

# An Algorithm for Solving Bi-Criteria Large Scale Transshipment Problems

Jasem AlRajhi

*Received: 10 December 2013 Accepted: 3 January 2014 Published: 15 January 2014*

---

## Abstract

This paper describes an algorithm for solving a certain class of bi-criteria multistage transportation problems with transshipment (BMTSP). A several bi-criteria multistage transportation problem with transshipment are formulated. The presented algorithm is mainly based on application of the methods of solving bi-criteria single stage transportation problems, utilizing available decomposition techniques for solving large-scale linear programming problems, and the methods of treating the transshipment problems. The mathematical formulation of the presented class does not affect the special structure of the transshipment problem for each of the individual stages. An illustrative example is introduced to validate that the implementation of the algorithm.

---

**Index terms**— large scale transportation problem, transshipment problem, multi-objective, decision making, decomposition technique of linear programming.

## 1 Introduction

When shipments go directly from a supply point to a demand point, i.e. shipments do not take place between origins or between destinations nor from destinations to origins, it is called a classical transportation problem. In many real life situations, shipments are allowed between supply points or between demand points. There are many points (called transshipment points) through which goods can be transshipped on their journey from a supply point to a demand point. Shipping problems with any or all of these characteristics are considered as transshipment problems. It was first introduced by Ford (1965) [1] in which he introduced an extension of the original transportation problem to include the possibility of transshipment.

The problem of determining simultaneously the flow of primary products through processors to the market of final products has been formulated alternatively as a transshipment model by multi-regional, multi-product, and multi-plant problem formulated in the form of general linear programming model has been proposed by Judge et al (1965) [4].

Later, various alternative formulations of the transshipment problem within the framework of the transportation model that permits solution of problems of the type discussed by King and Logan without the need for subtraction of artificial variables were discussed by Hurt and Tramel (1965) [5]. On the other hand, Farah and Prakash (1985) [6] studied time minimizing transshipment problem. Then dynamic transshipment problem was studied by Herer and Tzur (2001) [7]. Ozdemir (2006) studied Multi location transshipment problem with capacitated production and lost sales afterwards [8]. Furthermore, Osman et al (1984) [9] introduced an algorithm for solving bi-criteria multistage transportation problems.

Recently, Khurana et al (2011) [10] studied a transshipment problem with mixed constraints. He introduced an algorithm for solving time minimizing capacitated transshipment problem [11]. Abo-elnaga et al (2012) [12] introduced a trust region globalization strategy to solve multi-objective transportation, assignment, and transshipment problems. Khurana (2013) [13] introduced a Multi-index fixed charge bi-criterion transshipment problem. Rajendran et al [14] (2012) presented A new method namely, splitting method, to solve fully interval transshipment problems. Zaki et al [15] (2012) used the genetic algorithm for solving transportation, assignment, and transshipment problems. Ojha et al [16] (2011) formulated single and multi-objective transportation models with fuzzy relations under the fuzzy logic. Saraj et al [17] (2010) solved the multi objective transportation



## 7 d) Bi-Criteria Multistage Transportation Problem wth transshipment of the Fourth Kind (BMTSP 4)

This case represents bi-criteria multistage transshipment problems in which the difference between the input and output transportation commodity is known at the sources (destinations) of each intermediate stage.

The assumed transportation restrictions in this case affect the transshipment formulation of each individual stage.

The mathematical formulation of the problem representing this case is given as:

Where and for the quantity shipped from the source to itself and from the destination to itself:  $k = 1, 2, ?, N$

$$k j i k k c k j i k k d k i k S k j k D 1 1 1 1 1 1, \dots, 2, 1, 1 1 1 1 1 1 1 1 n j a x x j j j n j i i j ? ? ? ? ? 1 1$$

$$1 1 1 1, \dots, 2, 1, 1 1 1 1 1 1 1 1 n j b x x j j j n j i j i ? ? ? ? ? \dots k k k j k j j n j i k j j n j a x x k k k$$

$$k k k k k k, \dots, 2, 1, 1 ? ? ? ? ? k k k j k j j n j i j k j i n j b x x k k k k k k k k, \dots, 2, 1, 1 ? ? ? ? ? .$$

$$\dots N N N j N j j n j i j N i j n j a x x N N N N N N N N, \dots, 2, 1, 1 ? ? ? ? ? N N N j N j j n j i j N j i n$$

$$j b x x N N N N N N N N, \dots, 2, 1, 1 ? ? ? ? ? , 0 ) , ( 1 1 1 1 1 1 ? ? ? ? ? k j i k j i k j i r k k k k$$

$$k k k x x x F 0, \dots 0, 1 1 1 ? ? ? N j i k j i j i N N k k x x x ; \dots, \dots, ; \dots, \dots, 1 1 N K N K j j j i i i N k F$$

$$k r, \dots, 2, 1, ? \dots \dots 1 1 1 1 1 1 1 1 1 1 1 1 ? ? ? ? ? ? ? ? ? ? k k k k k k k k n i n j k j i k j i n i n j$$

$$j i j i x c x c Z \text{ Min } ? ? ? ? ? N N N N N N N n i n j N j i N j i x c 1 1 \dots \dots 1 1 1 1 1 1 2 1 1 1 1 1 1 1 ?$$

$$? ? ? ? ? ? ? ? ? k k k k k k k k n i n j k j i k j i n i n j j i j i x d x d Z ? ? ? ? ? N N N N N N N n i n j$$

$$N j i N j i x d 1 1$$

$$\dots k j i k k c 0 ? k j i k k d k i k S k j k D 1 1 1 1 1 1, \dots, 2, 1, 1 1 1 1 1 1 1 1 n j a x x j j j n j i i j ? ? ?$$

$$? ? ? 2 2 1 1 1 1 2 2 1 1 1, \dots 2, 1 ; \dots, 2, 1, ) ( 1 2 2 2 2 2 2 2 1 1 1 1 1 1 1 1 n j n j b x x x x j n j i j j i j$$

$$j j n j i j i ? ? ? ? ? ? ? ? ? ? \dots k k k k n j i k j k j j k j j n j i j k j i n j n j b x x x x k k k k k k$$

$$k k k k k k k k k k, \dots 2, 1 ; \dots, 2, 1, ) ( 1 1 1 1 1 1 1 1 1 1 1 1 1 1 ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ?$$

$$? ? ? ? ? N N N N n j i j N j N j j N j j n j i j N j i n j n j b x x x x N N N N N N N N N N N N N N N N$$

$$N N, \dots 2, 1 ; \dots, 2, 1, ) ( 1 1 1 1 1 1 1 1 1 1 1 1 1 1 ?$$

$$N N N j N j j n j i j N j i n j b x x N N N N N N N N, \dots, 2, 1, 1 ? ? ? ? ? 0 ? k j i k k x \text{ for all } i k, j k ;$$

$$k=1, 2, ?, N$$

(BMTSP 1) is solved as a bi-criteria single stage transshipment problem.

(BMTSP 2) can be solved as  $N$  single stage bicriteria transshipment problems and the minimum value of the total transport costs and deteriorations are obtained as the sum of the minimum transportation costs and deteriorations for each individual stage.

(BMTSP 3) can be solved using the decomposition technique utilizing the special nature of transshipment problems. The next section will be devoted to the solution of this type of problems.

(BMTSP 4) is solved using any method for solving bi-criteria linear programming problems.

## 8 e) An Algorithm for Solving BMTSP 3

The decomposition technique of linear programming can be used to solve the bi-criteria multistage transshipment problems especially for the (BMTSP 3) type. This type of bi-criteria multistage transshipment problems decomposed into [2,3,5,8]:

? Sub problems corresponding to every stage. The following section presents an algorithm for determining all non dominated extreme points for the (BMTSP 3) model from which the solution for (BMTSP 1) and (BMTSP 2) models can be deduced from it as special cases. Assuming that independent constraints are:  $D_k$ ,  $k = 1, 2, ?, N$  is the technological matrix of the  $k$ th stage activity.  $D_k$  is  $(m_k + n_k) * (m_k + n_k)$  matrix,  $N$  is the number of stages,  $m_k$  is the number of sources at  $k$ th stage,  $n_k$  is the number of destinations.  $b_k$  is the column vector consisting of the availabilities and requirements of the  $k$ th sub-problem,  $b_k$  is  $(m_k + n_k) * 1$  column vector. It follows that each set of independent constraints can be written as:  $D_k x_k = b_k$ ,  $k = 1, 2, ?, N$ .

$x_k$  represent the vector of the corresponding variables,  $x_k$  is  $(m_k + n_k) * 1$  column vector.

## 9 Assuming that common constraints are:

$A_k$  which represents the technological matrix of  $k$  stage activity,  $A_k$  is  $m_0 * (m_k * n_k)$  matrix,  $m_0$  is the number of common constraints.  $b_0$  is the corresponding common resources vector which can be written as  $m_0 * 1$ . This gives:  $A_1 x_1 + A_2 x_2 + \dots + A_k x_k + \dots + A_N x_N = b_0$  Assuming that the objective functions are:  $c_k$  which represent the vector of the first criterion coefficients for the  $k$ th stage activity,  $c_k$  is  $1 * (m_k * n_k)$  row vector. The algorithm presented here is divided into two phases. Phase 1: To determine the non-dominated extreme points in the objective space. This algorithm is validated by the following theorem [1].

## 10 ? Theorem

Point  $z(q) =$  in a non-dominated extreme point is the objective space if and only if  $z(q)$  is recorded by the algorithm.

Phase II: The decomposition algorithm can be found in [7]. Since the special structure of the (BMTSP 3) model may allow the determination of the optimal solution by decomposing the problem into small sub-problems then by solving those sub-problems almost independently, and the decomposition algorithm for solving large scale

## 14 CONCLUSION

linear programming problems utilizing the special nature of transshipment problem can be used to solve it. ? ? ?  
? ? ? q q z z 2 1 ,  
An Algorithm for Solving Bi-Criteria Large Scale Transshipment Problems © 2014 Global Journals Inc. (US)  
th

### 11 Phase I:

Step 1: From phase II, we can find: and are obtained and q is set to 1. Similarly, we can find: and set  $L = L$   
{(r,q)}, (q,s)} and go to step 3. ? ? M x z Min z ? / . 1 ) 1 ( 1 ? ? ? . / . ) 1 ( 1 1 2 ) 1 ( 2 M x and z z z Min z  
? ? ? ) 1 ( 1 z ) 1 ( 2 z ? ? M x z Min z ? / . 2 ) 2 ( 2 ? ? ? . / . ) 2 ( 2 2 1 ) 2 ( 1 M x and z z z Min z ? ? ?  
If . stop ) , , ( ) , ( ) 1 ( 2 ) 1 ( 1 ) 2 ( 2 ) 2 ( 1 z z z z ? Global Journal of  
Step 3 : Set  $L = L - \{(r-s)\}$ . If  $L =$  , stop. Otherwise go to step 2.  
Phase II:  
Step 1 : Reduce the original problem to the modified form in terms of the new variables  
Step 2 : Find an initial basic feasible solution to the modified problem.  
Step 3 : Solve the sub-problems Subject to:  
Note: c k is used with the first criteria, and d k is used with the second criteria.  
In order to obtain and by using the transportation technique, go to step 4.  
Step 4 : For the current iteration, find: Then determine

### 12 If

, the current solution is optimal and the process can be terminated, the optimal solution to multistage  
transportation problem is:  
Otherwise, go to step 5.  
Step 5 : Introduce the variable corresponding to into the basic solution. Determine the leaving variable using  
the feasibility condition and compute the next B -1 using the revised simplex method technique, go to step 3.

### 13 ? Illustrative Example

The suggested algorithm for solving problem of the type BMTSP 3 is illustrated in the following example:  
Consider the following bi-criteria two-stage transshipment problem. For each stage the availabilities,  
requirements, costs and deteriorations for each stage are given by: An Algorithm for Solving Bi-Criteria Large  
Scale Transshipment Problems One requirement is added to the above problem:= q+1 ? , ? ) ( 2 ) ( 2 ) , ( 2 ( 2  
z ( ( q r s s r z z a ? ? ) ( 1 ) ( 1 ) , ( 1 r s s r z z a ? ? k ( x , k=1,2,...,N) k j i k j i s r k j i s r j i N k k k k k k k  
k k x d a c e ) ( ) , 2 ? ? ? N k o x M x k k , , , 2 , 1 , , ? ? ? k x , k=1,2,...,N, . ) min ( , 1 k j i k j i j i N k k k k k k  
k k x c ? ? ? ? Let 1 z = k j i k j i j i N k k k k k k k x c ? ? ? ? , 1 2 z = k j i k j i j i N k k k k k k k x d ? ?  
? ? , 1 1 z , 2 z ( ( ) , ( ) ( 2 ) ( 1 s s z z ) , ( ) ( 2 ) ( 1 r r z z ? ) ( 2 ) ( 1 , q q z z ( ( , 1 , 2 ) ( 2 1 ) ( 1 ? ? ?  
? q q z z z z q q ? ? ? k k k o B k k k x A R c d O R c w ) ( ? ? , k k k b x D ? . , , , , 2 , 1 , N k o x k ? ? k l x  
? k w \* , , , , , 2 , 1 , \* N k v c w k B k k ? ? ? ? ) ( k k Min ? ? ? ? ? o ? ? N k x l x K K L L L k , , , , 2 , 1 , 1 ? ? ?  
? ? ? k L ? ? 1 1 a = 6 , 1 2 a = 4 , 1 3 a = 2 , 1 1 b = 2 1 a = 9 , 1 2 b = 2 2 a = 3 , ( 2 ) , ( 1 , 1 ?

It is required that the quantity shipped from the first source to the first destination in the first stage is equal to  
the quantity shipped from the first source to the first destination in the second stage. The mathematical model  
is given as follows:

$$\text{Minimize } 1 = 5x_{11} + 4x_{12} + 0x_{13} + 2x_{14}$$

### 14 Conclusion

An algorithm for solving a certain class of bicriteria multistage transportation problems with transshipment  
(BMTSP) is presented. The presented algorithm enables solving such problems more realistically. It can be used  
for determining all efficient extreme points. The main advantage of this approach is that the bi-criteria two stage  
transshipment problem can be solved using the standard form of a transshipment problem at each iteration.  
Goods transportation may not operate always directly among suppliers and customers. In such problems, it is  
possible to optimize the transshipment problem into two stages. From the application, decision maker will have  
all efficient extreme points and their related distributions. Therefore, any point can be chosen, which will provide  
their policy.

<sup>1</sup>© 2014 Global Journals Inc. (US)

<sup>2</sup>14=11, X 2 22=2, X 2 24=1, X 2 25=12, X 2 31=12, X 2 42=12, X 2 53=12. 2 Z 2 =(127,140) X 1 11=6, X  
1 12=2, X 1 13=10, X 1 21=3, X 1 22=1, X 1 24=12, X 1 33=2, X 1 35=12, X 1 41=12, X 1 52=12, X 2 11=6,  
X 2 13=3, X 2 14=12, X 2 22=2, X 2 23=1, X 2 25=12, X 2 31=12, X 2 42=12, X 2 53=12. 3 Z 3 = (121,141)  
X 1 11=6, X 1 12=3, X 1 13=9, X 1 21=3, X 1 23=1, X 1 24=12, X 1 33=2, X 1 35=12, X 1 41=12, X 1 52=12,  
X 2 11=6, X 2 13=4, X 2 14=11, X 2 22=2, X 2 24=1, X 2 25=12, X 2 31=12, X 2 42=12, X 2 53=12. 4 Z 4 =  
(115,149) X 1 11=6, X 1 12=3, X 1 13=9, X 1 21=1, X 1 23=3, X 1 24=12, X 1 31=2, X 1 35=12, X 1 41=12, X  
1 52=12, X 2 11=6, X 2 13=4, X 2 14=11, X 2 22=2, X 2 24=1, X 2 25=12, X 2 31=12, X 2 42=12, X 2 53=12.  
5 Z 5 = (124,140) X 1 11=6, X 1 12=3, X 1 13=9, X 1 21=3, X 1 23=1, X 1 24=12, X 1 33=2, X 1 35=12, X 1



Figure 1:



Figure 2:

11

Figure 3: Table 1 - 1 :

12

Figure 4: Table 1 - 2 :

21

An Algorithm for Solving Bi-Criteria Large Scale Transshipment Problems  
 Subject to:  
 Table 2-2 : Deterioration cost at stages (2)

Figure 5: Table 2 - 1 :

3

$1\ 14 + x\ 1\ 15\ 23 + 0x\ 1\ 22 + x\ 1\ 21 + 8x\ 1 + 10x\ 1\ 24 + 4x\ 1\ 25 + 9x\ 1\ 31 +$   
 $9x\ 1\ 32 + 3x\ 1\ 33 + 2x\ 1\ 34 + 0x\ 1\ 35 + 0x\ 1\ 41 + x\ 1\ 42 + 5x\ 1\ 43 + 9x\ 1\ 44$   
 $+ 9x\ 1\ 45 + 3x\ 1\ 51 + 0x\ 1\ 52 + 4x\ 1\ 53 + 6x\ 1\ 54 + 7x\ 1\ 55 + 4x\ 2\ 11 + 3x\ 2$   
 $12 + 2x\ 2\ 13 + 0x\ 2\ 14 + 3x\ 2\ 15 + 8x\ 2\ 21 + 4x\ 2\ 22 + 7x\ 2\ 23 + 2x\ 2\ 24 + 0x$   
 $2\ 25 + 0x\ 2\ 31 + 2x\ 2\ 32 + 4x\ 2\ 33 + 8x\ 2\ 34 + 7x\ 2\ 35 + 4x\ 2\ 41 + 0x\ 2\ 42 +$   
 $3x\ 2\ 43 + 3x\ 2\ 44 + 5x\ 2\ 45 + 3x\ 2\ 51 + 4x\ 2\ 52 + 0x\ 2\ 53 + 4x\ 2\ 54 + 9x\ 2\ 55$

Figure 6: Table 3 :

4

Year 2014  
 7  
 IV Version I  
 ( ) B Volume XIV Issue  
 Global Journal of Researches in Engineering

Figure 7: Table 4 :

---

$41=12, X\ 1\ 52=12, X\ 2\ 11=6, X\ 2\ 13=3, X\ 2\ 14=12, X\ 2\ 22=2, X\ 2\ 23=1, X\ 2\ 25=12, X\ 2\ 31=12, X\ 2\ 42=12,$   
 $X\ 2\ 53=12. 6\ Z\ 6 = (124,140) X\ 1\ 11=6, X\ 1\ 12=3, X\ 1\ 13=9, X\ 1\ 21=3, X\ 1\ 23=1, X\ 1\ 24=12, X\ 1\ 33=2, X\ 1$   
 $35=12, X\ 1\ 41=12, X\ 1\ 52=12, X\ 2\ 11=6, X\ 2\ 13=3, X\ 2\ 14=12, X\ 2\ 22=2, X\ 2\ 23=1, X\ 2\ 25=12, X\ 2\ 31=12,$   
 $X\ 2\ 42=12, X\ 2\ 53=12. 7\ Z\ 7 = (124,140) X\ 1\ 11=6, X\ 1\ 12=3, X\ 1\ 13=9, X\ 1\ 21=3, X\ 1\ 23=1, X\ 1\ 24=12, X$   
 $1\ 33=2, X\ 1\ 35=12, X\ 1\ 41=12, X\ 1\ 52=12, X\ 2\ 11=6, X\ 2\ 13=3, X\ 2\ 14=12, X\ 2\ 22=2, X\ 2\ 23=1, X\ 2\ 25=12,$   
 $X\ 2\ 31=12, X\ 2\ 42=12, X\ 2\ 53=12. 8\ Z\ 8 = (115,149) X\ 1\ 11=6, X\ 1\ 12=3, X\ 1\ 13=9, X\ 1\ 21=1, X\ 1\ 23=3, X$   
 $1\ 24=12, X\ 1\ 31=2, X\ 1\ 35=12, X\ 1\ 41=12, X\ 1\ 52=12, X\ 2\ 11=6, X\ 2\ 13=4, X\ 2\ 14=11, X\ 2\ 22=2, X\ 2\ 24=1,$   
 $X\ 2\ 25=12, X\ 2\ 31=12, X\ 2\ 42=12, X\ 2\ 53=12. 9\ Z\ 9 = (113,156) X\ 1\ 11=6, X\ 1\ 12=4, X\ 1\ 13=8, X\ 1\ 23=4, X$   
 $1\ 24=12, X\ 1\ 31=2, X\ 1\ 35=12, X\ 1\ 41=12, X\ 1\ 51=1, X\ 1\ 52=11, X\ 2\ 11=6, X\ 2\ 13=4, X\ 2\ 14=11, X\ 2\ 22=2,$   
 $X\ 2\ 24=1, X\ 2\ 25=12, X\ 2\ 31=12, X\ 2\ 42=12, X\ 2\ 53=12.III.$

- 
- [Waiel (2001)] ‘A multiobjective transportation problem under fuzziness’. F Waiel . *Fuzzy Sets and Systems*, January (2001). 117 p. .
- [Hurt and Tramel ()] ‘Alternative formulation of the transshipment problem’. V Hurt , T Tramel . *Journal of Farm Economics* 1965. 47 (3) p. .
- [Khurana et al. ()] ‘An algorithm for solving time minimizing capacitated transshipment problem’. A Khurana , V Tripti , S Arora . *International Journal of Management Science and Engineering Management* 2012. 7 (3) p. .
- [Judge et al. ()] ‘An interregional model: Its formulation and application to the live-stock industry’. G Judge , J Hsvlicek , R Rizek . *Agriculture and Economy and Revision* 1965. 7 p. .
- [Sayed et al. ()] ‘Efficient Multiobjective Genetic Algorithm for Solving Transportation, Assignment, and Transshipment Problems’. A Sayed , Abd Zaki , Allah A Mousa , M Hamdy , Adel Y Geneedi , Elmekawy . ID: 16769. *Applied Mathematics* 2012. 3 (1) .
- [Rhody ()] *Interregional competitive position of the hog-pork industry in the southeast United States*, D Rhody . 1963. Iowa State University (Ph.D. thesis)
- [Ozdemir et al. ()] *Multi location transshipment problem with capacitated production and lost sales*, D Ozdemir , Yucesan , Y Here . 2006. 2006. (Proceeding of the)
- [Khurana (2013)] *Multi-index fixed charge bi-criterion transshipment problem*, Archana Khurana . June (2013). 50 p. .
- [Das et al. (1999)] ‘Multiobjective transportation problem with interval cost, source and destination parameters’. S K Das , A Goswami , S S Alam . *European Journal of Operational Research* August (1999). 117 (1) p. .
- [Osman and Ellaimony ()] ‘On bicriteria multistage transportation problems’. M S A Osman , E E M Ellaimony . *First Conference on Operations Research and its Military Applications*, 1984. p. .
- [King and Logan ()] ‘Optimum location, number, and size of processing plants with raw product and final product shipments’. G King , S Logan . *Journal of Farm Economics* 1964. 46 p. .
- [Sarajl and Mashkoorzadeh (2010)] ‘Solving a Multi Objective Transportation Problem (MOTP) Under Fuzziness on Using Interval Numbers’. Mansour Sarajl , Feryal Mashkoorzadeh . *AIP Conf. Proc. 1281*, (Rhodes (Greece) September (2010). 1137.
- [Rajendran and Pandian ()] ‘Solving Fully Interval Transshipment Problems’. P Rajendran , P Pandian . *International Mathematical Forum* 2012. 7 (41) p. .
- [Khurana and Arora ()] ‘Solving transshipment problems with mixed constraints’. A Khurana , S Arora . *International Journal of Management Science and Engineering Management* 2011. 6 (4) p. .
- [Here and Tzura ()] ‘The dynamic transshipment problem’. Y Here , M Tzura . *Naval Research Logistics Quarterly* 2001. 48 p. .
- [Grag and Parakash ()] ‘Time minimizing transshipment problem’. R Grag , S Parakash . *Indian Journal of Pure and Applied Mathematics* 1985. 16 (5) p. .
- [Ojha et al. ()] ‘Transportation policies for single and multi-objective transportation problem using fuzzy logic’. Anupam Ojha , Kr , Manoranjan Mondal , Maiti . *Mathematical and Computer Modelling* 2011. 2011. 01 p. .
- [Orden ()] ‘Transshipment problem’. A Orden . *Management Science* 1956. (3) p. .
- [Yousria et al. ()] ‘Trust region algorithm for multi-objective transportation, assignment, and transshipment problems’. A Yousria , E Bothina , Z Hanadi . *Life Science Journal* 2012. 9 (3) p. .
- [Winter Simulation Conference] *Winter Simulation Conference*, p. .