

Modeling, Simulation and Control of 2-R Robot

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Abstract

This article presents a study of Three PID controller technique of a 2-Revolute joint robot. First we present Denavit- Hartenberg parameters for 2-R robot. Then we studied the dynamics of the 2-R robot and derived the nonlinear equations of motion. A PID controller has been implemented for three types of modeling technique: model based on linearization about equilibrium point, model based on Autodesk Inventor and Matlab/Simulink software's, and lastly model based on feedback linearization of the robot. A comparison between the three controllers is presented showing the effectiveness of each technique.

Index terms— robotics, 2-R robot, dynamic, modeling, simulation, control and PID.

1 Introduction

Robotics is the science that deals with robot's design, modeling and controlling. Nowadays robots are used everywhere in everyday life. It has accompanied people in most of industry and daily life jobs. (Gouasmi, Ouali, Fernini, & Meghatria, 2012).

The range of robot utilization is very wide. A large family of robots is used in industry and manufacturing. Robots are used in supplying the motion required in manufacturing processes such as pick and place, assembly, painting, milling, cutting, welding, drilling, etc.

Because of different types of tasks different manipulator configurations are available such as rectangular, cylindrical, spherical, revolute and horizontal jointed (Gouasmi et al., 2012).

A two revolute joint robot configuration with two degrees of freedom is generally well-suited for small parts insertion and assembly, like electronic components. Although the final goal is to design and manufacture real robotics, it is very useful to perform simulations prior to investigations with real robots. Simulations are easier to setup, less expensive, faster and more convenient to use. It allows better design exploration and helps you enhance your final real robot by selecting suitable parameters for the system you want to design (Lajpah, 2008).

There are many control techniques used to control a robot arm. The most used ones are the PID control, optimal control, adaptive control and robust control. "There are many kinds of controllers that can be used to cause a designed robot arm to move along a desired trajectory" (Sukvichai, 2008). The simplest which we used in this paper to control the robot arm is the PID controller. The Denavit-Hartenberg (D-H) parameters for the 2-R robot will be defined as in the table below. The initial position (at $t = 0$) from the homogeneous transformation matrix where $\theta_1 = 0^\circ$ and $\theta_2 = 0^\circ$ are shown in figure (2).

2 II.

3 Problem Formulation

4 Robot Dynamics

Description of x and y in terms of θ_1 and θ_2 in term of linear displacement: $x = l_1 \sin \theta_1 + l_2 \sin(\theta_1 + \theta_2)$ and $y = l_1 \cos \theta_1 + l_2 \cos(\theta_1 + \theta_2)$

So, Kinetic Energy could be formed as: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} J_2 \dot{\theta}_2^2$ (1)

10 CONCLUSION

Substitute for v_1 and v_2 KE $\frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 l_2^2 \dot{\theta}_2^2 + \frac{1}{2} m_1 l_1 \dot{\theta}_1^2 + \frac{1}{2} m_2 l_2 \dot{\theta}_2^2 \cos^2 \theta_2$
+ $\frac{1}{2} m_2 l_2^2 \dot{\theta}_2^2 + \frac{1}{2} m_2 l_2 \dot{\theta}_1 \dot{\theta}_2 + \frac{1}{2} m_2 l_2 \dot{\theta}_1 \dot{\theta}_2 + \frac{1}{2} m_2 l_2 \dot{\theta}_1 \dot{\theta}_2$ (2)
And Potential Energy is $PE = m_1 g l_1 \sin \theta_1 + m_2 g (l_1 \sin \theta_1 + l_2 \sin (\theta_1 + \theta_2))$ (3)
a) Equations of motion
The Lagrangian of a dynamic system is defined as the difference between the kinetic and potential energy at an arbitrary instant (N.Jazar, 2010).

5 $L = KE - PE$ So, by Lagrange Dynamics, we form the Lagrangian

$L = \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 l_2^2 \dot{\theta}_2^2 + \frac{1}{2} m_1 l_1 \dot{\theta}_1^2 + \frac{1}{2} m_2 l_2 \dot{\theta}_2^2 \cos^2 \theta_2 + \frac{1}{2} m_2 l_2^2 \dot{\theta}_2^2 + \frac{1}{2} m_2 l_2 \dot{\theta}_1 \dot{\theta}_2 + \frac{1}{2} m_2 l_2 \dot{\theta}_1 \dot{\theta}_2 + \frac{1}{2} m_2 l_2 \dot{\theta}_1 \dot{\theta}_2$ (4)
Using Lagrange to form generalized equations of motion in matrix form as: $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_1} \right) - \frac{\partial L}{\partial \theta_1} = \tau_1$ and $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_2} \right) - \frac{\partial L}{\partial \theta_2} = \tau_2$
 $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_1} \right) = m_1 l_1 \dot{\theta}_1 + m_2 l_2 \dot{\theta}_2 \cos \theta_2$ and $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_2} \right) = m_2 l_2 \dot{\theta}_2 \cos \theta_2 + m_2 l_2 \dot{\theta}_1 \sin \theta_2$
 $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_1} \right) + \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_2} \right) = (m_1 l_1 + m_2 l_2) \dot{\theta}_1 + m_2 l_2 \dot{\theta}_2 \cos \theta_2 = M \dot{\theta}$ (5)
And the general form is: $H(\theta) \ddot{\theta} + C(\dot{\theta}, \theta) \dot{\theta} + g(\theta) = M \ddot{\theta}$ IV.

6 Pid Controller based on Linear Model

We define new variables in order to convert the 2-R robot to an equivalent linear model. $x_1 = \theta_1$, $x_2 = \dot{\theta}_1$, $x_3 = \theta_2$, $x_4 = \dot{\theta}_2$
 $\dot{x}_1 = x_2$, $\dot{x}_2 = x_3$, $\dot{x}_3 = x_4$, $\dot{x}_4 = x_5$
Rewrite the equation of motion using these variables, and use new constants c_1 to c_6 function of robot specifications to make equations in simple form $\ddot{x}_1 = c_1 x_2 + c_2 x_3 + c_3 x_4 + c_4 x_5$ and $\ddot{x}_3 = c_5 x_4 + c_6 x_5$
 $\ddot{x}_1 = c_1 x_2 + c_2 x_3 + c_3 x_4 + c_4 x_5$ (6) and $\ddot{x}_3 = c_5 x_4 + c_6 x_5$ (7)
 $x_1 = x_3$ (8) and $x_2 = x_4$ (9)
 $Y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \end{bmatrix} D$ a) Linearized model
We substitute values of constants c_1 to c_6 into the state-space model to get the state space matrices:

7 H

Now we can write the state-space model using linearization about the equilibrium point: $\theta_1 = 0$, $\dot{\theta}_1 = 0$, $\theta_2 = 0$, $\dot{\theta}_2 = 0$
 $M_1 = 0$, $M_2 = 0$
We Perform Taylor series expansion of the nonlinear functions and neglect high-order terms, to get the linearized model. At equilibrium point: Linearization of the variable x_1 with respect to other variables: $\frac{\partial L}{\partial x_1} = 0$, $\frac{\partial L}{\partial x_2} = 0$, $\frac{\partial L}{\partial x_3} = 1$, $\frac{\partial L}{\partial x_4} = 0$
Linearization of the variable x_1 with respect to other variables: $\frac{\partial L}{\partial x_2} = 0$, $\frac{\partial L}{\partial x_3} = 0$, $\frac{\partial L}{\partial x_4} = 0$, $\frac{\partial L}{\partial x_5} = 0$
Linearization of the variable x_1 with respect to other variables: Linearization of the variable x_1 and x_2 with respect to input torques: $\frac{\partial L}{\partial \tau_1} = 0$, $\frac{\partial L}{\partial \tau_2} = 0$, $\frac{\partial L}{\partial \tau_1} = 0$, $\frac{\partial L}{\partial \tau_2} = 0$, $\frac{\partial L}{\partial \tau_1} = c_5$, $\frac{\partial L}{\partial \tau_2} = c_6$
We can write the state-space model: $\dot{x} = A x + B u$ and $y = C x + D u$
 $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$, $D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

8 Pid Controller based on Feedback Linearization

Having system's equation $H(\theta) \ddot{\theta} + C(\dot{\theta}, \theta) \dot{\theta} + g(\theta) = M \ddot{\theta}$ While: $M = H$
This way, we decoupled the system to have the (non-physical) torque input: $M \ddot{\theta} = H \ddot{\theta}$
However, the physical torque inputs to the system are: $M \ddot{\theta} = H \ddot{\theta}$
To design the feedback PID controller, error signals are assumed to be: We notice that the response is following the control signal with relatively good manner. And errors of θ_1 and θ_2 are equal to zero in a short time.

9 VII.

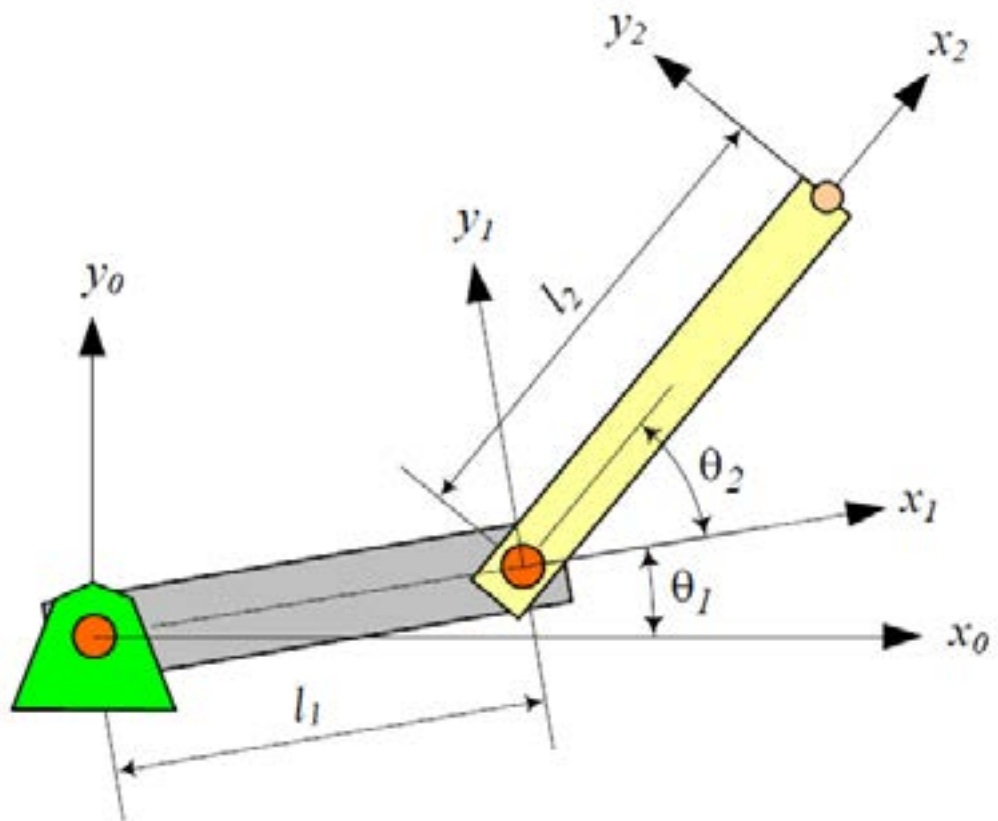
10 Conclusion

The main content of this paper is about modeling a 2-R robot using two methods: first is mathematical modeling using Lagrange dynamic equations and the second is using Autodesk Inventor and Simulink software's to develop the model. After that we used PID controller to validate the models and to notice the difference in accuracy achieved by each technique. Linearization about working point is valid in one point only, while it is no longer



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Figure 1: ?" 1 ?" 2



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Figure 2: Fig. 1 :

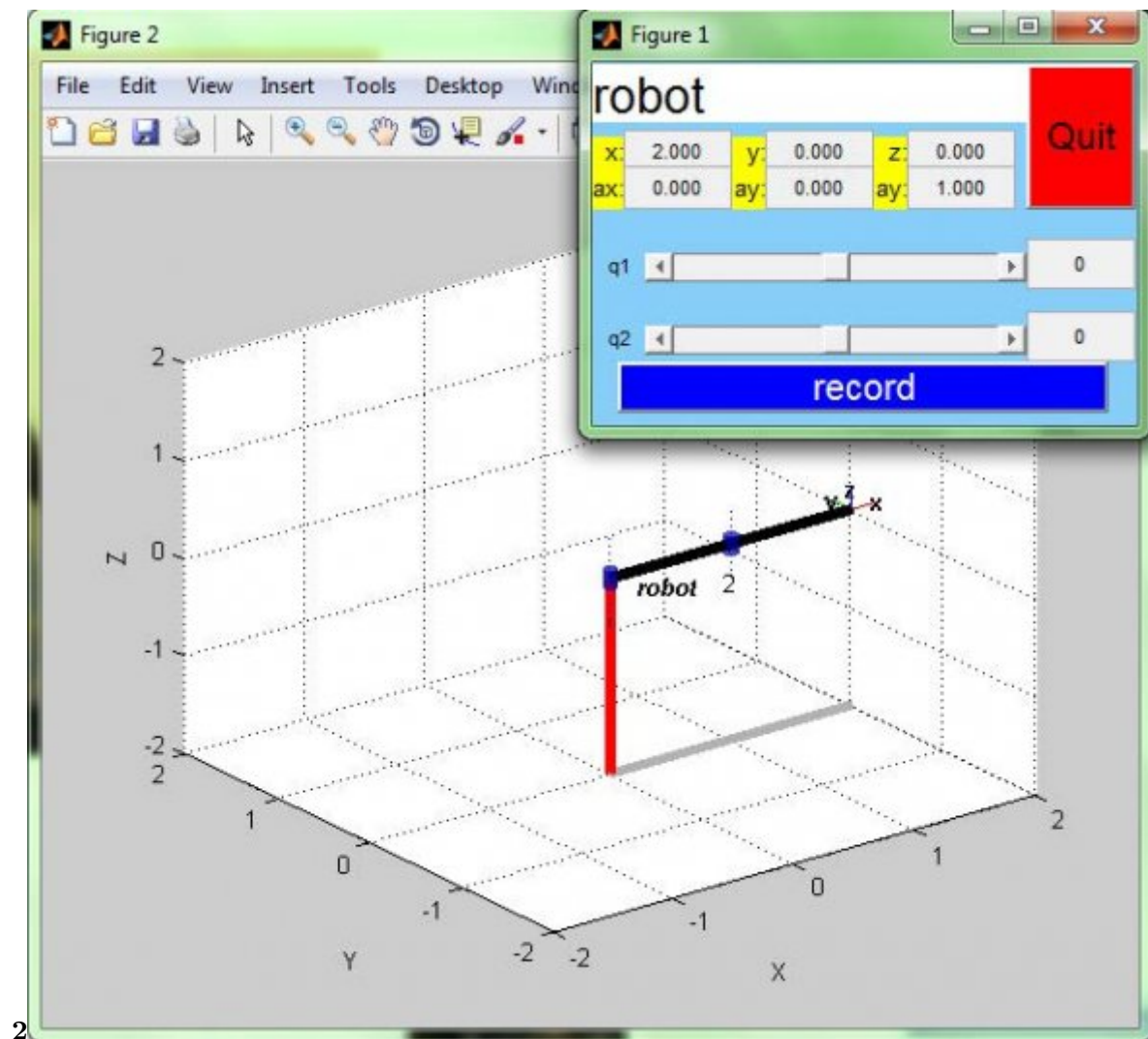


Figure 3: Fig. 2 :

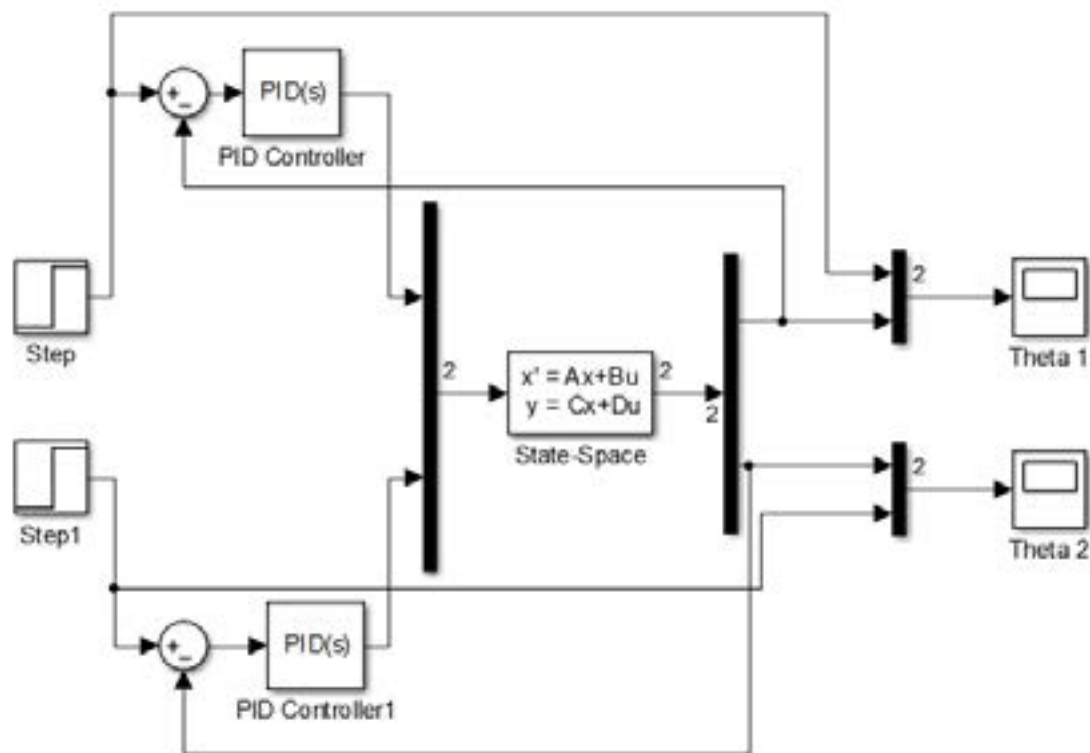
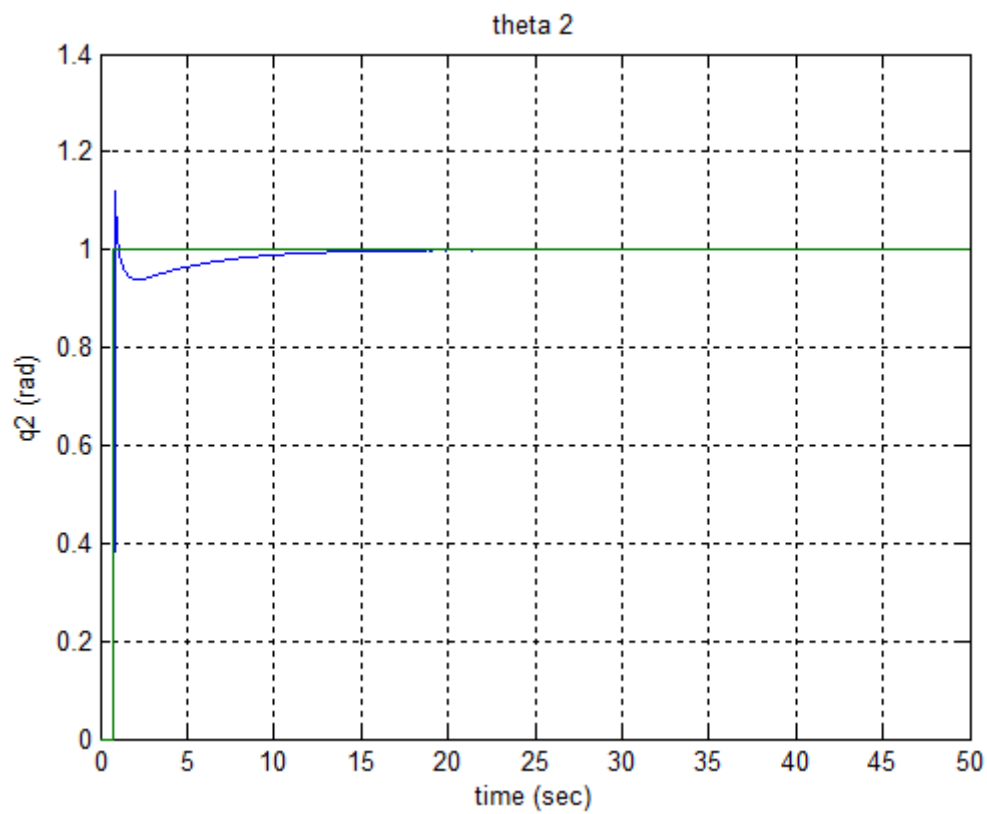
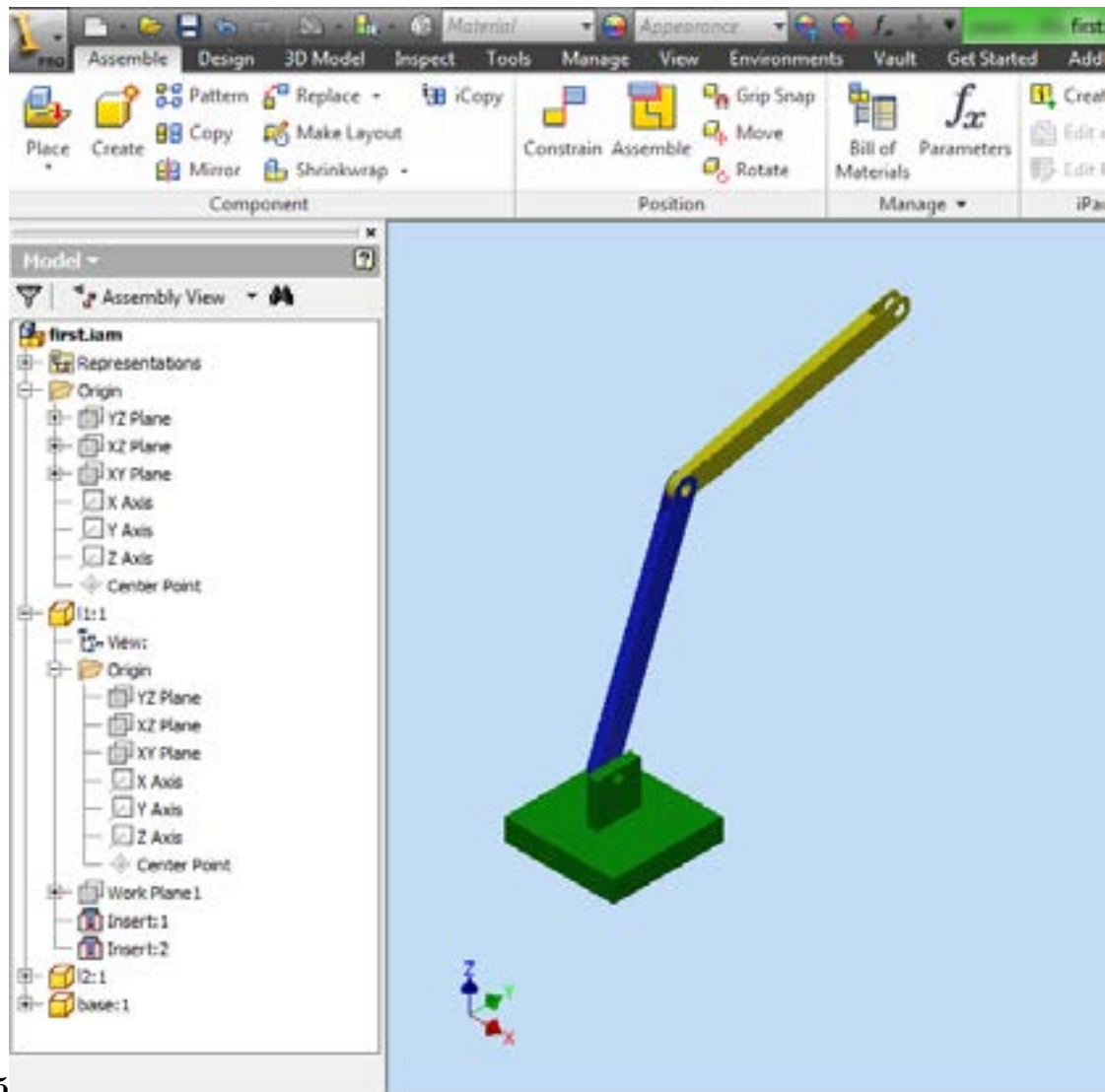


Figure 4:



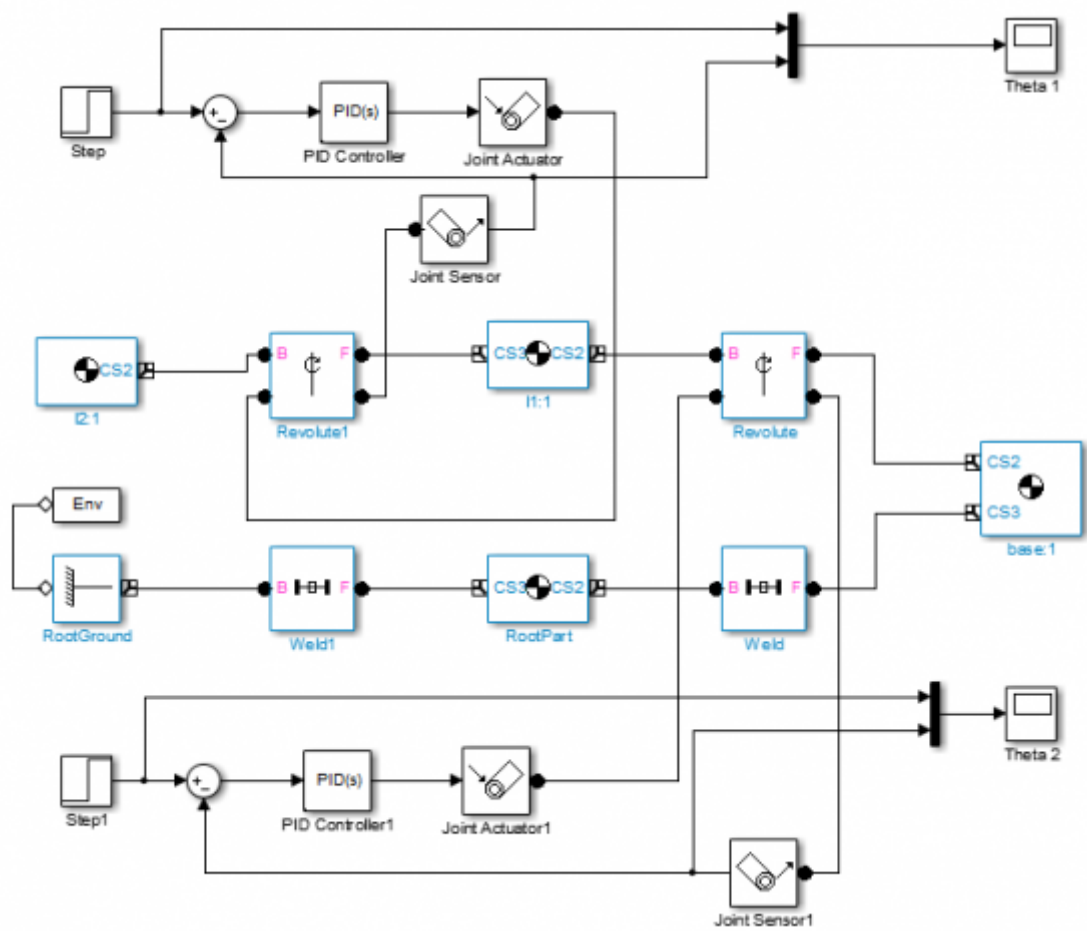
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Figure 5: Fig. 3 :



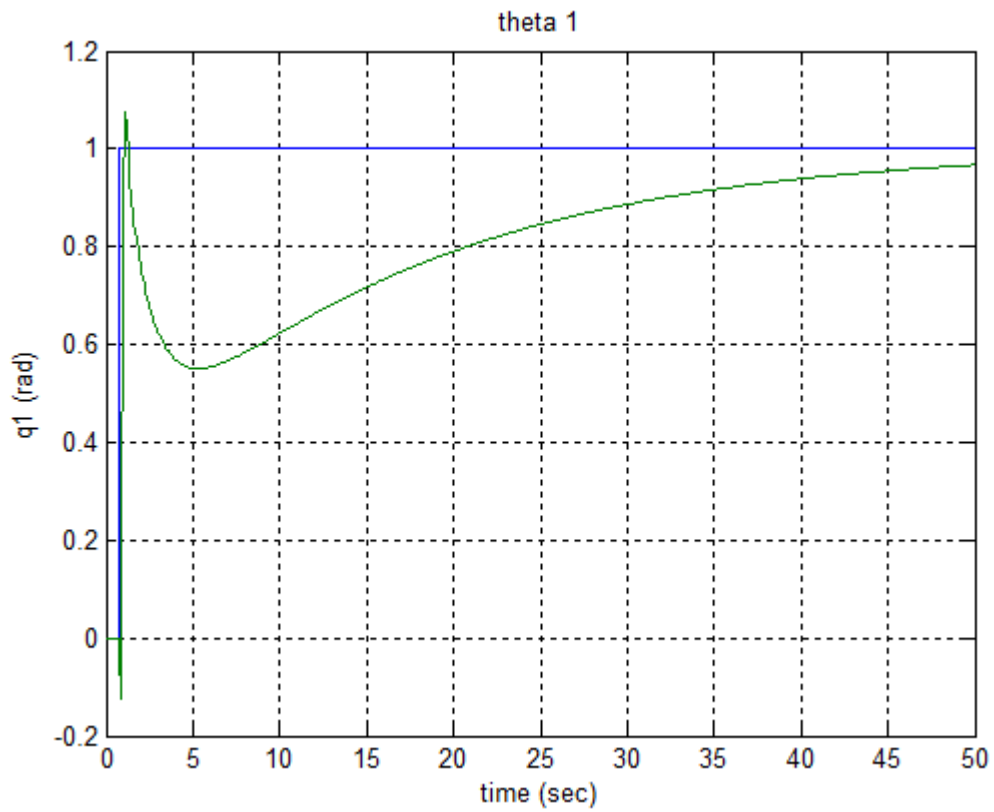
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Figure 6: Fig. 4 :Fig. 5 :

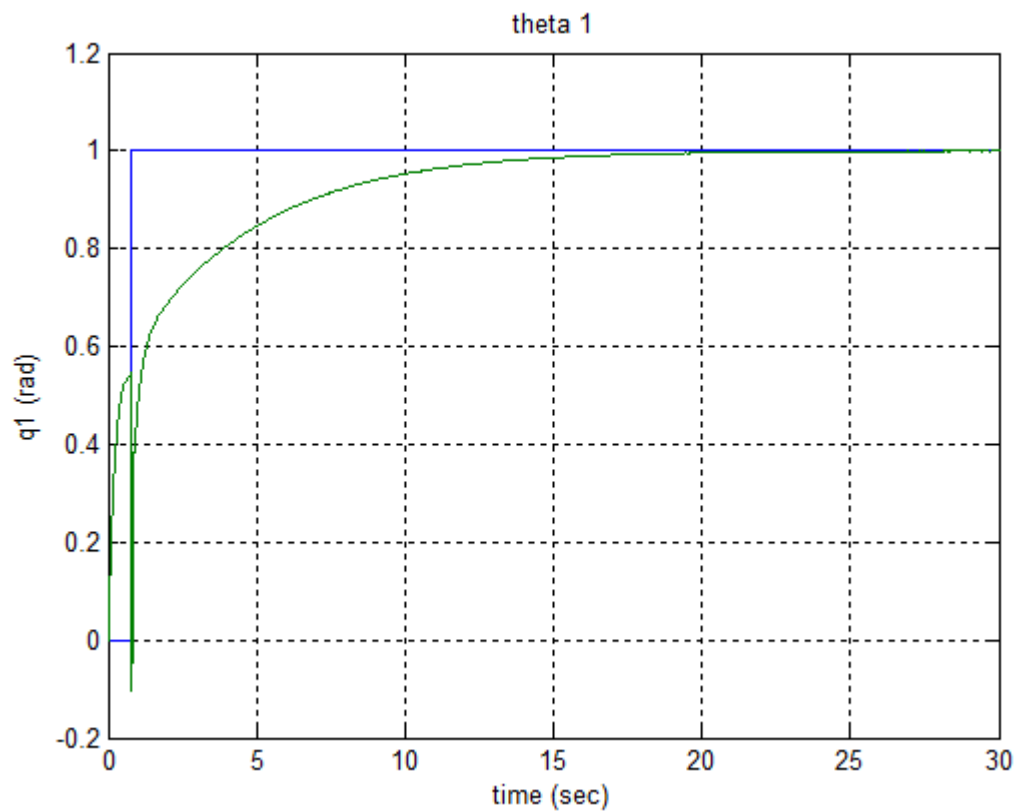


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Figure 7: Fig. 6



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Figure 8: $e^{-1} = 4$, $\tau = 2f = \pi/4$


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Figure 9: Fig. 11 :

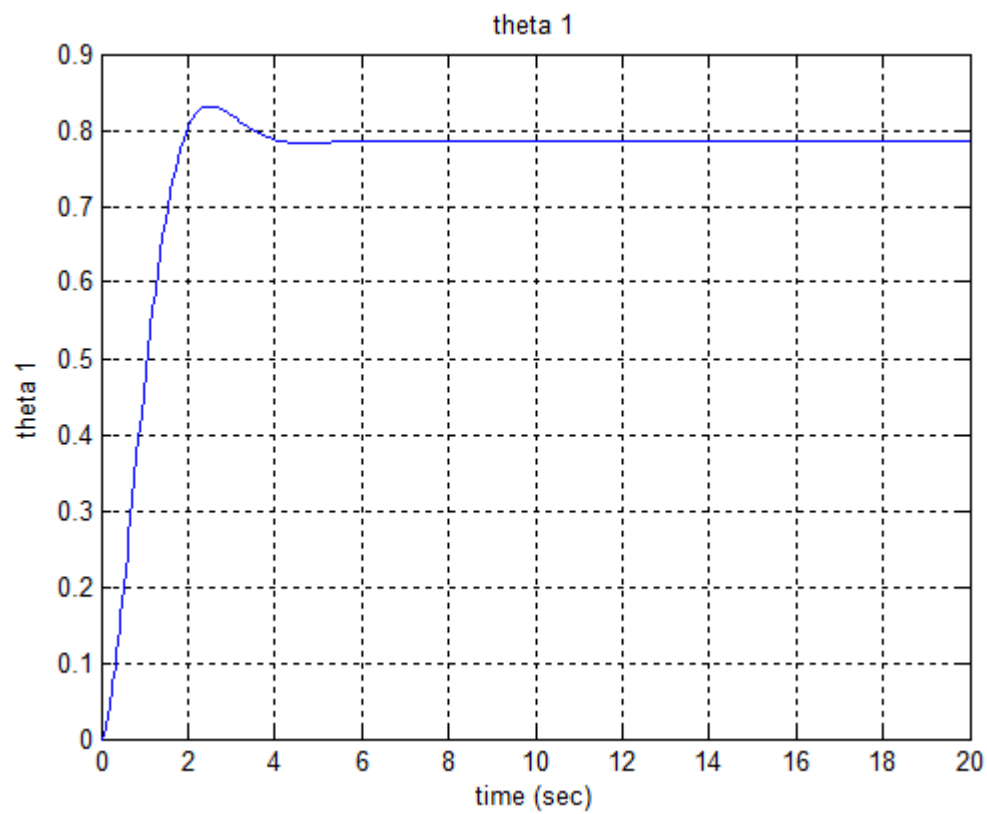


Figure 10:

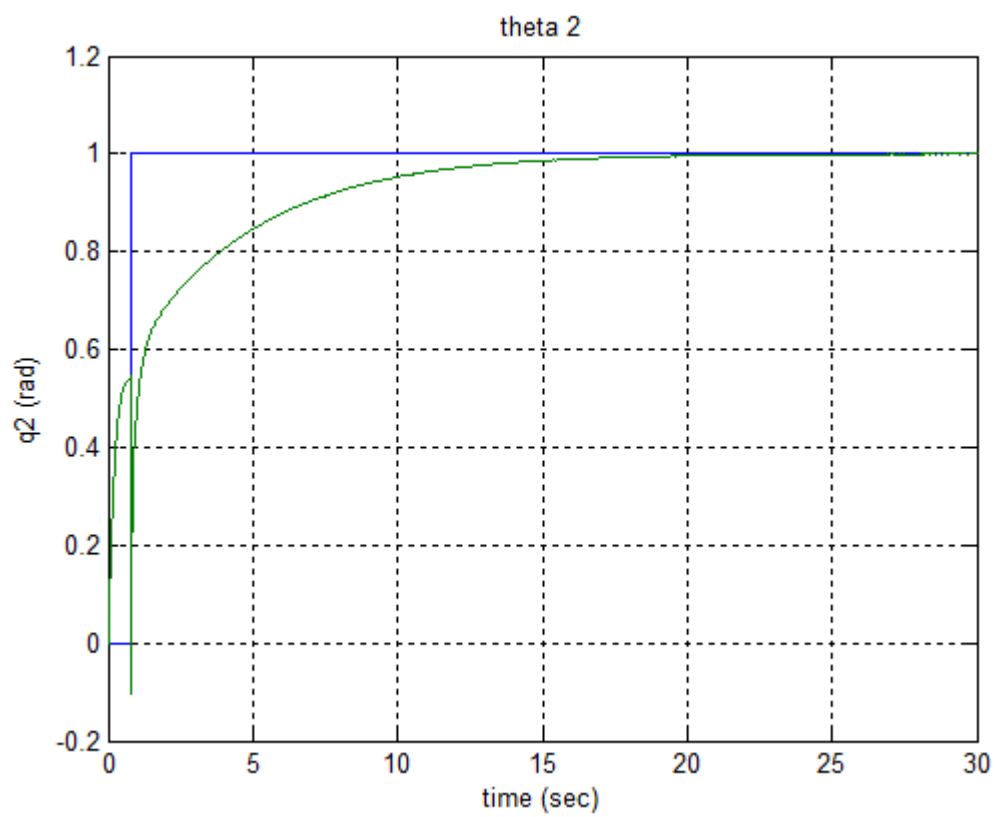


Figure 11:

1

| D-H parameters of 2-R Robot | | | | |
|-----------------------------|-------|-------|-------|-------|
| Frame No. | ?? ?? | ?? ?? | ?? ?? | ?? ?? |
| 1 | L 1 | 0 | 0 | ? 1 |
| 2 | L 2 | 0 | 0 | ? 2 |

Figure 12: Table 1 :

93 valid for other points. The model designed from Autodesk Inventor and Simulink software's is giving better and
 94 reasonable response. Good results are found when using feedback linearization. ^{1 2}

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