_0,t_N) = \int_{t_0}^{t_N}\phi_1(t_0,\sigma(s))K(s)K^*(s)\phi_1^*(t_o,\sigma(s))\Delta s$ is non singular.

Then the control U (t) defined by

$$U(t) = -K^*(t)$$

$$\phi_1^*(t_0,\sigma(t))W_1^{-1}(t_0,t_N)\left[X_0 - \phi_1(t_0,t_N)X_f\phi_2^*(t_0,t_N)\right]\phi_2^{*-1}(t_0,t)$$

defined for $t_0 < t < t_N$ transfers $X(t_0) = X_0$ to $X(t_N) = X_f$.

Proof : First we suppose that $W_1(t_0,t_N)$ is non - singular. Then U (t) defined as above exists. We know that any solution of (4.1) has the form

$$X(t) = \phi_1(t,t_0)$$

$$\left[X_0 + \int_{t_0}^{t}\phi_1(t_0,\sigma(s))K(s)U(s)\phi_2^*(t_0,s)\Delta s\right]\phi_2^*(t,t_0)$$

put $t = t_N$ and substitute U(t) defined as above we will get $X(t_N) = X_f$ and hence S_2 is controllable.

Conversely, suppose that S_2 is controllable. We have to show that $W_1(t_0,t_N)$ is non-singular. Since $W_1(t_0,t_N)$ is symmetric, clearly it is positive semi definite.

Now suppose that there exists some column vector $\Omega_1 \neq 0$ such that $\Omega_1^* W_1(t_0,t_N)\Omega_1 = 0$

$$\Rightarrow \int_{t_0}^{t_N}\Omega_1^*\phi_1(t_0,\sigma(s))K(s)k^*(s)\phi_1^*(t_0,\sigma(s))\Omega_1\Delta s = 0$$

$$\Rightarrow \int_{t_0}^{t_N}\theta_1^*(s,t_0)\theta_1(s,t_0)\Delta s = 0, \text{ where } \theta_1(s,t_0) =$$

$$K^*(s)\phi_1^*(t_0,\sigma(s))\Omega_1$$

$$\Rightarrow \int_{t_0}^{t_N}\|\theta_1\|^2\Delta s = 0. \quad \text{Hence } \theta_1 \equiv 0 \text{ on } [t_0, t_N].$$

By our assumption since S_2 is completely controllable, there exists a control V (t) (say) making $X(t_N) = 0$ if $X(t_0)$ $\Omega_1 g$ where g is any non zero 1xn matrix.

$$X(t_N) = 0 \Rightarrow X(t_0) + \int_{t_0}^{t_N}\phi_1(t_0,\sigma(s))K(s)V(s)\phi_2^*(t_0,s)\Delta s = 0$$

$$\Rightarrow \Omega_1 g = -\int_{t_0}^{t_N}\phi_1(t_0,\sigma(s))K(s)V(s)\phi_2^*(t_0,s)\Delta s \text{ Now}$$

$$\|\Omega_1 g\|^2 = (\Omega_1 g)^*(\Omega_1 g) = -$$

$$\int_{t_0}^{t_N}\phi_2(t_0,s)V^*(s)K^*(s)\phi_1^T(t_0,\sigma(s))\Omega_1 g\Delta s = 0.\Omega_1 g =$$

$0 \Rightarrow \Omega_1 = 0$ which is a contradiction.

Therefore $W_1(t_0, t_N)$ is positive definite, consequently it is non - singular.

Theorem 4.3 : The system S_2 is completely observable if and only if the symmetric observability matrix

$V(t_0, t_N) = \int_{t_0}^{t_N}\phi_1^*(s,t_0)C^*(s)C(s)\phi_1(s,t_0)\Delta s$ is non singular.

Proof: Suppose $V(t_0, t_N)$ is nonsingular. Without loss of generality suppose that $U(t) \equiv 0\ \forall t \in [t_0, t_N]$ then $X(t) = \phi_1(t,t_0)X_0\ \phi_2^*(t,t_0)$.

For this the out put is $Y(t) = C(t)\ \phi_1(t,t_0)X_0\ (t,t_0)$

$$\Rightarrow \phi_1^*(t,t_0)\ C^*(t)\ Y(t)\ \phi_2^{*-1}(t,t_0) = \phi_1^*(t,t_0)C^*(t)C(t)\ \phi_1^*(t,t_0)X_0 .$$

Integrating from t_0 to t_N we get

$$\int_{t_0}^{t_N}\phi_1^*(s,t_0)C^*(s)Y(s)\phi_2^{*-1}(s,t_0)\Delta s = V(t_0, t_N)X_0.$$

$$X_0 = V^{-1}(t_0,t_N)\int_{t_0}^{t_N}\phi_1^*(s,t_0)C^*(s)Y(s)\phi_2^{*-1}(s,t_0)\Delta s$$

Therefore S_2 is observable. Conversely suppose S_2 is completely observable. We will show that $V(t_0, t_N)$ is non singular. Since $V(t_0, t_N)$ is symmetric, clearly it is positive semidefinite. If possible suppose that there exists a column vector $\Omega_1 \neq 0$ such that

$\Omega_1^*\ V\ (t_0, t_N)\Omega_1 = 0.$ Then

$$\int_{t_0}^{t_N}\|C(s)\phi_1(s,t_0)\Omega_1\|^2\Delta s = 0$$

$\Rightarrow C(s)\phi_1(s,t_0)\ \Omega_1 = 0\ \ \forall\ s \in [t_0, t_N]$. If $X_0 = \Omega_1 k$ where k is row vector of order 1xn. Then the output is $Y(t) = = C(t)\phi_1(t,t_0)X_0\phi_2^*(t,t_0)$

$= C(t)\phi_1(t,t_0)\Omega_1 k\ \phi_2^*(t,t_0) = 0$.

i.e X_0 can not determined with the knowledge of Y(t) in this case. This contradicts our assumption that S_3 is completely observable. Therefore $V(t_0, t_N)$ is positive definite and hence $V(t_0, t_N)$ is non - singular.

Observation 4.1: From the theorem (4.2) it is observed that the controllability matrix is independent of ϕ_2. The fundamental matrix ϕ_1 alone determines the controllability criterion of the dynamical system S_2. We also observe that the controllability criterion can be determined using the fundamental matrix ϕ_2 alone. In this case the controllability matrix is given by $W_2(t_0,t_N) = \int_{t_0}^{t_N}(\phi_2(t_0,\sigma(s))K(s)K^*(s)\phi_2^*(t_o,\sigma(s))\Delta s$, and the control U(t) defined by

$$U(t) = -K^*(t)$$

$$\left(K(t)K^*(t)\right)^{-1}\phi_1^{-1}(t_0,\sigma(t))\left[X_0 - \phi_1(t_0,t_N)X_f\phi_2^*(t_0,t_N)\right]$$

$$W_2^{-1}(t_0,t_N)\phi_2(t_0,t)K(t)K^*(t)$$

defined for t0 < t < t_N transfers $X(t_0) = X_0$ to $X(t_N) = Xf$.

Observation 4.2 : From the theorem (4.3) it is observed that the observability matrix is independent of ϕ_2. The fundamental matrix ϕ_1 alone determines the observability criterion of the dynamical system S_2. We can also determine the observability criterion of S_2 using the fundamental matrix ϕ_2 alone.

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