

Kinematics, Localization and Control of Differential Drive Mobile Robot

Sandeep Kumar Malu¹ and Jharna Majumdar²

¹ Nitte Meenakshi Institute of Technology

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Abstract

The present work focuses on Kinematics, Localization and closed loop motion control of a differential drive mobile robot which is capable of navigating to a desired goal location in an obstacle free static indoor environment. Two trajectory planning approaches are made (i) the robot is rotated to eliminate orientation error and then translate to overcome distance error (ii) Both rotational and translational motion is given to the robot to overcome orientation and distance error simultaneously. Localization is estimated by integrating the robot movement in a fixed sampling frequency. The control law is based on kinematics model which provides updated reference speed to the high frequency PID control of DC motor. Stability of proposed control law is validated by Lyapunov Criterion. Both experimental and simulation results confirm the effectiveness of the achieved control algorithms and their efficient implementation on a two wheeled differential drive mobile robot using an 8-bit microcontroller.

Index terms— kinematics, odometric localization, PID speed control, differential drive robot, lyapunov stability theory.

1 INTRODUCTION

Autonomous mobile robots have various applications in the field of industry, military and security environment. The problem of autonomous motion planning and control of wheeled mobile robots have attracted lot of research interest in the field of robotics. Consequently engineers working on design of mobile robots have proposed various drive mechanisms to drive such robots. However the most common way to build a mobile robot is to use two-wheel drive with differential steering and a free balancing wheel (castor). Controlling the two motors independently make such robots to have good manoeuvring and work well in indoor environment. Mobile robots with such drive systems are a typical example of non-holonomic mechanisms due to the perfect rolling constraints on a wheel motion (no longitudinal or lateral slipping).

An asymptotic stable controller using Backstepping method for posture tracking and its stability has been validated [1] [7]. An Adaptive Controller [2] to compensate errors can further improve its stability. A non-linear control design [6] using feedback linearization can provide better performance than conventional linearized controller. Many authors [2] [3] have proposed methods to reduce odometry error caused by kinematic imperfection. A different approach of localization using RFID technology [5] is efficient, fast and cheap.

A neural network [4] based reactive navigation algorithm for mobile robot in unstructured environment while avoiding obstacles is found to be optimized in computation.

In the present paper, kinematics model and localization using optical encoder coupled with the DC motor of a differential drive robot is presented. This model itself is used as a motion controller in a closed loop control scheme. In the absence of workspace obstacles, the basic motion tasks assigned to wheeled mobile robots may be reduced to moving between two robot postures and following a given trajectory.

The paper is organized as follows: In section II, basic equations of Kinematics and Motion Model of the robot are reported. The Localization in indoor environment using optical encoder coupled with the motor is described

44 in brief. In section III, Control law is proposed and its stability analysis is carried out based on Lyapunov theory.
 45 In section IV, PID speed control of motor is presented. Section V includes some simulation results. Section VI
 46 highlights implementation strategies of control and localization using an 8-bit ATmega 32 microcontroller while
 47 Section VI contains conclusion and future work.

48 2 II. KINEMATICS OF DIFFERENTIAL DRIVE ROBOT a) 49 Motion Model

50 Let Inertial Reference Frame is $\{X I, Y I\}$ and Robot Frame is $\{X R, Y R\}$. The Robot position $P [x y]^T$ is
 51 expressed in cartesian co-ordinate system of inertial frame. The relationship between Inertial and Robot frames
 52 is represented using basic transformation matrix as follows: $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x_r \\ y_r \end{bmatrix} + \begin{bmatrix} x_c \\ y_c \end{bmatrix}$ (1)

53 Where, The robot under consideration is a two wheeled differential drive robot, where each wheel is driven
 54 independently. Forward motion is achieved when both wheels are driven at the same rate, turning right is
 55 achieved by driving the left wheel at a higher rate than the right wheel and vice-versa for turning left. This type
 56 of mobile robot can turn on the spot by driving one wheel forward and second wheel in opposite direction at the
 57 same rate. Third wheel is a castor wheel needed for the stability of mobile robot. $\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} v \cos \theta \\ v \sin \theta \\ \omega \end{bmatrix}$
 58 $\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} + \begin{bmatrix} x_c \\ y_c \end{bmatrix}$ A Global Journal of Researches in Engineering

59 Each individual wheel contributes to the robot's motion and at the same time, imposes constraints on robot
 60 motion. It is assumed that the wheels of the robot do not slide. It is expressed by Non Holonomic Constraint.

$$61 \quad \dot{x} \sin \theta - \dot{y} \cos \theta = 0 \quad (2)$$

62 Also the measure of the traveled distance travelled by each wheel is not sufficient to calculate the final position
 63 of the robot. One needs to know how this movement is executed as a function of time. This can be illustrated
 64 in Fig. 2. $\dot{x} = v \cos \theta$; $\dot{y} = v \sin \theta$; $\dot{\theta} = \omega$; $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} \int v \cos \theta dt \\ \int v \sin \theta dt \end{bmatrix}$; $\theta = \theta_0 + \int \omega dt$; $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \begin{bmatrix} v \cos \theta_0 t \\ v \sin \theta_0 t \end{bmatrix} + \begin{bmatrix} \omega t^2 \sin \theta_0 \\ \omega t^2 \cos \theta_0 \end{bmatrix}$

65 Fig. 2 : Dependence of Robot position on its velocities as function of time velocities v and ω . First consider
 66 the contribution of each wheel's spinning speed to the translation speed at P in the direction of +X R . If one
 67 wheel spins ($v_1 = \omega r$) while the other wheel contributes nothing and is stationary ($v_2 = 0$), Since P
 68 is halfway between the two wheels, it will move instantaneously with half the speed i.e. $\dot{x} = (1/2)v_1 \cos \theta$.
 69 In a differential drive robot, these two contributions can simply be added to calculate the \dot{x} . Consider, for
 70 example, a differential robot in which each wheel spins with equal speed but in opposite directions.

71 The result is a stationary, spinning robot. As expected, \dot{x} will be zero in this case. The value of \dot{y}
 72 is even simpler to calculate. Neither wheel can contribute to sideways motion in the robot's reference frame,
 73 hence \dot{y} is always zero. Finally, we must compute the rotational component $\dot{\theta}$ of the robot. Once again the
 74 contributions of each wheel can be computed independently and just added. Consider the right wheel (we will
 75 call this wheel 1). Forward spin of this wheel results in counter clockwise rotation at point P. Recall that if wheel
 76 1 spins alone, the robot pivots around wheel 2. The rotation velocity $\dot{\theta}_1$ at P can be computed because the
 77 wheel is instantaneously moving along the arc of a circle of radius d. $\dot{\theta}_1 = \frac{v_1}{d}$. The same calculation
 78 applies to the left wheel, with the exception that forward spin results in clockwise-rotation at point P : $\dot{\theta}_2 = -\frac{v_2}{d}$
 79 $\dot{\theta} = \dot{\theta}_1 + \dot{\theta}_2$.

80 Mapping between Robot velocities to wheel velocities is given as follows: $\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ r & r \end{bmatrix}$
 81 $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -r & r \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$; $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -r & r \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$ (3)

82 Where, r = radius of wheel and d = axial distance between wheels.

83 3 b) Kinematic Equations

84 Kinematics is the most basic study of how mechanical systems behave. In mobile robotics, we need to understand
 85 the mechanical behaviour of the robot both in order to design appropriate mobile robots for desired tasks and
 86 to understand how to build control software.

87 Consider a differential drive robot at some t is valid when $\dot{\theta} \neq 0$ and above set of equations will be employed
 88 in establishing feedback control law for robot manoeuvring as discussed later in section III.

89 4 c) Odometric Localization

90 Localization is one of the most fundamental aspects of a mobile robot. All mobile robot system has to answer
 91 the fundamental question, which is "Where has to be obtained, so that the robot can easily move from source
 92 to destination. There are number of localization techniques with respect to mobile robot, however in the present
 93 work we have used dead reckoning method for localization. Dead reckoning method uses odometry to measure
 94 the movement of the robot. In the present work, we obtain the data from an incremental encoder (odometry),
 95 which is fitted along with a motor of the mobile robot. Incremental encoders measure the rotation of the wheels,
 96 which in turn, calculates robot position and orientation using integration approaches of kinematic model over [t
 97 k, t k+1].

98 Assuming Robot configuration q_k

99 $\begin{bmatrix} x \\ y \end{bmatrix}$ and constant velocity inputs V_k and ω_k are known at discrete time t_k , then using
 100 Euler integration $q_{k+1} = q_k + \begin{bmatrix} V_k \cos \theta_k \\ V_k \sin \theta_k \\ \omega_k \end{bmatrix} \Delta t$; $q_{k+1} = q_k + \begin{bmatrix} V_k \cos \theta_k \\ V_k \sin \theta_k \\ \omega_k \end{bmatrix} \Delta t =$
 101 $\begin{bmatrix} x_k \\ y_k \end{bmatrix} + \begin{bmatrix} V_k \cos \theta_k \\ V_k \sin \theta_k \\ \omega_k \end{bmatrix} \Delta t$ (6)

102 where, $\theta_k = \theta_{k-1} + \Delta\theta_k$ and $\theta_k = \theta_{k-1} + \Delta\theta_k$ and $\theta_k = \theta_{k-1} + \Delta\theta_k$

103 The reconstruction of the current robot configuration is based on the incremental encoder data (odometry).

104 Let θ_k and ϕ_k be the no. of wheel rotations measured during the sampling time T_s by the encoders.

105 Linear and angular displacements of the robot is given as $\Delta x_k = 2r(\theta_k - \theta_{k-1})$, $\Delta y_k = 2r(\phi_k - \phi_{k-1})$

106 $\Delta\theta_k = r(\theta_k - \theta_{k-1})$ (7)

107 Where, r = radius of wheel and d = axial distance between wheels.

108 For a differential-drive robot the position can be estimated starting from a known position by integrating the

109 movement (summing the increment travel distances). The estimate of robot configuration at time t_k is computed

110 as: $\begin{bmatrix} x_k \\ y_k \\ \theta_k \end{bmatrix} = \begin{bmatrix} x_{k-1} \\ y_{k-1} \\ \theta_{k-1} \end{bmatrix} + \begin{bmatrix} \cos\theta_{k-1} \Delta x_k \\ \sin\theta_{k-1} \Delta x_k \\ \Delta\theta_k \end{bmatrix}$ (8)

111 Robot localization using the above odometric prediction (commonly referred to as dead reckoning) is accurate

112 enough in the absence of wheel slippage and backlash.

113 III.

114 5 CONTROL LAW AND STABILITY

115 The control algorithm must now be designed to drive the robot from its current configuration; say (x_k, y_k, θ_k)

116 (x_k, y_k, θ_k) to the goal position (x_g, y_g, θ_g) . Here the objective is to find a control $u = [u_1, u_2]$

117 so that the robot's goal position is reached in finite interval of time. The proposed control law is state dependent

118 i.e.

119 $u = \delta x, \delta y, \delta\theta$ which guarantees the state to be asymptotically driven to $[0, 0, 0]$ without

120 attaining $\dot{x} = 0$ in finite time. One of the most commonly used methods to study the asymptotic behaviour

121 is based on the Lyapunov stability theory. Consider a simple positive definite quadratic form of Lyapunov

122 function: $V = \frac{1}{2} \dot{x}^2 + \frac{1}{2} \dot{y}^2 + \frac{1}{2} \dot{\theta}^2$ (9)

123 Where the parameters V_1, V_2 represent one half of the squared weighted norms of the "distance error

124 vector" \dot{x} and "orientation error vector" $\dot{\theta}$ exhibited by the robot between its current position and goal position

125 defined with respect to the Reference Inertial Frame. Its time derivative is given by: $\dot{V} = \dot{x}\ddot{x} + \dot{y}\ddot{y} + \dot{\theta}\ddot{\theta}$

126 $= \dot{x}\ddot{x} + \dot{y}\ddot{y} + \dot{\theta}\ddot{\theta}$ Using kinematics equation (10), $\dot{V} = \dot{x}(\ddot{x} \cos\theta - \dot{y} \sin\theta) + \dot{y}(\ddot{y} \sin\theta + \dot{x} \cos\theta) + \dot{\theta}\ddot{\theta}$ (10)

127 From the equation (10), the first term can be made non-positive by letting the linear velocity of the form: $\ddot{x} = -k_1 \dot{x}$

128 $= -k_1 \dot{x} \cos\theta - k_2 \dot{y} \sin\theta > 0$ (11) then, $\ddot{x} = -k_1 \dot{x} \cos\theta - k_2 \dot{y} \sin\theta$ (12)

129 This means that \dot{x} term is always nonincreasing in time and consequently, since it is asymptotically

130 converges to non-negative finite limit.

131 Similarly the second term \dot{y} can be made be non-positive by letting the angular velocity $\ddot{\theta}$ take the

132 form of: The result in (15) is a negative semi-definite form. By applying Barbalat's Lemma, it follows that \dot{V}

133 necessary converges to zero for increasing time; thus in turn implying convergence of the state vector $[x, y, \theta]$ to

134 $[0, 0, 0]$. Hence it can be concluded that control vectors expressed by (11) and (12) $\dot{V} = -k_1 \dot{x}^2 - k_2 \dot{y}^2 - k_3 \dot{\theta}^2 < 0$

135 $+ k_1 \dot{x}^2 + k_2 \dot{y}^2 + k_3 \dot{\theta}^2 > 0$

136 6 LOW LEVEL PID SPEED CONTROL

137 The speed of DC motor can be adjusted to a great extent so as to provide easy control and high performance.

138 There are several conventional and numeric controller types intended for controlling the DC motor speed.

139 Recently, many modern control methodologies such as nonlinear control [8], optimal control [9], variable structure

140 control [10] and adaptive control [11] have been extensively proposed for DC motors. However, these approaches

141 are either complex in theory or difficult to implement. PID controller algorithm involves three parameters denoted

142 P, I and D interpreted in terms of time. P depends on present error, I on the accumulation of past errors, and

143 D is a prediction of future errors. PID control with its three term functionality covering both transient and

144 steady-states response, offers the simplest and yet most efficient solution for many real world control problems.

145 In spite of the simple structure and robustness of this method, optimally tuning gains of PID controllers have

146 been quite difficult.

147 The electric equivalent circuit of the armature and the free-body diagram of the rotor are shown in the Fig.

148 ?? below The PID control design criteria are (i) less settling time (<1 s) (ii) overshoot less than 5% (iii) steady

149 state error less than 1%. The PID speed control design is incorporated into the system. The transfer function

150 for a PID controller given is by: $G(s) = K_p + \frac{K_i}{s} + K_d s$ (17)

151 Where, K_p, K_i and K_d are gains. PID is simulated in Matlab and its response is plotted. Further, gain

152 values are tuned manually to obtain desired response.

153 V.

154 7 SIMULATION RESULTS

155 Simulation results of DC motor characteristics incorporating PID for unit step input at different gains are shown

156 in figure 7:

157 8 IMPLEMENTATION

158 The robot which we used for our experiments is fabricated in-house. It has a differential-drive mechanism

159 consisting of high torque Dynaflex DC motors each of which is equipped with high resolution Jayashree-15S

9 CONCLUSION AND FUTURE WORK

160 optical quadrature shaft encoder for precise position and speed feedback to controller. The robot uses two 12V
161 DC batteries in series to power motors, while 5V for Microcontroller development board. The power electronics
162 module used to drive the DC motor is Super Hercules 9V-24V, 15A Motor Driver from Nex-robotics. The
163 algorithm of localization and control has been implemented on 8-bit Atmega32 Microcontroller.

164 The output signal from one channel of encoder is fed to the rising edge enabled external interrupt pin. On
165 interrupt, the status of other channel decides the direction and equivalent rotation counts of robot motion. These
166 counts on accumulation at fixed sampling frequency (in our case 33 Hz) resolve robot position estimation using
167 the expression for odometric localization explained in section II. Also these counts acts as feedback to PID speed
168 control executing at 100 Hz frequency.

169 The control algorithm runs at 33 Hz wherein distance error vector and orientation error vector between robot
170 instantaneous position and goal (target) position are calculated and robot control vectors viz. Two approaches
171 to reach the goal position have been implemented. In first one, robot is made to rotate until orientation error
172 is eliminated and then translated to overcome distance error. While in other method, robot exhibits both linear
173 and angular velocities to overcome distance and orientation error simultaneously.

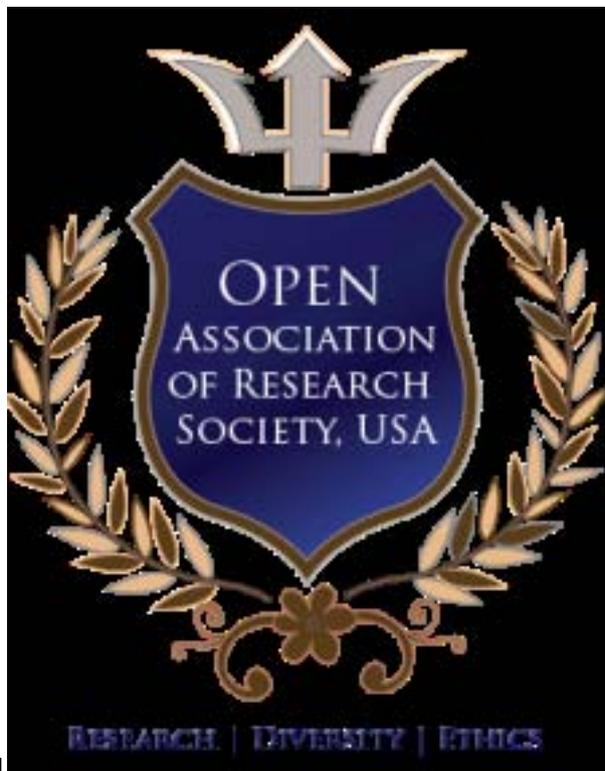
174 Using Kinematics motion models the linear and angular velocities of the robot are transformed to right and
175 left wheel speeds and fed as reference speed to PID speed control. The upper limit of robot motion is set to 40
176 RPM.

177 9 CONCLUSION AND FUTURE WORK

178 In this paper, Kinematics feedback path controller for a differential drive mobile robot has been presented. Both
179 the approach of Go-to-Goal motion is implemented using 8-bit Atmega 32 microcontroller. The robot motion
180 behaviour in real time is in line with the behaviour observed in simulation. Also the proposed control law is
181 validated as per Lyapunov stability criterion. The control algorithm proposed in this paper will be extended for
182 human following application, where set points (goal locations) are updated regularly through computer vision
183 algorithm of detecting and tracking human.

184 The present work will be extended to include obstacle avoidance in Go-to-Goal motion and dynamics
185 consideration of robot for control which can enhance better stabilization. Further, Umbmark calibration needs to
186 be performed to avoid localization error due to irregular wheel diameter and wheel axial alignment. Also Attitude
187 compensation has to be incorporated in odometric localization whenever robot travels in irregular surface.

VIII. ^{1 2 3}



1

Figure 1: Fig. 1 :

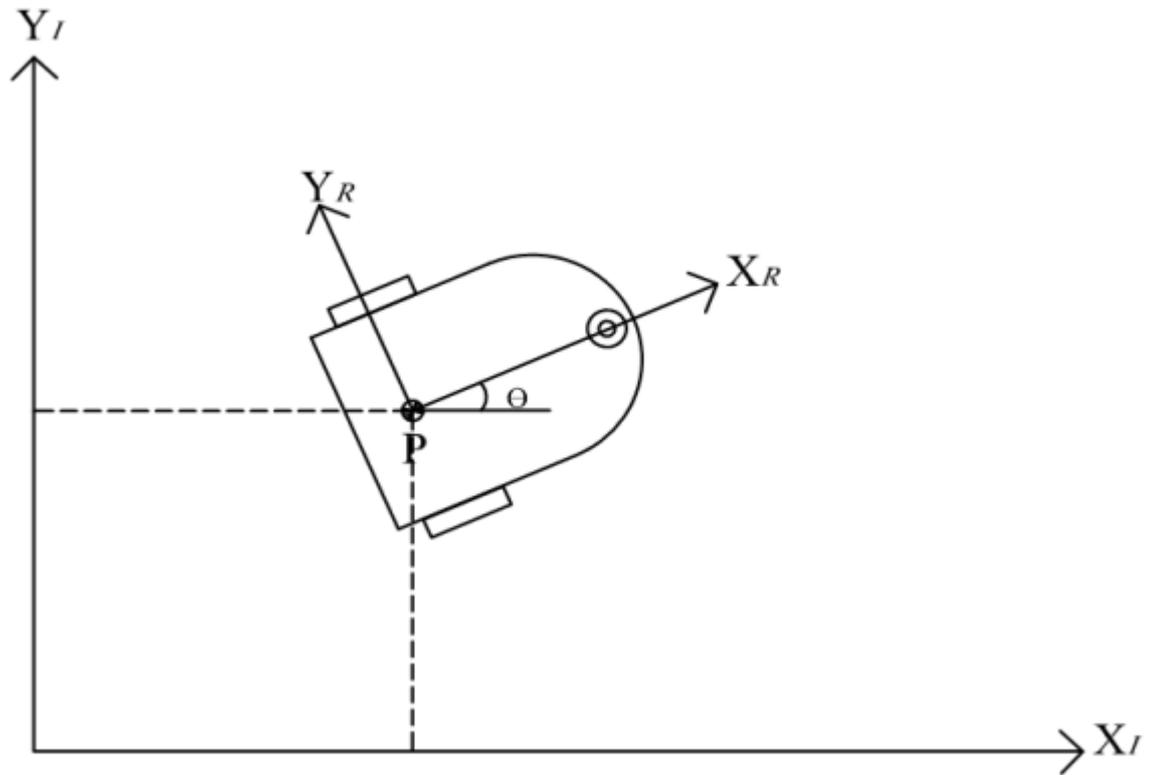
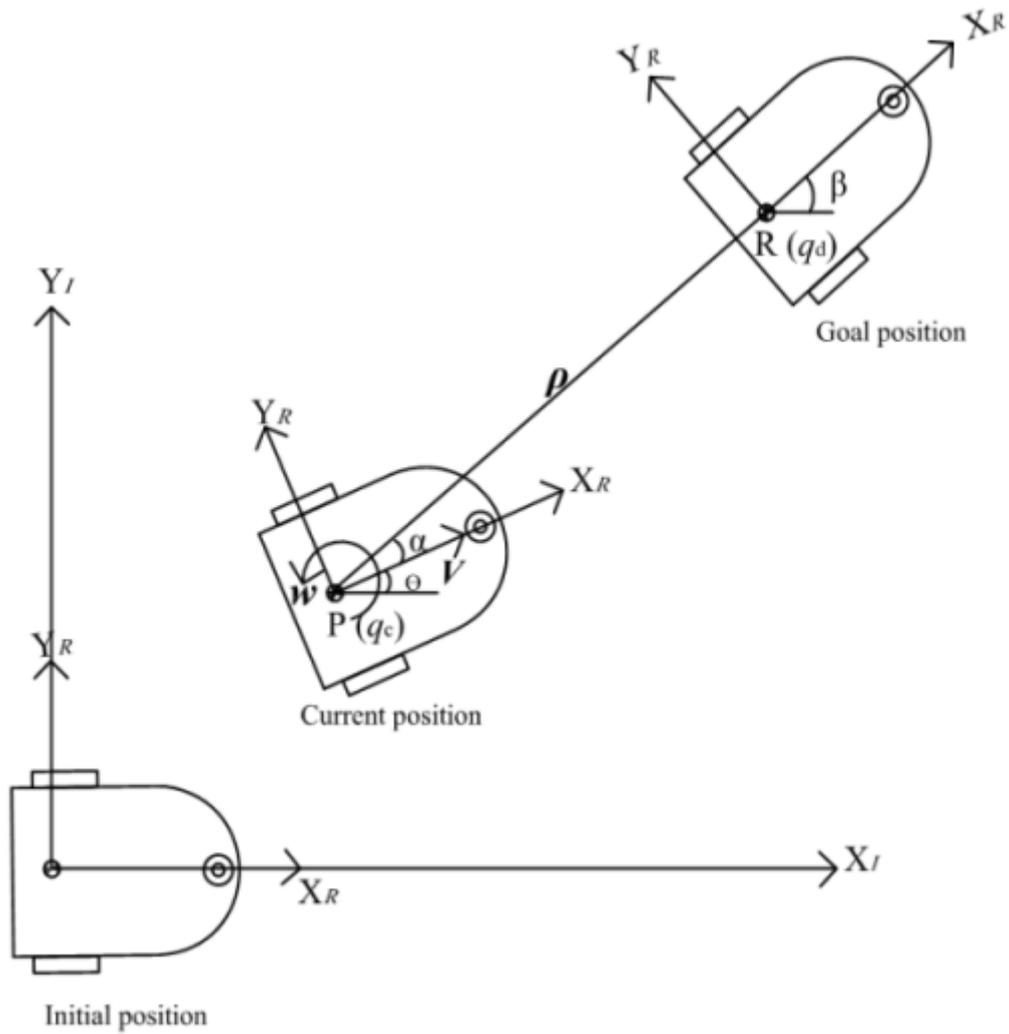


Figure 2:

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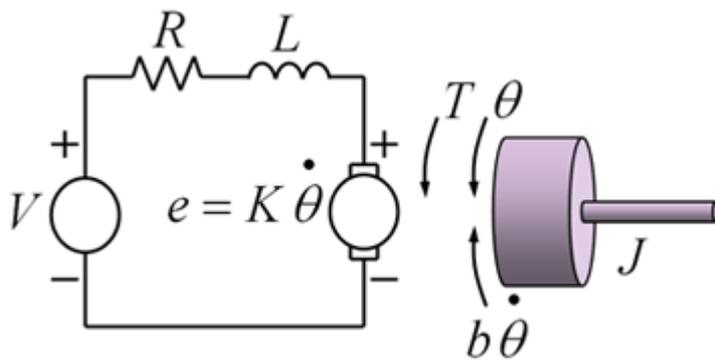
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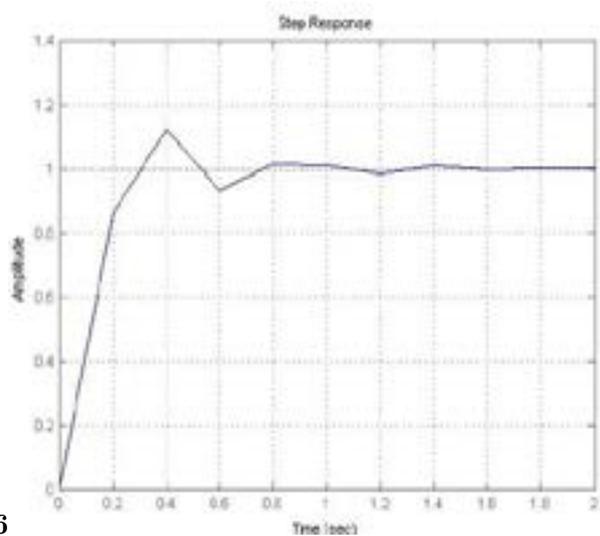
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Figure 4: 2



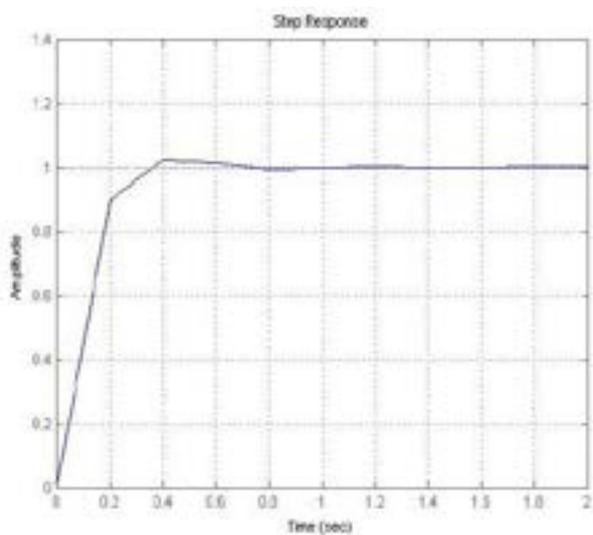
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Figure 5: Fig. 4 :



56

Figure 6: Fig. 5 :Fig. 6 :



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Figure 7: Fig. 7 :

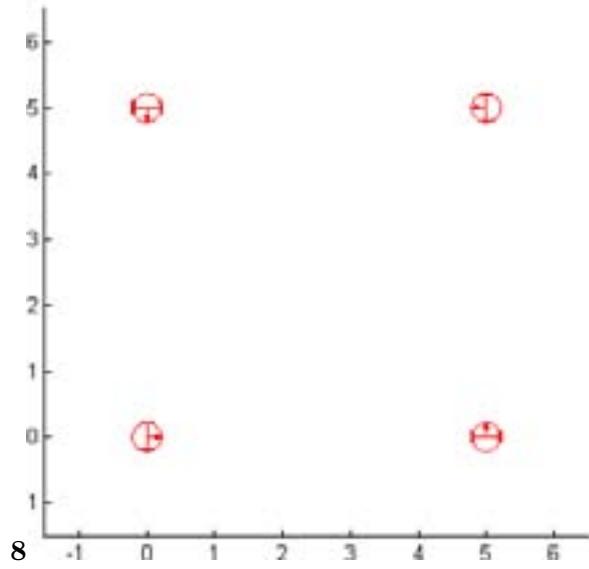


Figure 8: Fig. 8 :

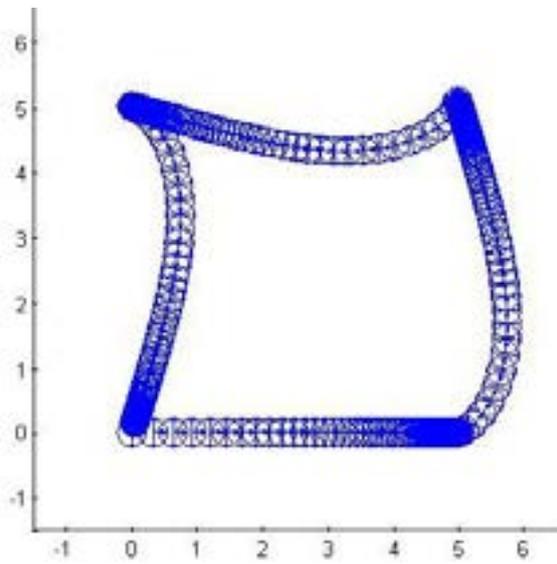


Figure 9: Global

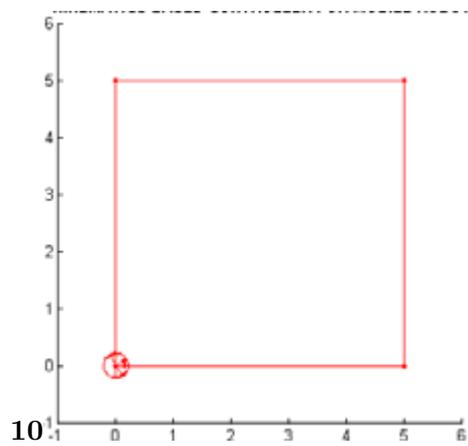


Figure 10: Fig. 10 :

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193 .2 Global

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