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An Algorithm for Solving Bi-Criteria Large Scale Transshipment Problems

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ANALGORITHMFORSOLVINGBICRITERIALARGESCALETRANSSHIPMENTPROBLEMS

An Algorithm for Solving Bi-Criteria Large Scale Transshipment Problems

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I. Introduction

When shipments go directly from a supply point to a demand point, i.e. shipments do not take place between origins or between destinations nor from destinations to origins, it is called a classical transportation problem. In many real life situations, shipments are allowed between supply points or between demand points. There are many points (called transshipment points) through which goods can be transshipped on their journey from a supply point to a demand point. Shipping problems with any or all of these characteristics are considered as transshipment problems. It was first introduced by Orden (1965) [1] in which he introduced an extension of the original transportation problem to include the possibility of transshipment. The problem of determining simultaneously the flow of primary products through processors to the market of final products has been formulated alternatively as a transshipment model by multi-regional, multi-product, and multi-plant problem formulated in the form of general linear programming model has been proposed by Judge et al (1965) [4].

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Later, various alternative formulations of the transshipment problem within the framework of the transportation model that permits solution of problems of the type discussed by King and Logan without the need for subtraction of artificial variables were discussed by Hurt and Tramel (1965) [5]. On the other hand, Grag and Prakash (1985) [6] studied time minimizing transshipment problem. Then dynamic transshipment problem was studied by Herer and Tzur (2001) [7]. Ozdemir (2006) studied Multi location transshipment problem with capacitated production and lost sales afterwards [8]. Furthermore, Osman et al (1984) [9] introduced an algorithm for solving bi-criteria multistage transportation problems.

Recently, Khurana et al (2011) [10] studied a transshipment problem with mixed constraints. He introduced an algorithm for solving time minimizing capacitated transshipment problem [11]. Abo-elnaga et al (2012) [12] introduced a trust region globalization strategy to solve multi-objective transportation, assignment, and transshipment problems. Khurana (2013) [13] introduced a Multi-index fixed charge bi-criterion transshipment problem. Rajendran et al [14] (2012) presented A new method namely, splitting method, to solve fully interval transshipment problems. Zaki et al [15] (2012) used the genetic algorithm for solving transportation, assignment, and transshipment problems. Ojha et al [16] (2011) formulated single and multi-objective transportation models with fuzzy relations under the fuzzy logic. Saraj et al [17] (2010) solved the multi objective transportation problem (MOTP) under fuzziness using interval numbers. Abd El-Wahed [18] (2001) presented a multi-objective transportation problem under fuzziness. Das et al [19] (1999) introduced a multi-objective transportation problem with interval cost, source and destination parameters.

In this paper a formulation of different structures of bi-criteria large-scale transshipment problems and an algorithm for solving a class of them, which can be solved using the decomposition technique of linear programming by utilizing the special nature of transshipment problems, is presented. The new algorithm determines the points of the non-dominated set in the objective space. The method consists of solving the same multistage transshipment problem repeatedly but with different objectives and each

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iteration gives either new non-dominated extreme point or changes the direction of search in the objective space. An illustrative example is presented in this paper.

II. Formulation of Bi-Criteria Multistage Transshipment Problems

The formulation of different bi-criteria multistage transportation problems with transshipment presented in this paper covers several real situations as shown in the following cases.

a) *Bi-Criteria Multistage Transportation Problem with transshipment of the First Kind (BMTSP 1)*

This case represents multistage transshipment problems without any restrictions on intermediate stages. In order to develop a mathematical formulation of the problems, it is assumed that the availabilities are "aj", where j= 1, 2, 3, ..n and "n" is the number of sources and destinations. Where as the requirements are "b_j", j= 1, 2, 3, ….., n. The minimum transportation costs and deteriorations from i to j are "c_{ij}","d_{ij}" where i and j= 1, 2, 3, …., n. X_{ij} denotes the quantity shipped from i to j; and "xj_{jj}" is the neat amount transshipped through point j where $x_{ij} \geq 0$. Then the problem takes the form:

$$Min.Z_1 = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$$

$$Z_2 = \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}x_{ij}$$

Where $c_{ij} = 0$ for the quantity shipped from the source "Si" to itself and from destination "Dj" to itself:.

$$\sum_{\substack{i=1\\ i\neq j}}^{n} x_{ji} - x_{jj} = a_j,\ j = 1,2,\ldots,n.$$

$$\sum_{\substack{i=1\\ i\neq j}}^{n} x_{ij} - x_{jj} = b_j,\ j = 1,2,\ldots,n.$$

$x_{ij} \geq 0$ for all i, j.

b) *Bi-criteria Multistage Transportation Problem with Transshipment of the Second Kind (BMTSP 2):*

This case represents bi-criteria multistage transshipment problems in which the transportation at any stage is independent of the transportation of the other stages. In order to obtain the mathematical formulation of the problem which represents this case, it is assumed that for kth stage, k=1,2,3,…N. The availabilities are: $(a^k_{jk}),\ j_k = 1,2,3,\ldots n_k$, n_k is the number of sources and destinations at the kth stage; the requirements are: $(b^k_{jk}),\ j_k = 1,2,3,\ldots n_k$ the transportation costs and deteriorations are $c^k_{i_k j_k}$ and $d^k_{i_k j_k}$ where i_k= 1,2, 3, ., nk; jk= 1, 2, 3, ., nk. $x^k_{i_k j_k}$ denotes the quantity shipped from i_k to j_k and $x^k_{j\,j}$ is the net amount transshipped through point j_k, $x^k_{j_k j_k} \geq 0$.

Then the problem takes the form:

$$Min.Z^k_1 = \sum_{i_k=1}^{n_k}\sum_{j_k=1}^{n_k} c^k_{i_k j_k} x^k_{i_k j_k}$$

$$Z^k_2 = \sum_{i_k=1}^{n_k}\sum_{j_k=1}^{n_k} d^k_{i_k j_k} x^k_{i_k j_k}$$

Where $c^k_{i_k j_k} = 0$ for the quantity shipped from the source (S_i) to itself and from the destination (D_j) to itself as follows:.

$$\sum_{\substack{i_k=1\\ i_k\neq j_k}}^{n_k} x^k_{j_k i_k} - x^k_{j_k j_k} = a^k_{j_k},\ j_k = 1,2,\ldots,n.$$

$$\sum_{\substack{i_k=1\\ i_k\neq j_k}}^{n_k} x^k_{i_k j_k} - x^k_{j_k j_k} = b^k_{j_k},\ j_k = 1,2,\ldots,n.$$

$x_{ij} \geq 0$ for all i_k, j_k.

and the minimum transportation cost is given by:

$$MinZ = \sum_{k=1}^{n} MinZ^k$$

c) *Bi-Criteria Multistage Transportation Problem With Transshipment of the Third Kind (BMTSP 3):*

This case represents bi-criteria multistage transshipment problems with some additional transportation restrictions on the intermediate stages which does not affect the transshipment problem formulation at each stage. The mathematical formulation of the problem representing this case is given as:

$$Min.Z_1 = \sum_{i_1=1}^{n_1}\sum_{j_1=1}^{n_1} c^1_{i_1 j_1} x^1_{i_1 j_1} + \ldots + \sum_{i_k=1}^{n_k}\sum_{j_k=1}^{n_k} c^k_{i_k j_k} x^k_{i_k j_k} + \ldots$$

$$\ldots + \sum_{i_N=1}^{n_N}\sum_{j_N=1}^{n_N} c^N_{i_N j_N} x^N_{i_N j_N}$$

$$Z_2 = \sum_{i_1=1}^{n_1}\sum_{j_1=1}^{n_1} d^1_{i_1 j_1} x^1_{i_1 j_1} + \ldots + \sum_{i_k=1}^{n_k}\sum_{j_k=1}^{n_k} d^k_{i_k j_k} x^k_{i_k j_k} + \ldots$$

$$\ldots + \sum_{i_N=1}^{n_N}\sum_{j_N=1}^{n_N} d^N_{i_N j_N} x^N_{i_N j_N}$$

Where $c^k_{i_k j_k}$ and $d^k_{i_k j_k} = 0$ for the quantity shipped from the source $S^k_{i_k}$ to itself and from the destination $D^k_{j_k}$ to itself: k = 1, 2,…, N

$$\sum_{\substack{i_1=1\\ i_1\neq j_1}}^{n_1} x^1_{j_1 i_1} - x^1_{j_1 j_1} = a^1_{j_1}, j_1 = 1,2,...,n_1$$

$$\sum_{\substack{i_1\neq j_1\\ i_1=1}}^{n_1} x^1_{i_1 j_1} - x^1_{j_1 j_1} = b^1_{j_1}, j_1 = 1,2,...,n_1$$

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$$\sum_{\substack{i_k\neq j_k\\ i_k=1}}^{n_k} x^k_{j_k j_k} - x^k_{j_k j_k} = a^k_{j_k}, j_k = 1,2,...,n_k$$

$$\sum_{\substack{i_k\neq j_k\\ i_k=1}}^{n_k} x^k_{i_k j_k} - x^k_{j_k j_k} = b^k_{j_k}, j_k = 1,2,...,n_k$$

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$$\sum_{\substack{i_N\neq j_N\\ i_N=1}}^{n_N} x^N_{j_N i_N} - x^N_{j_N j_N} = a^N_{j_N}, j_N = 1,2,...,n_N$$

$$\sum_{\substack{i_N\neq j_N\\ i_N=1}}^{n_N} x^N_{i_N j_N} - x^N_{j_N j_N} = b^N_{j_N}, j_N = 1,2,...,n_N$$

$$F_{r_k}(x^{k-1}_{i_{k-1}j_{k-1}}, x^k_{i_k j_k}, x^{k+1}_{i_{k+1}j_{k+1}}) = 0,$$

$$x^1_{i_1 j_1} \geq 0,.., x^k_{i_k j_k} \geq 0,..x^N_{i_N j_N} \geq 0$$

For all $i_1,...,i_K,...,i_N; j_1,..., j_K,..., j_N;$

where: $F_{r_k}, k = 1,2,..., N$ are linear functions representing the additional transportation restrictions and r_k is the number of this linear functions at the k^{th} stage.

d) Bi-Criteria Multistage Transportation Problem wth transshipment of the Fourth Kind (BMTSP 4)

This case represents bi-criteria multistage transshipment problems in which the difference between the input and output transportation commodity is known at the sources (destinations) of each intermediate stage. The assumed transportation restrictions in this case affect the transshipment formulation of each individual stage.

The mathematical formulation of the problem representing this case is given as:

$$Min.Z_1 = \sum_{i_1=1}^{n_1}\sum_{j_1=1}^{n_1} c^1_{i_1 j_1} x^1_{i_1 j_1} + ... + \sum_{i_k=1}^{n_k}\sum_{j_k=1}^{n_k} c^k_{i_k j_k} x^k_{i_k j_k} + ...$$
$$... + \sum_{i_N=1}^{n_N}\sum_{j_N=1}^{n_N} c^N_{i_N j_N} x^N_{i_N j_N}$$

$$Z_2 = \sum_{i_1=1}^{n_1}\sum_{j_1=1}^{n_1} d^1_{i_1 j_1} x^1_{i_1 j_1} + ... + \sum_{i_k=1}^{n_k}\sum_{j_k=1}^{n_k} d^k_{i_k j_k} x^k_{i_k j_k} + ...$$
$$... + \sum_{i_N=1}^{n_N}\sum_{j_N=1}^{n_N} d^N_{i_N j_N} x^N_{i_N j_N}$$

Where $c^k_{i_k j_k}$ and $d^k_{i_k j_k} = 0$ for the quantity shipped from the source $S^k_{i_k}$ to itself and from the destination $D^k_{j_k}$ to itself: k = 1, 2, …, N

$$\sum_{\substack{i_1\neq j_1\\ i_1=1}}^{n_1} x^1_{j_1 i_1} - x^1_{j_1 j_1} = a^1_{j_1}, j_1 = 1,2,...,n_1$$

$$\sum_{\substack{i_1\neq j_1\\ i_1=1}}^{n_1} x^1_{i_1 j_1} - x^1_{j_1 j_1} - (\sum_{\substack{i_2\neq j_2\\ i_2=1}}^{n_2} x^2_{j_2 i_2} - x^2_{j_2 j_2}) = b^1_{j_1}, j_1 = 1,2,..., n_1; j_2 = 1,2,...n_2$$

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$$\sum_{\substack{i_{k-1}\neq j_{k-1}\\ i_{k-1}=1}}^{n_{k-1}} x^{k-1}_{i_{k-1}j_{k-1}} - x^{k-1}_{j_{k-1}j_{k-1}} - (\sum_{\substack{i_k\neq j_k\\ i_k=1}}^{n_k} x^k_{j_k i_k} - x^k_{j_k j_k}) = b^{k-1}_{j_{k-1}}, j_{k-1} = 1,2,..., n_{k-1}; j_k = 1,2,...n_k$$

$$\sum_{\substack{i_{N-1}\neq j_{N-1}\\ i_{N-1}=1}}^{n_{N-1}} x^{N-1}_{i_{N-1}j_{N-1}} - x^{N-1}_{j_{N-1}j_{N-1}} - (\sum_{\substack{i_N\neq j_N\\ i_N=1}}^{n_N} x^N_{j_N i_N} - x^N_{j_N j_N}) = b^{N-1}_{j_{N-1}},$$

$$j_{N-1} = 1,2,..., n_{N-1}; j_N = 1,2,...n_N$$

$$\sum_{\substack{i_N\neq j_N\\ i_N=1}}^{n_N} x^N_{i_N j_N} - x^N_{j_N j_N} = b^N_{j_N}, j_N = 1,2,...,n_N$$

$x^k_{i_k j_k} \geq 0$ for all i_k, j_k; k=1, 2, …, N

(BMTSP 1) is solved as a bi-criteria single stage transshipment problem.

(BMTSP 2) can be solved as N single stage bi-criteria transshipment problems and the minimum value of the total transport costs and deteriorations are obtained as the sum of the minimum transportation costs and deteriorations for each individual stage.

(BMTSP 3) can be solved using the decomposition technique utilizing the special nature of transshipment problems. The next section will be devoted to the solution of this type of problems.

(BMTSP 4) is solved using any method for solving bi-criteria linear programming problems.

e) An Algorithm for Solving BMTSP 3

The decomposition technique of linear programming can be used to
solve the bi-criteria multistage transshipment problems especially for the (BMTSP 3) type. This type of bi-criteria multistage transshipment problems decomposed into [2, 3, 5, 8]:

- Sub problems corresponding to every stage.
- A master program which ties together the sub problems.

Let:

D^k be the matrix consisting of the coefficients of k^{th} sub-problem constraints.

A^k the matrix consisting of the coefficients of k^{th} stage tie-in constraints.

b the vector of constant coefficients in the tie-in constraints.

b^k the vector consisting of the availabilities and requirements of
k^{th} sub-problem.

R_o the matrix consisting of the first m_o columns of B^{-1}, m^o denotes the number of elements of b, B be the current basis matrix.

c^k the vector of first objective coefficients of k^{th} sub-problem .

d^k the vector of second objective coefficients of k^{th} sub-problem .

c_B the corresponding vector of basic variables coefficients.

N the number of sub- problems.

The following section presents an algorithm for determining all non dominated extreme points for the (BMTSP 3) model from which the solution for (BMTSP 1) and (BMTSP 2) models can be deduced from it as special cases.

Assuming that independent constraints are:

D^k, k= 1,2,…,N is the technological matrix of the kth stage activity. D^k is $(m_k + n_k) * (m_k + n_k)$ matrix, N is the number of stages, mk is the number of sources at k^{th} stage, n^k is the number of destinations.

b^k is the column vector consisting of the availabilities and requirements of the k^{th} sub-problem, bk is $(m_k + n_k)$ * 1 column vector. It follows that each set of independent constraints can be written as:

$D^k x^k = b^k$, k = 1,2,…,N. x^k represent the vector of the corresponding variables, x^k is $(m_k + n_k)$ *1 column vector.

Assuming that common constraints are:

A^k which represents the technogical matrix of k^{th} stage activity, A_k is $m_0 * (m_k * n_k)$ matrix, m_0 is the number of common constraints. b^0 is the corresponding common resources vector which canbe written as m_o*1.

This gives: $A^1 x^1 + A^2x^2 + \ldots\ldots + A^k x^k + \ldots\ldots + A^N x^N = b^0$

Assuming that the objective functions are:

c^K which represent the vector of the first criterion coefficients for the k^{th} stage activity, c^k is $1*(m_k*n_k)$ row vector.

d^k represent the vector of the second criterion coefficients for the k^{th} stage activity, d^k is $1*(m_k*n_k)$ row vector.

Let: For the master program:

B be the basic matrix associated with the current basic solution, B is (mo*N) * (m_o+N) matrix.

C_B the row vector of the corresponding coefficients in the objective function, C_B is $1*(m_o+N)$ row vector.

R_o the matrix of size $(m_o + N)*m_o$ consisting of the first mo columns of B^{-1}, and

v_j the $(m_o + j)$th column of the same matrix B^{-1}

The algorithm presented here is divided into two phases.

Phase 1: To determine the non-dominated extreme points in the objective space. This algorithm is validated by the following theorem [1].

- *Theorem*

Point $z^{(q)} = \left(z_1^{(q)}, z_2^{(q)}\right)$ in a non-dominated extreme point is the objective space if and only if z(q) is recorded by the algorithm.

Phase II: The decomposition algorithm can be found in [7]. Since the special structure of the (BMTSP 3) model may allow the determination of the optimal solution by decomposing the problem into small sub-problems then by solving those sub-problems almost independently, and the decomposition algorithm for solving large scale linear programming problems utilizing the special nature of transshipment problem can be used to solve it.

Phase I:

Step 1: From phase II, we can find:

$$z_1^{(1)} = Min.\ \left(z_1 / x\ \varepsilon\ M\right)$$

$$z_2^{(1)} = Min.\ \left(z_2 / z_1 = z_1^{(1)}\ and\ x\ \varepsilon\ M\right)$$

$z_1^{(1)}$ and $z_2^{(1)}$ are obtained and q is set to 1. Similarly, we can find:

$$z_2^{(2)} = Min.\ \left(z_2 / x\ \varepsilon\ M\right)$$

$$z_1^{(2)} = Min.\ \left(z_1 / z_2 = z_2^{(2)}\ and\ x\ \varepsilon\ M\right)$$

$$\text{If } (z_1^{(2)}, z_2^{(2)}) = (z_1^{(1)}, z_2^{(1)}), \text{ stop.}$$

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Otherwise record $(z_1^{(2)}, z_2^{(2)})$ and set $q = q+1$

Defines sets L = {(1,2)} and E = ϕ, and go to step 2.

Step 2: Choose an element $(r,s) \in L$ and set

$$a_1^{(r,s)} = |z_2^{(s)} - z_2^{(r)}| \quad \text{and}$$

$$a_2^{(r,s)} = |z_1^{(s)} - z_1^{(r)}| \quad \text{and}$$

Go to phase II to obtain the optimal solution $(\overline{x^k}, k=1,2,..,N)$ to the multistage transshipment problem.

$$\text{Minimize} \sum_{k=1}^{N} \sum_{i_k, j_k} (e_1^{(r,s)} c_{i_k j_k}^k + a_2^{(r,s)} d_{i_k j_k}^k) \, x_{i_k j_k}^k$$

Subject to

$$x^k \,\varepsilon\, M, x^k \geq o, \quad k = 1,2,..,N$$

If there are alternative optima, choose an optimal solution $\overline{x^k}$, k=1,2,..,N, for which

$$\sum_{k=1}^{N} \sum_{i_k, j_k} (c_{i_k j_k}^k \, x_{i_k j_k}^{-k} \ \min.)$$

$$\text{Let } \overline{z_1} = \sum_{k=1}^{N} \sum_{i_k, j_k} c_{i_k j_k}^k \, x_{i_k j_k}^{-k} \quad \text{and}$$

$$\overline{z_2} = \sum_{k=1}^{N} \sum_{i_k, j_k} d_{i_k j_k}^k \, x_{i_k j_k}^{-k}$$

If $(\overline{z_1}, \overline{z_2})$ is equal to $(z_1^{(r)}, z_2^{(r)})$ or $(z_1^{(s)}, z_2^{(s)})$

Set E = E ∪ {(r,s)} and go to step 3.

Otherwise record $(z_1^{(q)}, z_2^{(q)})$ such that

$z_1^{(q)} = \overline{z_1}$, $z_2^{(q)} = \overline{z}_2$ *and set* $q = q+1$,

L = L ∪ {(r,q)}, (q,s)} and go to step 3.

Step 3 : Set L = L - {(r-s)}. If L = ϕ, stop.

Otherwise go to step 2.

Phase II:

Step 1 : Reduce the original problem to the modified form in terms of the new variables β^k

Step 2 : Find an initial basic feasible solution to the modified problem.

Step 3 : Solve the sub-problems

$$w^k = (c^k OR\ d^k - c_B \ R_o \ A^k) \ x^k$$

Subject to:

$$D^k \ x^k = b^k,$$

$$x^k \geq o, k = 1,2,..., N.$$

Note: c^k is used with the first criteria, and d^k is used with the second criteria.

In order to obtain $\hat{x}_l^k$ and w^{*k} by using the transportation technique, go to step 4.

Step 4 : For the current iteration, find:

$$\rho^k = \overset{*k}{w} - c_B \ v^k \ , k = 1,2,..., N,$$

Then determine $\rho = \underset{k}{Min}\,(\rho^k)$

If $\rho \geq o$, the current solution is optimal and the process can be terminated, the optimal solution to multistage transportation problem is:

$$x^k = \sum_{L=1}^{L} \beta_L^K \ \overset{\wedge K}{xl}, \ k = 1,2,..., N$$

Otherwise, go to step 5.

Step 5 : Introduce the variable β_L^k corresponding to ρ into the basic solution. Determine the leaving variable using the feasibility condition and compute the next B^{-1} using the revised simplex method technique, go to step 3.

- *Illustrative Example*

The suggested algorithm for solving problem of the type BMTSP 3 is illustrated in the following example:

Consider the following bi-criteria two-stage transshipment problem. For each stage the availabilities, requirements, costs and deteriorations for each stage are given by:

$a_1^1 = 6$, $a_2^1 = 4$, $a_3^1 = 2$, $b_1^1 = a_1^2 = 9$, $b_2^1 = a_2^2 = 3$,

$b_1^2 = 6$, $b_2^2 = 2$, $b_3^2 = 4$

Table 1-1 : Transportation cost at stages (1)

	D^1_1	D^1_2	S^1_1	S^1_2	S^1_3
S^1_1	5	4	0	2	1
S^1_2	10	8	1	0	4
S^1_3	9	9	3	2	0
D^1_1	0	1	5	9	9
D^1_2	3	0	4	6	7

Table 1-2 : Transportation cost at stages (2)

	D^2_1	D^2_2	D^2_3	S^2_1	S^2_2
S^2_1	4	3	3	0	3
S^2_2	8	4	7	2	0
D^2_1	0	2	4	8	7
D^2_2	4	0	3	3	5
D^2_3	3	4	0	4	9

Table 2-1 : Deterioration cost at stages (1)

	D^1_1	D^1_2	S^1_1	S^1_2	S^1_3
S^1_1	3	6	0	1	4
S^1_2	7	9	3	0	6
S^1_3	12	11	4	6	0
D^1_1	0	3	7	11	12
D^1_2	5	0	7	8	8

Table 2-2 : Deterioration cost at stages (2)

	D^2_1	D^2_2	D^2_3	S^2_1	S^2_2
S^2_1	6	5	5	0	6
S^2_2	11	6	9	5	0
D^2_1	0	4	6	11	9
D^2_2	6	0	5	4	7
D^2_3	5	7	0	6	11

One requirement is added to the above problem:

It is required that the quantity shipped from the first source to the first destination in the first stage is equal to the quantity shipped from the first source to the first destination in the second stage.

The mathematical model is given as follows:

Minimize
$$
\begin{aligned}
{}^{1} = \; & 5x^1_{11} + 4x^1_{12} + 0x^1_{13} + 2x^1_{14} + x^1_{15} \\
& + 10x^1_{21} + 8x^1_{22} + x^1_{23} + 0x^1_{24} + 4x^1_{25} \\
& + 9x^1_{31} + 9x^1_{32} + 3x^1_{33} + 2x^1_{34} + 0x^1_{35} \\
& + 0x^1_{41} + x^1_{42} + 5x^1_{43} + 9x^1_{44} + 9x^1_{45} \\
& + 3x^1_{51} + 0x^1_{52} + 4x^1_{53} + 6x^1_{54} + 7x^1_{55} \\
& + 4x^2_{11} + 3x^2_{12} + 2x^2_{13} + 0x^2_{14} + 3x^2_{15} \\
& + 8x^2_{21} + 4x^2_{22} + 7x^2_{23} + 2x^2_{24} + 0x^2_{25} \\
& + 0x^2_{31} + 2x^2_{32} + 4x^2_{33} + 8x^2_{34} + 7x^2_{35} \\
& + 4x^2_{41} + 0x^2_{42} + 3x^2_{43} + 3x^2_{44} + 5x^2_{45} \\
& + 3x^2_{51} + 4x^2_{52} + 0x^2_{53} + 4x^2_{54} + 9x^2_{55}
\end{aligned}
$$

$$
\begin{aligned}
Z^2 = \; & 3x^1_{11} + 6x^1_{12} + 0x^1_{13} + 1x^1_{14} + 4x^1_{15} \\
& + 7x^1_{21} + 9x^1_{22} + 3x^1_{23} + 0x^1_{24} + 6x^1_{25} \\
& + 12x^1_{31} + 11x^1_{32} + 4x^1_{33} + 6x^1_{34} + 0x^1_{35} \\
& + 0x^1_{41} + 3x^1_{42} + 7x^1_{43} + 11x^1_{44} + 12x^1_{45} \\
& + 5x^1_{51} + 0x^1_{52} + 7x^1_{53} + 8x^1_{54} + 8x^1_{55} \\
& + 6x^2_{11} + 5x^2_{12} + 5x^2_{13} + 0x^2_{14} + 6x^2_{15} \\
& + 11x^2_{21} + 6x^2_{22} + 9x^2_{23} + 5x^2_{24} + 0x^2_{25} \\
& + 0x^2_{31} + 4x^2_{32} + 6x^2_{33} + 11x^2_{34} + 9x^2_{35} \\
& + 6x^2_{41} + 0x^2_{42} + 5x^2_{43} + 4x^2_{44} + 7x^2_{45} \\
& + 5x^2_{51} + 7x^2_{52} + 0x^2_{53} + 6x^2_{54} + 11x^2_{55}
\end{aligned}
$$

Subject to:

$$
\begin{aligned}
& x^1_{11} = x^2_{11} \\
& x^1_{11} + x^1_{12} + x^1_{13} + x^1_{14} + x^1_{15} = 18 \\
& x^1_{21} + x^1_{22} + x^1_{23} + x^1_{24} + x^1_{25} = 16 \\
& x^1_{31} + x^1_{32} + x^1_{33} + x^1_{34} + x^1_{35} = 14 \\
& x^1_{41} + x^1_{42} + x^1_{43} + x^1_{44} + x^1_{45} = 12 \\
& x^1_{51} + x^1_{52} + x^1_{53} + x^1_{54} + x^1_{55} = 12 \\
& x^1_{11} + x^1_{21} + x^1_{31} + x^1_{41} + x^1_{51} = 21 \\
& x^1_{12} + x^1_{22} + x^1_{32} + x^1_{42} + x^1_{52} = 15 \\
& x^1_{13} + x^1_{23} + x^1_{33} + x^1_{43} + x^1_{53} = 12 \\
& x^1_{14} + x^1_{24} + x^1_{34} + x^1_{44} + x^1_{54} = 12 \\
& x^1_{15} + x^1_{25} + x^1_{35} + x^1_{45} + x^1_{55} = 12 \\
& x^2_{11} + x^2_{12} + x^2_{13} + x^2_{14} + x^2_{15} = 21 \\
& x^2_{21} + x^2_{22} + x^2_{23} + x^2_{24} + x^2_{25} = 15 \\
& x^2_{31} + x^2_{32} + x^2_{33} + x^2_{34} + x^2_{35} = 12 \\
& x^2_{41} + x^2_{42} + x^2_{43} + x^2_{44} + x^2_{45} = 12 \\
& x^2_{51} + x^2_{52} + x^2_{53} + x^2_{54} + x^2_{55} = 12 \\
& x^2_{11} + x^2_{21} + x^2_{31} + x^2_{41} + x^2_{51} = 18 \\
& x^2_{12} + x^2_{22} + x^2_{32} + x^2_{42} + x^2_{52} = 14 \\
& x^2_{13} + x^2_{23} + x^2_{33} + x^2_{43} + x^2_{53} = 16 \\
& x^2_{14} + x^2_{24} + x^2_{34} + x^2_{44} + x^2_{54} = 12 \\
& x^2_{15} + x^2_{25} + x^2_{35} + x^2_{45} + x^2_{55} = 12
\end{aligned}
$$

Table 3 : Set of non dominated extreme points

Iteration	L	E	Recorded Point
1	{(1,2)}	ϕ	Z^1=(113,156)
2	{(1,2)}	ϕ	Z^2=(127,140)
3	{(1,3), (3,2)}	ϕ	Z^3=(121,141)
4	{(3,2), (1,4), (4,3)}	ϕ	Z^4=(115,149)
5	{(1,4), (4,3), (3,5), (5,2)}	ϕ	Z^5=(124,140)
6	{(1,4), (4,3), (3,5)}	{(5,2)}	Z^6=(124,140)
7	{(1,4), (4,3)}	{(5,2), (3,5)}	Z^7=(124,140)
8	{(1,4)}	{(5,2), (3,5), (4,3)}	Z^8=(115,149)
9	ϕ	{(5,2), (3,5), (4,3), (1,4)}	Z^9=(113,156)

Table 4 : Non zero value of X_{ij} for each non dominated point

Iteration	Non dominated (Z_1, Z_2)	Non zero value of X_{ij}
1	$Z^1=(113,156)$	$X^1_{11}=6, X^1_{12}=4, X^1_{13}=8, X^1_{23}=4, X^1_{24}=12, X^1_{31}=2, X^1_{35}=12, X^1_{41}=12, X^1_{51}=1, X^1_{52}=11, X^2_{11}=6, X^2_{13}=4, X^2_{14}=11, X^2_{22}=2, X^2_{24}=1, X^2_{25}=12, X^2_{31}=12, X^2_{42}=12, X^2_{53}=12.$
2	$Z^2=(127,140)$	$X^1_{11}=6, X^1_{12}=2, X^1_{13}=10, X^1_{21}=3, X^1_{22}=1, X^1_{24}=12, X^1_{33}=2, X^1_{35}=12, X^1_{41}=12, X^1_{52}=12, X^2_{11}=6, X^2_{13}=3, X^2_{14}=12, X^2_{22}=2, X^2_{23}=1, X^2_{25}=12, X^2_{31}=12, X^2_{42}=12, X^2_{53}=12.$
3	$Z^3=(121,141)$	$X^1_{11}=6, X^1_{12}=3, X^1_{13}=9, X^1_{21}=3, X^1_{23}=1, X^1_{24}=12, X^1_{33}=2, X^1_{35}=12, X^1_{41}=12, X^1_{52}=12, X^2_{11}=6, X^2_{13}=4, X^2_{14}=11, X^2_{22}=2, X^2_{24}=1, X^2_{25}=12, X^2_{31}=12, X^2_{42}=12, X^2_{53}=12.$
4	$Z^4=(115,149)$	$X^1_{11}=6, X^1_{12}=3, X^1_{13}=9, X^1_{21}=1, X^1_{23}=3, X^1_{24}=12, X^1_{31}=2, X^1_{35}=12, X^1_{41}=12, X^1_{52}=12, X^2_{11}=6, X^2_{13}=4, X^2_{14}=11, X^2_{22}=2, X^2_{24}=1, X^2_{25}=12, X^2_{31}=12, X^2_{42}=12, X^2_{53}=12.$
5	$Z^5=(124,140)$	$X^1_{11}=6, X^1_{12}=3, X^1_{13}=9, X^1_{21}=3, X^1_{23}=1, X^1_{24}=12, X^1_{33}=2, X^1_{35}=12, X^1_{41}=12, X^1_{52}=12, X^2_{11}=6, X^2_{13}=3, X^2_{14}=12, X^2_{22}=2, X^2_{23}=1, X^2_{25}=12, X^2_{31}=12, X^2_{42}=12, X^2_{53}=12.$
6	$Z^6=(124,140)$	$X^1_{11}=6, X^1_{12}=3, X^1_{13}=9, X^1_{21}=3, X^1_{23}=1, X^1_{24}=12, X^1_{33}=2, X^1_{35}=12, X^1_{41}=12, X^1_{52}=12, X^2_{11}=6, X^2_{13}=3, X^2_{14}=12, X^2_{22}=2, X^2_{23}=1, X^2_{25}=12, X^2_{31}=12, X^2_{42}=12, X^2_{53}=12.$
7	$Z^7=(124,140)$	$X^1_{11}=6, X^1_{12}=3, X^1_{13}=9, X^1_{21}=3, X^1_{23}=1, X^1_{24}=12, X^1_{33}=2, X^1_{35}=12, X^1_{41}=12, X^1_{52}=12, X^2_{11}=6, X^2_{13}=3, X^2_{14}=12, X^2_{22}=2, X^2_{23}=1, X^2_{25}=12, X^2_{31}=12, X^2_{42}=12, X^2_{53}=12.$
8	$Z^8=(115,149)$	$X^1_{11}=6, X^1_{12}=3, X^1_{13}=9, X^1_{21}=1, X^1_{23}=3, X^1_{24}=12, X^1_{31}=2, X^1_{35}=12, X^1_{41}=12, X^1_{52}=12, X^2_{11}=6, X^2_{13}=4, X^2_{14}=11, X^2_{22}=2, X^2_{24}=1, X^2_{25}=12, X^2_{31}=12, X^2_{42}=12, X^2_{53}=12.$
9	$Z^9=(113,156)$	$X^1_{11}=6, X^1_{12}=4, X^1_{13}=8, X^1_{23}=4, X^1_{24}=12, X^1_{31}=2, X^1_{35}=12, X^1_{41}=12, X^1_{51}=1, X^1_{52}=11, X^2_{11}=6, X^2_{13}=4, X^2_{14}=11, X^2_{22}=2, X^2_{24}=1, X^2_{25}=12, X^2_{31}=12, X^2_{42}=12, X^2_{53}=12.$

III. Conclusion

An algorithm for solving a certain class of bi-criteria multistage transportation problems with transshipment (BMTSP) is presented. The presented algorithm enables solving such problems more realistically. It can be used for determining all efficient extreme points. The main advantage of this approach is that the bi-criteria two stage transshipment problem can be solved using the standard form of a transshipment problem at each iteration. Goods transportation may not operate always directly among suppliers and customers. In such problems, it is possible to optimize the transshipment problem into two stages. From the application, decision maker will have all efficient extreme points and their related distributions. Therefore, any point can be chosen, which will provide their policy.

References Références Referencias

1. Orden, A. (1956). "Transshipment problem", Management Science, (3): 276-285.
2. King, G. Logan, S. (1964). "Optimum location, number, and size of processing plants with raw product and final product shipments", Journal of Farm Economics, 46: 94-108.
3. Rhody, D. (1963). "Interregional competitive position of the hog-pork industry in the southeast United States", Ph.D. thesis, Iowa State University.
4. Judge, G., Hsvlicek J., and Rizek, R. (1965). "An interregional model: Its formulation and application to the live-stock industry", Agriculture and Economy and Revision, 7 :1-9.
5. Hurt, V. and Tramel, T. (1965). "Alternative formulation of the transshipment problem", Journal of Farm Economics, 47 (3): 763-773.
6. Grag, R. and Parakash, S. (1985). "Time minimizing transshipment problem", Indian Journal of Pure and Applied Mathematics, 16 (5): 449-460.
7. Here, Y. and Tzura, M. (2001). "The dynamic transshipment problem", Naval Research Logistics Quarterly, 48: 386-408.
8. Ozdemir, D. Yucesan, E and Here, Y. (2006). "Multi location transshipment problem with capacitated production and lost sales", Proceeding of the 2006 Winter Simulation Conference, Pages 1470-1476.
9. Osman, M.S.A. and Ellaimony, E.E.M. (1984). "On bicriteria multistage transportation problems", First Conference on Operations Research and its Military Applications, Page 143-157.
10. Khurana A. and Arora S. (2011). "Solving transshipment problems with mixed constraints", International Journal of Management Science and Engineering Management, 6 (4): Page 292-297.
11. Khurana A., Tripti V. and Arora S. (2012). "An algorithm for solving time minimizing capacitated transshipment problem", International Journal of Management Science and Engineering Management, 7 (3): Page 192-199.
12. Yousria A., Bothina E. and Hanadi Z. (2012). "Trust region algorithm for multi-objective transportation,

assignment, and transshipment problems", Life Science Journal, 9 (3): Page 1765-1772.
13. Archana Khurana, June (2013). "Multi-index fixed charge bi-criterion transshipment problem", OPSEARCH, Volume 50, Issue 2, pp 229-249.
14. P. Rajendran and P. Pandian, (2012). "Solving Fully Interval Transshipment Problems", International Mathematical Forum, Vol. 7, no. 41, 2027 – 2035.
15. Sayed A. Zaki, Abd Allah A. Mousa, Hamdy M. Geneedi, Adel Y. Elmekawy, (2012). "Efficient Multiobjective Genetic Algorithm for Solving Transportation, Assignment, and Transshipment Problems", Applied Mathematics, Vol. 3 No. 1, Article ID: 16769, 8 pages.
16. Anupam Ojha, Shyamal Kr. Mondal, Manoranjan Maiti, (2011), "Transportation policies for single and multi-objective transportation problem using fuzzy logic", Mathematical and Computer Modelling 01/2011; 53:1637-1646.
17. Mansour Saraj1 and Feryal Mashkoorzadeh, September (2010), "Solving a Multi Objective Transportation Problem (MOTP) Under Fuzziness on Using Interval Numbers", AIP Conf. Proc. 1281, 1137, Rhodes (Greece).
18. Waiel F. Abd El-Wahed, January (2001), "A multi-objective transportation problem under fuzziness", Fuzzy Sets and Systems, Volume 117, Issue 1, Pages 27–33.
19. S.K. Das, A. Goswami, S.S. Alam, August (1999), "Multiobjective transportation problem with interval cost, source and destination parameters", European Journal of Operational Research, Volume 117, Issue 1, Pages 100–112.