Calculation of the Strength Reliability of Parts under Random Loading

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Abstract

The most important characteristics of many engineering objects are their strength and reliability. To determine the probability of an object failure according to the strength criterion, it is necessary to know the laws of distribution of the actual and ultimate stresses. However the processing of actual data on the acting and ultimate stresses indicates that these data cannot be described by the known laws of the parametric statistical theory.This paper proposes a new approach to the solution of the problem, which is based on the application of the mathematical apparatus of nonparametric statistics. The considered approach of calculating the probability of a failure and the quantile estimates of the safety factor of machine parts are universal. They allow estimation of the strength reliability of items regardless of the complexity of the laws of distribution of random values of the actual and ultimate stresses.

Index terms— stresses, strength, reliability, failure, methods of nonparametric statistics, distribution density function, parzen-rosenblatt method.

1 Introduction

t present, when solving the problems associated with an increase in the manufacturing efficiency, improvement of the diagnostics of diseases, statistical data processing in insurance and financial mathematics, one has to deal with experimental data. The distribution density functions (DDFs) for these data are most frequently unknown and are not described by the laws of distribution of random quantities that were developed in the theory of mathematical statistics. Therefore, the main trend in the development of the statistical science involves the elaboration of methods for processing experimental data that allow the actual laws of distribution of random quantities to be taken into account. In the second half of the last century, an approach to estimation of many functionals on the basis of a nonparametric estimate of the probability density was proposed in [1,2,3]. To date, owing to the development of the computer engineering, this approach has gained significant development for solving various problems in economics and medicine [4,5,6,7]. Nonparametric methods became widespread in solving identification and regression-analysis problems [8,9, **??**0].

The most important characteristics of numerous engineering objects are their strength and reliability. Up to now, these characteristics are determined on the basis of the laws that are considered in the theory of parametric statistics [11,12]. At the same time, it was shown in [13,14,15] that the DDFs of the actual and ultimate stresses, on the basis of which the probability of no-failure operation of an item is determined, are seldom described by the laws that were studied in the statistical theory. This study considers the solution of the problem of calculating the probability of no-failure operation of several engineering objects on the basis of applying methods of nonparametric statistics.

2 II.

3 Statement of the Problem

Calculated estimates of the strength reliability of parts are currently obtained using two fundamentally different approaches. According to the first one [11,12], the probability of a failure of a part is calculated as[ ] 0 s) - (? = ? y Pr(**1**)

where? -are the effective stresses (MPa) at a hazardous place of the part, s -the permissible stresses (MPa) for its material.

Problem du dt t f t u f F F Q s s ? ? ? ? ? ? ? ? ? ? ? ? ? ? + ? = 0 0) ( ) (1 ? ? (2)

where? ? = 0) (du u f F ? ? ? ? = 0) (du u f F s s .

It is conventionally assumed that the density functions) (? ? f and) (s f S

are distributed according to a normal law, thus allowing the problem (1) to be solved on the basis of tables of the normal distribution. Papers [11,12] presents the solutions of problem (1) for several laws of distribution of the random quantities? and s that were studied in the theory of parametric statistics.

Despite the versatility of this approach, it is not always possible to obtain a quantitative estimate of the strength reliability of a studied part within its framework. This is confirmed by Fig. 1, which shows the functions It can be easily seen in Fig. 1 that the calculation of the probability of failure using formula (1) results here in the zero value of the probability of a failure. In this case, the problem of estimating the technical state of a part can be solved via realization of the second approach, which implies the calculation of quantile (? ? n) estimates of the safety margin (? n) at a specified probability ? via the numerical solution of the equation ( ) Here,) (? n f n is the DDF for ? n , which is calculated from the dependence ? ? s n = (4)

As a rule, when the safety margin is calculated, the random character of ? and s is disregarded and only their average values are used. However, for a number of reasons, determining the characteristics of the random quantity ? n on the basis of formula (**??**) is not a trivial problem [13]. For actual conditions of the use of parts, the random quantity ? is not described by the laws that were considered within the framework of parametric statistics. The analysis of the results of processing experimental data (yield stress, ultimate stress of pipe steels), which are used to calculate allowable stresses, shows that the use of a normal distribution law is not always correct here and more "flexible" laws should be used, e. corresponds to the Cauchy distribution, for which a mean value and a variance are generally absent. For the reasons that were presented above, problem (3) can be solved using conventional methods of parametric statistics only under serious assumptions. As a result, the correct calculation results are not guaranteed. Here, more powerful algorithms that operate regardless of the complexity of the functions) (? ? f ,) (s f s

, and) (s f s must be applied. Exactly such algorithms, the possibility of realization of which is provided only by the achievements of the modern computer engineering and computer simulation methods, were developed within the framework of the theory of nonparametric statistics [13,14,15].

4 III.

5 Used Theoretical Methods

For determination of probability of part failure in accordance with equation (2) it is necessary to solve two auxiliary problems.

6 Problem 1: Reconstruction of an unknown DDF on the basis of a sample of values of a random quantity

According to [13], on the basis of a sample of stressesm i i , 1 , = ? , the estimate of a left ({ } i i ? ? min min =) and right ({ } i i ? ? max max =) censored

unknown DDF for stresses is represented in the form of the expansion (Parzen-Rosenblatt estimate with a normal kernel): .? ? ? ? ? ? ? ? ? c h h m f m i i 1 5 , 0 exp 2 1) (1 2 ? = ? = (5) ? ? ? ? ? ? ? ? ? ? d h h m ? m i i ? ? = ? = max min 1 2 5 , 0 exp 2 1 in which ? ? ? ? ? ? ? ? ? d h h m ? m i i ? ? = ? =? = ? ? = ? ? m i m i j j i h h h h m m J 1 1 2 5 , 0 exp 2 1) 1 (1 ln 1 max max ? ? ? ? ? ? ? (6)

The solution of problem (6) allows determination of all parameters that are included in (5) and, thus, reconstruction of the function) (? ? f .

For a kernel function with a normal kernel, a close-to-optimal value of the parameter is defined from the dependence ( ) ( ) ( ) [ ] ( ) ( ) [ ] s s s s s s s c u u u u u s f 1 3 5 24 6 1 2 exp 2 1) (2 4 4 3 3 2 2 ? ? ? ? ? ? ? + ? ? ? + × × ? ? ? ? ? ? ? ? = ? ? ? ?(**9**)

where ( ) Let us specify a random quantity V with a normal distribution law. To obtain the random quantity s with the distribution function) (s F s , it is necessary to use the equation [13]:( ) ( ) [ ] ( ) ( ) [ ] ds u u u u u c s s s s s s s s ? ? ? ? ? ? ? ? ? ? ? ? + ? ? ? + × × ? ? ? ? ? ? ? ? ? ? ? ? = ?V s F s =) ((10) Because ds s f s F s s s) ( ) (0 ? =

, on the basis of dependence (9), we obtainF s s c s P s F /) ( ) (= , (**11**)

Where . This procedure is repeated, and the sample s is extended to the required size.? = max min) (s s s F ds s P c ; 2 1 ? ? ? = s u s ; [ ] × ? ? ? = ? ? ? ? ? ? ? 2 1 1 6) 5 , 0 exp(2 1) (

The algorithm for generating a sample of a random quantity that, e.g., has the DDF in the form of (**??**) is constructed quite analogously. This algorithm is called the nonparametric random-number generator [13,14,15].

7 IV.

8 Computer Experiments Realizing Developed Approach

Example 1. It is required to determine the probability of a failure of a pipe that is exposed to an internal pressure and a temperature during operation. The pipe diameter is 1420 mm, its wall thickness is 16.5 mm, the pipe material is 17GS steel, and the permissible stresses for the pipe material obey a normal distribution law.

In order to reconstruct the DDF ? + ? ? ? = H W H W H H K u u b T a Z 3 * 1 3) 1 (1 10 13 , 6 ? , (**13**)

where Z H is the coefficient that accounts for the shapes of the mated surfaces; a W is the interaxial distance of the helical gearing (mm); b W is the working width of the gear rim (mm); u is the gear ratio; and K H? is the load factor, which is related to * 1H T via a nonlinear dependence.

It follows from (13) that the dependence of

9 Results and Discussion

In the conventional approach to the solution of the considered problems for each random quantity using the fitting criteria (chi-square, omega-square, Kolmogorov-Smirnov), a distribution law must be selected. However, this law can be adopted only with a certain probability. The value of this probability is not a priori known. In this case, there is a risk of adopting a distribution law that is actually not realized (error of the second kind). Thus, the reliability of the result of solving the problem is an uncertain value.

The use of methods of nonparametric statistics for solving problems makes it possible to eliminate the aforementioned uncertainty.

10 VI.

11 Conclusion

The approach considered in this study and the mathematical apparatus for calculating the probability of no-failure operation or a failure and quantile estimates of the safety margin of machine components and structures is universal. It allows estimation of the strength reliability of articles regardless of the complexity of the laws of distribution of random values of actual and limiting stresses. [1]

1

Figure 1: 1)

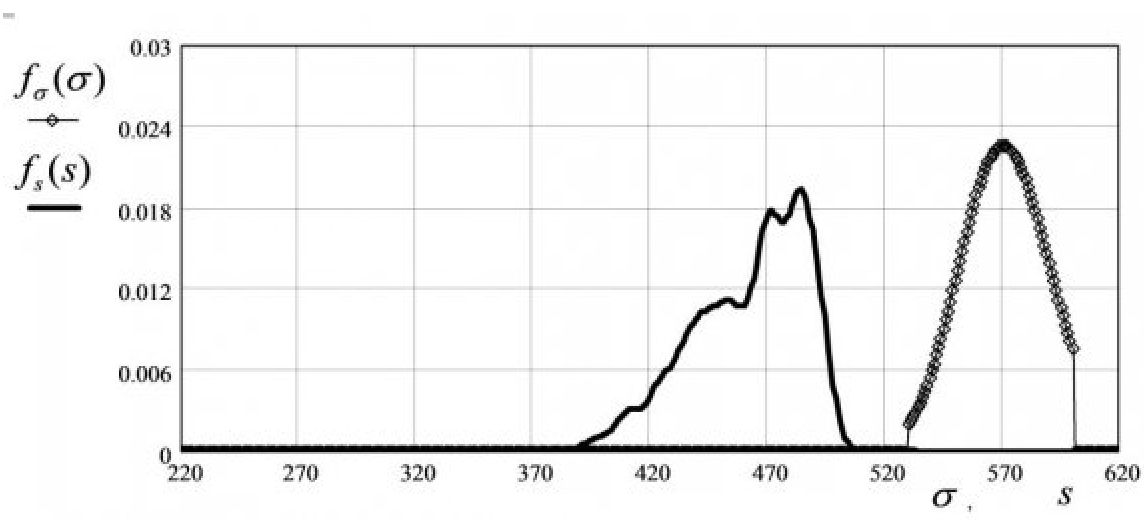
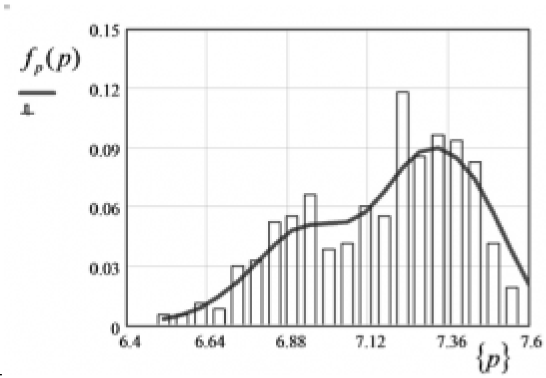


Figure 2:



1

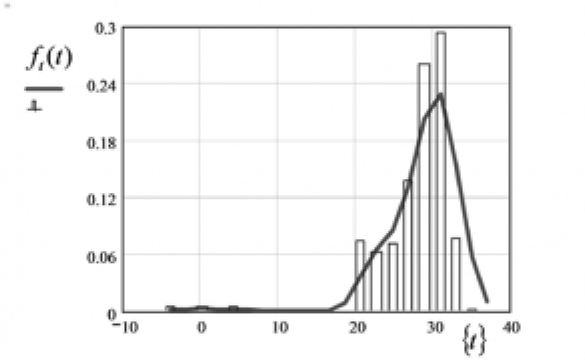


Figure 3: Fig. 1

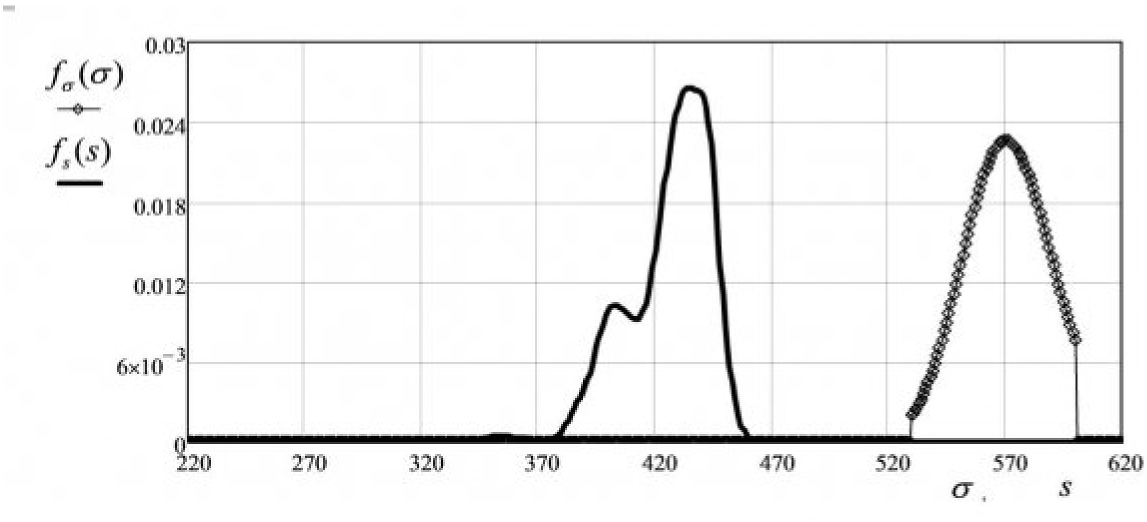


Figure 4:

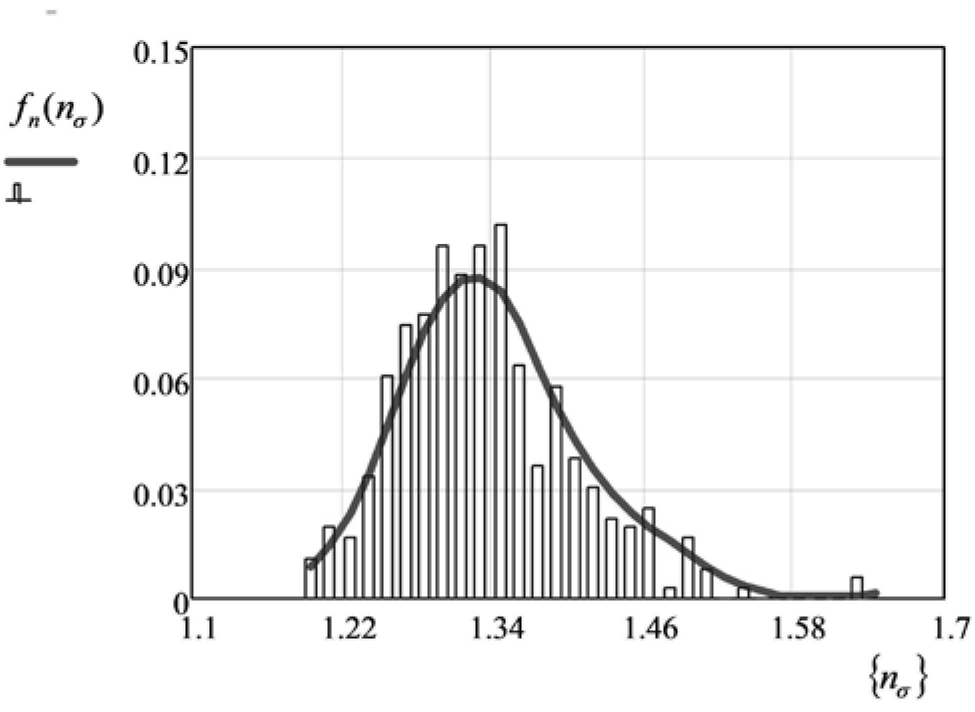


Figure 5:

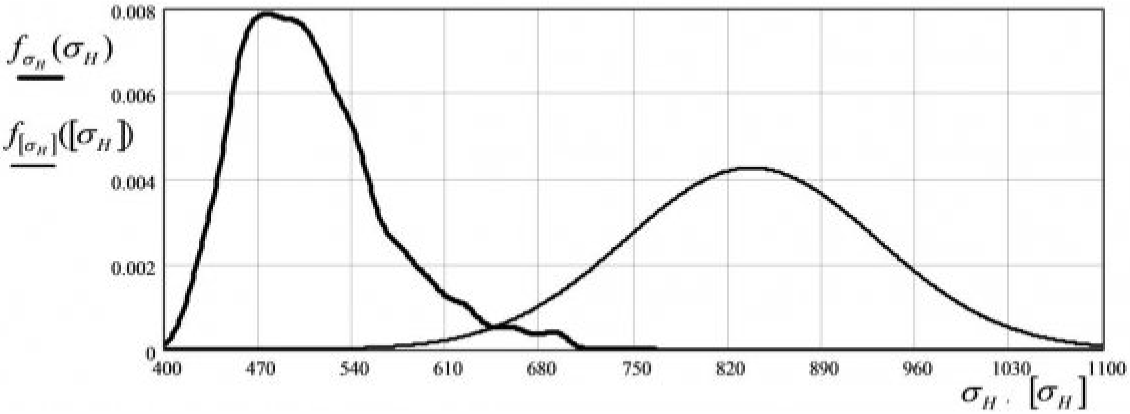


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Figure 6: DProblem 2 :

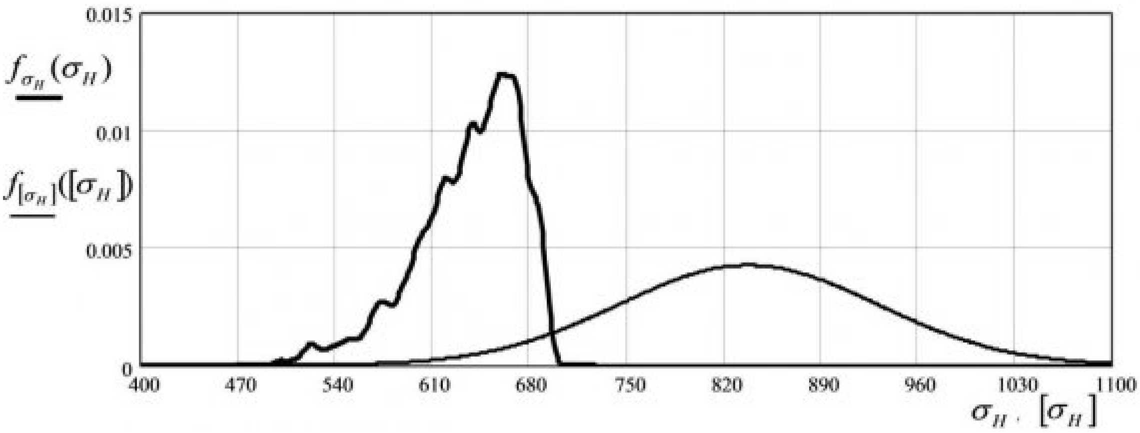

Figure 7:

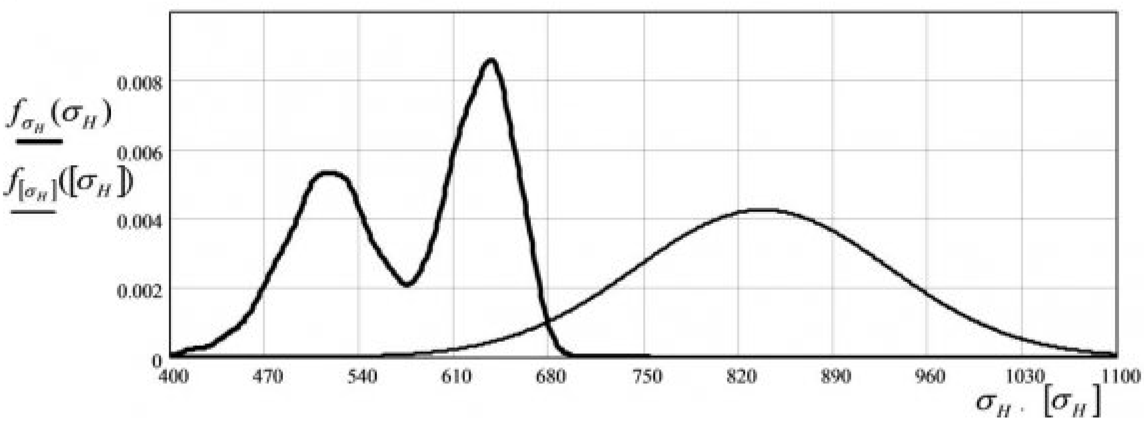

Figure 8:

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